Laser-assisted internal conversion

P. Kálmán
Department of Experimental Physics, Institute of Physics, Technical University of Budapest, H-1521 Budapest, Hungary
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A general formula for the laser-assisted internal conversion coefficient valid for all electronic shells and an approximation for near threshold laser-assisted internal conversion coefficient are deduced. The weak laser field and low frequency limits of the result are discussed. Numerical examples of the approximated, near threshold laser-assisted internal conversion coefficients are also given.

In two recent articles[1,2] (hereafter denoted papers I and II), the laser-assisted internal conversion process (LA-ICP) was investigated in a simple, nonrelativistic model, where the interaction Hamiltonian governing internal conversion was supposed to be of Coulomb type. In these papers the problem of having no solution for the electronic states which would take into account the joint action of the Coulomb potential and the intense radiation field was solved approximately. We took the laser-modified free-particle solution (Volkov solution) for the final electronic state and a hydrogen-type bound state for the initial one. However, there is a better approximation for solving this problem used in a variety of laser-assisted processes.[3] Using this better approximation as a final state, good results were obtained for multiphoton ionization of hydrogen by a strong field,[4] and very simple and clear expressions were deduced for laser-assisted x-ray absorption in more recent works.[5(a),5(b)] Therefore, we believe that the application of this state in the LA-ICP may also be successful.

This computation is mainly the same as those in papers I and II. Thus, we will not repeat the details. The initial and final electronic states are

$$\psi_i=\phi_{n\lambda\mu}(\mathbf{R})\exp\frac{ie\,\mathbf{A}(t)\cdot\mathbf{R}}{\hbar c}\exp(-iE_Bt/\hbar)\ , \tag{1}$$

$$\psi_f=\exp\left[-i\frac{1}{2m\hbar}\int^t[\hbar\mathbf{q}-e\,\mathbf{A}(t')/c]^2dt'\right]u^{(-)}(\mathbf{R})\ , \tag{2a}$$

with the validity condition for the latter

$$\hbar q \gg eE_0/\omega\ . \tag{2b}$$

Here $\phi_{n\lambda\mu}(\mathbf{R})$ is a hydrogen-type solution for quantum numbers n, λ, μ, and for energy eigenvalue E_B; $\mathbf{R}$ denotes the electronic coordinate; $\mathbf{q}$ is the wave vector of the outcoming electron; $\mathbf{A}(t)$ denotes the vector potential of the laser radiation; $u^{(-)}(\mathbf{R})$ is a Coulomb function; E_0 is the amplitude of the electric field strength of the laser; and ω is its angular frequency. We work in the radiation gauge.[6]

We notice that the exponential containing $\mathbf{A}(t)\cdot\mathbf{R}$ in Eq. (1) can be substituted for by unity as we investigate inner shells, and therefore the initial electronic state has a small characteristic volume where the wave function has considerable magnitude and where the $e\,\mathbf{A}\cdot\mathbf{R}/\hbar c \ll 1$ holds, yielding unity for the exponential. In this way the initial state is reduced to a simple hydrogenic-type bound state. Thus, the effect of the laser remained only in the time-dependent factor of Eq. (2a), with the result that the space-dependent and time-dependent parts of the S-matrix element governing the process can be separated. Furthermore, the space-dependent part is the same as in the laser-free case.

We investigate LA-ICP's where the laser has states of circular and linear polarization. We distinguish these two cases by writing circ and lin in the indexes. The corresponding vector potentials are

$$\mathbf{A}_{\rm circ}=(cE_0/\omega)(\hat{\mathbf{e}}_1\cos\omega t-\hat{\mathbf{e}}_2\sin\omega t) \tag{3}$$

and

$$\mathbf{A}_{\rm lin}=(cE_0/\omega)\hat{\mathbf{e}}_3\cos\omega t\ . \tag{4}$$

The unit vectors $\hat{\mathbf{e}}_1,\hat{\mathbf{e}}_2,\hat{\mathbf{e}}_3$ perpendicular to each other define the frame of reference.

After the use of the Jacobi-Anger formula[7] (circ case), the definition and properties of the generalized Bessel functions $J_N(b,d)$ (Ref. 8) (lin case), the required amount of algebra, and after dividing by the gamma transition rate, we obtain the laser-assisted internal conversion coefficient (LA-ICC) $\alpha_{nj\lambda}^{l,\rm las}$ of a transition of multipolarity l and for an electronic state of quantum numbers n,j,λ in the form

$$\alpha_{nj\lambda}^{l,\rm las}=\sum_{N>-r}\alpha_{nj\lambda}^{l,\rm free}(q_N)T(b_N)\ , \tag{5}$$

where $T(b_N)=[T_{\rm circ}(b_N)$ or $T_{\rm lin}(b_N)]$ with

$$T_{\rm circ}(b_N)=\frac{1}{2b_N}\int_0^{2b_N}J_{2|N|}(x)dx=\int_0^{\pi/2}J_N^2(b_N\sin\theta)\sin\theta\,d\theta \tag{6}$$

and

$$T_{\rm lin}(b_N)\int_0^1J_N^2(b_Nx,-d/4)dx\ . \tag{7}$$

Here $\alpha_{nj\lambda}^{l,\text{free}}(q_N)$ is the laser-free ICC at $q=q_N$; n, j, and λ are the principle, the total angular momentum, and the orbital momentum quantum numbers of the electronic state, respectively; l is the multipolarity of the gamma transition;

$$q_N=[(N+r)\hbar\omega/R]^{1/2}/a_B \ ;$$

a_B is the Bohr radius; $R=e^2/2a_B$; $r=r_0-d_{\text{pol}}$; $r_0=\Delta/\hbar\omega$; $\hbar\omega$ is the laser photon energy; $\Delta=E_B+\hbar\omega_{ab}$; $\hbar\omega_{ab}>0$ is the energy of the gamma transition; $J_{2|N|}(x)$ is a Bessel function of the first kind; N is an integer; $d_{\text{pol}}=(d_{\text{circ}}$ or $d_{\text{lin}})$ with

$$d_{\text{circ}}=d, \quad d_{\text{lin}}=d/2 \ ; \tag{8a}$$

$$d=e^2E_0^2/2m\hbar\omega^3; \quad b_N=eE_0q_N/m\omega^2=b_0(N+r)^{1/2} \ , \tag{8b}$$

with $b_0=1.07\times10^{-6}\times I^{1/2}(\hbar\omega)^{-3/2}$; I is the intensity of the laser in W/cm^2; and $\hbar\omega$ is in units of eV.

In the weak laser field region $(b_0\ll1)$ our general result [Eq. (5)] gives back the laser-free ICC.[9] In this case the $N=0$ term produces the main contribution to the sum because of Eq. (8), $d\ll1$, $b_N\ll1$, and thus in Eqs. (6) and (7) the Bessel and generalized Bessel functions can be approximated by their small argument expressions.[7,9] In consequence, in the $I\to0$ limit $T(b_0)=1$ and $T(b_N)=0$ $(N\neq0)$ for both states of polarization.

A very important case of LA-ICP is the low frequency $(\omega\to0)$ limit, which was investigated recently in a paper[10] where mainly the intense field-assisted beta decay was discussed. In Ref. 10 no field effect was found for beta decay, and it was also stated that the long wavelength limit of LA-ICC was field independent in each case investigated by the authors. Now we show that our general result [Eq. (5)] also has this feature in the $\omega\to0$ limit and in the case of a circularly polarized radiation field.[11]

We investigate the LA-ICP far from the threshold, i.e., for $\Delta\gg0$ where $r\sim r_0$ in consequence of Eq. (2b). Because of the large order characteristics of Bessel functions[7,12] we can find a large integer M fulfilling the $M\hbar\omega/R\ll\Delta/R$ condition in the $\omega\to0$ case while terms with $|N|>M$ can be neglected in Eq. (5). For $|N|<M$,

$$q_N=(N\hbar\omega/R+\Delta/R)^{1/2}/a_B\sim(\Delta/R)^{1/2}/a_B=q_0 \ , \tag{9a}$$

$$b_N=eE_0q_N/m\omega^2\sim eE_0q_0/m\omega^2=B \ , \tag{9b}$$

and Eq. (5) can be approximated as

$$\alpha_{nj\lambda}^{l,\text{las}}=\alpha_{nj\lambda}^{l,\text{free}}(q_0)\sum_{N=-M}^{N=M}T_N(B) \ , \tag{10}$$

with

$$T_N(B)=\int_0^{\pi/2}J_N^2(B\sin\theta)\sin\theta\,d\theta \ . \tag{11}$$

Using $J_N^2=J_{|N|}^2$ the sum in Eq. (10) can be written

$$\sum_{N=-M}^{N=M}T_N(B)=T_0(B)+2\sum_{N=1}^{\infty}T_N(B)-2\sum_{N=M+1}^{\infty}T_N(B) \ . \tag{12}$$

The last sum can be neglected because of the rapid decrease of J_N with increasing N. Using

$$J_0^2(z)+2\sum_{N=1}^{\infty}J_N^2(z)=1$$

and

$$\int_0^{\pi/2}\sin\theta\,d\theta=1 \ ,$$

we obtain

$$\alpha_{nj\lambda}^{l,\text{las}}(\omega\to0)=\alpha_{nj\lambda}^{l,\text{free}}(q_0) \ , \tag{13}$$

i.e., the low frequency limit of LA-ICC equals the laser-free ICC.

Now we deal with the near threshold LA-ICC. In papers I and II some special cases of near threshold LA-ICP were investigated where the presence of the laser field is essential, as the internal conversion process from the shell is energetically forbidden without laser. The reason for this is that the energy of the gamma transition $\hbar\omega_{ab}$ is less than the electronic binding energy $|E_B|$ on the shell $(\hbar\omega_{ab}<|E_B|)$. If the difference Δ is comparable to the laser photon energy $\hbar\omega$, then the result is that the LA-ICP can start on the shell after absorption of the demanded number of photons by energy conservation. Near the threshold, Eq. (5) can be approximated as

$$\alpha_{nj\lambda}^{l,\text{las}}=\alpha_{nj\lambda}^{l,\text{Th}}T \ , \tag{14}$$

where $\alpha_{nj\lambda}^{l,\text{Th}}$ is the threshold value of the laser-free ICC, $T=(T_{\text{circ}}$ or $T_{\text{lin}})$,

$$T_{\text{circ}}=\sum_{N>-r}T_{\text{circ}}(b_N) \ , \tag{15}$$

$$T_{\text{lin}}=\sum_{N>-r}T_{\text{lin}}(b_N) \ . \tag{16}$$

T_{circ} and T_{lin} depend on r_0 and b_0 only. Their curves were computed and published elsewhere for different values of parameters [$b_0<1$, imposed by Eq. (2b) near the threshold].[5] The weak field $(b_0\ll1)$ limit of Eqs. (15) and (16) can be obtained if we apply the small argument expressions of Bessel and generalized Bessel functions,[7,8]

$$T_{\text{circ}}=(b_K/2)^{2K}(2K!!)/[(K!)^2(2K+1)!!] \ , \tag{17}$$

$$T_{\text{lin}}=(b_K/2)^{2K}/[(K!)^2(2K+1)] \ , \tag{18}$$

where K is the minimum number of laser photons necessary to start the originally energetically forbidden process. As $b_K^2\sim I$ it means that $T_{\text{lin}}\sim I^K$ and $T_{\text{circ}}\sim I^K$, i.e., the process which is a K-order process from the point of view of laser photon absorption has the I^K laser intensity dependence, which fact is well known from multiphoton ionization phenomena.

Now we investigate numerically an isomeric transition, the ^{105m}Ag, $E3$ one, $\hbar\omega_{ab}=25.47$ keV, $t_{1/2}=7.23$ m with

$\Delta=-44$ eV for the K shell of $|E_B|=25.514$ keV,[13] which was also discussed in paper I and for which the above-mentioned approximation is valid. For ns states and in this model, the laser-free, threshold ICC can be written as

$$\alpha_{n\,1/2\,0}^{l,\text{th}}=D|i_l(n)|^2 \tag{19}$$

with

$$D=\frac{16\pi[(2l-1)!!]^2}{(l+1)}\left[\frac{\alpha mc^2}{\hbar\omega_{ab}}\right]^{2l+1}\left[\frac{Z_{\text{eff}}}{n}\right]^{2l}, \tag{20}$$

where α is the fine structure constant, mc^2 is the rest energy of the electron, $Z_{\text{eff}}/n=(|E_B|/R)^{1/2}$, and

$$i_l(n)=\int_0^\infty e^{-x}f_{n0}(x)J_{2l+1}[(8xn)^{1/2}]x^{1/2-l}dx\ , \tag{21}$$

where $f_{n0}(x)$ is characteristic for the shell investigated,[2] $f_{10}(x)=1$. These formulas can be obtained using the asymptotic form[14] of the Coulomb wave function[15] valid in the $q\to 0$ limit. Evaluating Eq. (21) by the aid of an integral formula[16] we can obtain $i_3(1)=2^{1/2}8e^{-2}{}_1F_1(6,8;2)/7!$. Here ${}_1F_1(a,b;c)$ denotes the confluent hypergeometric function. Thus, in our model $\alpha_{1\,1/2\,0}^{3,\text{th}}=152$ for the ^{105m}Ag transition. This value was compared to the result of a general, relativistic internal conversion theory,[17] where the threshold values of the laser-free ICC were given, and an agreement within 20% was found, which is fairly good taking into account the simplicity of our internal conversion model. In this way the error made by the use of this simplified model was estimated.

Furthermore, we deal with the case of circular polarization discussed also in papers I and II. In this case the other part of the approximated, near threshold, LA-ICC given by Eq. (14) is T_{circ} [Eq. (15)], which is plotted vs b_0 in Fig. 1 of Ref. 5(a). As Eq. (14) is factorized in two factors, which are the laser field independent, threshold ICC ($\alpha_{nj\lambda}^{l,\text{th}}$) and the only laser-dependent factor T, therefore characters of multiphoton ionization such as the above threshold (in laser intensity) ionization phenomenon appear because T shows all these features as was discussed in Ref. 5(a). Similarly, the hindering role of the ponderomotive potential, which was also observed directly in the above threshold ionization[18] and which was predicted in laser-assisted x-ray absorption,[5(b)] also occurs in LA-ICP as can be well seen in Figs. 1(a) and 1(b) of Ref. 5(b), where the $\log_{10}T_{\text{circ}}$ and $\log_{10}T_{\text{lin}}$ vs b_0 curves are given.

A nowadays available intense laser,[19] which has photon energy $\hbar\omega=6.42$ eV, produces $\alpha_{1\,1/2\,0}^{3,\text{las}}=7.4\times 10^{-14}$ for ^{105m}Ag with $I=2.3\times 10^{14}$ W/cm^2 [$b_0=1$, the corresponding T_{circ} value was obtained from Eq. (15)]. This LA-ICC value is unobservable. The rapid development of coherent ultraviolet[20] and soft-x-ray sources[21] perhaps will make it possible in the near future to produce coherent beams of required high intensity and of about 30–50 eV photon energy. In such a radiation field this LA-ICP is expected to increase strongly, as the absorption of fewer numbers (1 or 2) of photons enough to start the LA-ICP, e.g., a hypothetical laser of photon energy 30 eV and of intensity 2.4×10^{16} W/cm^2, can result in 0.16 for the LA-ICC of the above-mentioned process ($K=2$).

To summarize, for the case of an ordinary ICP, $\Delta\gg\hbar\omega$ and thus the low frequency limit [Eq. (13)] is valid, i.e., now change caused by the intense radiation field is obtained in accordance with Friar and Reiss.[10] The presence of the intense field modifies ICP only if the photon energy of the applied field is comparable to an energy characteristic of the process. Such processes are the ICP's near the threshold, where $|\Delta|\sim\hbar\omega$. It is found that the $\Delta<0$, near threshold cases have special interest; as for these transitions, the laser causes drastic changes because originally energetically forbidden processes can start in a laser of appropriate photon energy and intensity. In LA-ICP the x ray or gamma ray is originated from the nucleus of the same atom. The intense field can modify the in- and out-electronic states but it is supposed throughout the calculation that it leaves the nuclear states unchanged. Thus, the near threshold LA-ICP is similar to the near absorption edge, laser-assisted x-ray absorption process where the difference of the x-ray photon energy and the binding energy on the shell is comparable to the laser photon energy.[5]

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