Collisional width of giant resonances and interplay with Landau damping

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(Received 19 January 1989)

We present a semiclassical method to calculate the widths of giant resonances. We solve a mean-field kinetic equation (Vlasov equation) with collision terms treated within the relaxation time approximation to construct a damped strength distribution for collective motions. The relaxation time is evaluated from the time evolution of distortions in the nucleon momentum distribution using a test-particle approach. The importance of an energy dependent nucleon-nucleon cross section is stressed. Results are shown for isoscalar giant quadrupole and octupole motions. A quite important interplay between self-consistent (Landau) and collisional damping is revealed.

I. INTRODUCTION

The damping of giant resonances represents a long-standing problem, since we have several possible sources for the observed widths. In the random-phase approximation (RPA) (collisionless dynamics) we have the Landau damping ("fragmentation width") combined with nucleon emission in the continuum ("escape width"), but this is not enough to account for the experiments. Collisional damping must be included in developing a theory which goes beyond the RPA and implies a direct coupling of the $1p1h$ excitations to $2p2h$ modes. The resulting equations are very difficult to solve, and several approximate procedures have been developed in the last few years.[1,2,20–22]

In this paper we present a simple and transparent semiclassical approach to the problem, based on the solution of a linearized Vlasov equation with a collision term treated within the relaxation time approximation. The zero-temperature relaxation time is evaluated solving microscopically the evolution equation for the momentum distribution function, with energy-dependent nucleon-nucleon cross sections. We can satisfactorily reproduce the experimental behavior of the widths for isoscalar giant quadrupole and octupole resonances without free parameters. The main conclusions of this work are as follows:

(i) The relaxation time of giant resonances is strongly dependent on the multipolarity of the collective mode.

(ii) We have a quite important interplay between self-consistent (Landau damping) and collisional damping in building up the final observed width.

Theoretical uncertainties are coming from medium effects on the nucleon-nucleon cross sections, from the time variation of the Pauli blocking, i.e., the gain term in the collision integral, and from the use of nuclear matter parameters (ρ, p_F) also for finite nuclei. Here we limit our analysis to isoscalar motions, but the extension to isovector resonances can be easily done along the same line.

In Sec. II we evaluate the relaxation time starting from a quadrupole distorted distribution in momentum space. The response function is constructed in Sec. III, where we also compute the widths of strength distributions for quadrupole and octupole collective motions. Finally some conclusions are drawn in Sec. IV.

II. RELAXATION TIME FOR GIANT RESONANCES

At zero temperature the only possibility of having two-body collisions in the nuclear medium is related to distortions of the nucleon momentum distribution. Indeed if nucleons are sitting part of the time outside the Fermi sphere, they can collide, since the final momenta are not fully Pauli blocked.

This is the case for giant resonances (GR's) which can be very well described in phase space as scaling oscillations with momentum distortion of quadrupole type.[3–6]

For example the isoscalar giant quadrupole resonance corresponds to Fermi sphere deformations of the type

$$p_x \to p_x(1+2\alpha)\ ,$$
$$p_y \to p_y(1+2\alpha)\ , \tag{2.1}$$
$$p_z \to \frac{p_z}{(1+2\alpha)^2} \simeq p_z(1-4\alpha)\ ,$$

where $\alpha=\alpha(t)$ is the amplitude of the quadrupolar oscillation, with a periodic time behavior given by the giant resonance energy. Consistently the corresponding variation of the kinetic energy distribution univocally determines the energy of the collective motion.

The excitation energy per particle is given by the change of the mean kinetic energy in correspondence with the maximum value α_0 of the amplitude:

$$\frac{E^*_{\rm GR}}{A} = \epsilon_D = \frac{1}{2m}[\langle p_x^2\rangle + \langle p_y^2\rangle + \langle p_z^2\rangle]_{\rm distorted\ volume} - \tfrac{3}{5}\epsilon_F\ . \tag{2.2}$$

We will use an oblate ellipsoid geometry (see Fig. 1) for all the distortions in momentum space. Of course the results are the same for a prolate phase.

The lengths of the axis are $b=1+2\alpha$, $a=1/b^2$ (in p_F units), and the result of the integrations is

$$\frac{\epsilon_D}{\epsilon_F}=\frac{1}{5}(2b^2+a^2)-\frac{3}{5}\simeq\frac{48}{5}\alpha_0^2 . \tag{2.3}$$

This formula is used to evaluate the mean value of α^2 over an oscillation, $\overline{\alpha^2}=\frac{1}{2}\alpha_0^2$. Considering $\epsilon_D\simeq 65A^{-4/3}$ (for giant quadrupole) or $\epsilon_D\simeq 109A^{-4/3}$ (for giant octupole) we finally get

$$\overline{\alpha_{\rm GQR}}\simeq 0.3A^{-2/3}\sqrt{3/2} ,$$
$$\overline{\alpha_{\rm GQR}}\simeq 0.4A^{-2/3}\sqrt{3/2} , \tag{2.4}$$

where $\sqrt{3/2}$ comes from quantum corrections (zero-point motion). It is worth mentioning that the same values can be obtained in a classical harmonic oscillator picture along the collective variable assuming scaling estimations for the collective mass parameters.[5,7]

The amplitudes Eq. (2.4) will directly determine the relaxation time, as discussed in the following. Therefore the differences for various multipolarities will imply a clear multipole dependence of the relaxation time.

Fixing the initial distortion of the nucleon momentum distribution, we will follow the path to equilibrium solving microscopically the time evolution equation of the momentum distribution function

$$\frac{\partial g(\mathbf{p})}{\partial t}=I[g] , \tag{2.5}$$

where $I[g]$ is the collision integral, of Uehling-Uhlenbeck type.[8,9] We use the test-particle propagation method simulating the Uehling-Uhlenbeck integral by performing s-wave scatterings between these pseudoparticles.[10,11] At each time step, the procedure for a collision to take place is the following.[12,13]

(i) Two test particles (i,j) are randomly chosen in the occupied momentum space at time t.

(ii) A mean free path is defined as

$$\lambda=\frac{1}{\overline{\sigma}_{NN}\rho_{\rm test}/N}=\frac{1}{\overline{\sigma}_{NN}\rho} , \tag{2.6}$$

where $\overline{\sigma}_{NN}$ is the average nucleon nucleon cross section and $\rho=\rho_{\rm test}/N$ is the normal nuclear density. We will use $\rho=0.145\ {\rm fm}^{-3}$ consistent with $p_F=260$ MeV/c.

(iii) A collision probability is introduced as

$$\Pi_{ij}=\frac{\Delta t}{\Delta t_{\rm coll}} , \tag{2.7}$$

where Δt is the used time step interval and $\Delta t_{\rm coll}=\lambda/v_{ij}$ (v_{ij} is the relative velocity of the two test particles) is the mean time elapsed between two collisions. In order to take into account all possible collisions, one should choose $\Delta t\le\Delta t_{\rm coll}$. A minimum value of $\Delta t_{\rm coll}$, without Pauli effects, is $\Delta t_{\rm coll}\sim 1/\overline{\sigma}\rho 2v_F\sim 3$ fm/c.

(iv) A random number x, in the interval (0,1), is compared with Π_{ij}. If $x<\Pi_{ij}$, the collision can occur, and two final momenta p_{if}, p_{jf} are randomly chosen within the constraint of energy and momentum conservation. Conditions (iii) and (iv) ensure that test particles have on average at most a mean free path λ. The real mean free path is actually larger due to Pauli blocking. The collision is accepted only if the two final momenta p_{if}, p_{jf} are outside the initial ellipsoid. We remark that this frozen Pauli blocker eliminates any gain contribution leading to longer relaxation times.

In the case of constant $\overline{\sigma}_{NN}$ an exact result can be worked out for the relaxation time of a quadrupole deformation of the Fermi sphere.[8] The collisional procedure described here correctly reproduces this result.[13] As a measure of equilibration at each time step we compute the ratio

$$R=\frac{2\langle p_z^2\rangle}{\langle p_x^2\rangle+\langle p_y^2\rangle} . \tag{2.8}$$

Figure 2 shows the collisional relaxation times for a giant quadrupole resonance as a function of the mass number. The dashed line represents the results obtained with a fixed $\overline{\sigma}_{NN}=40$ mb, corresponding to an average free

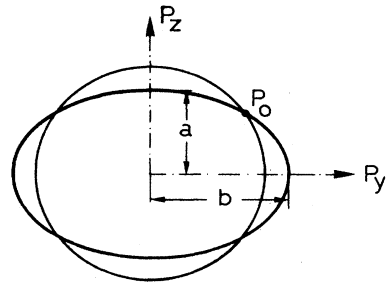

FIG. 1. Geometry of distortions in momentum space.

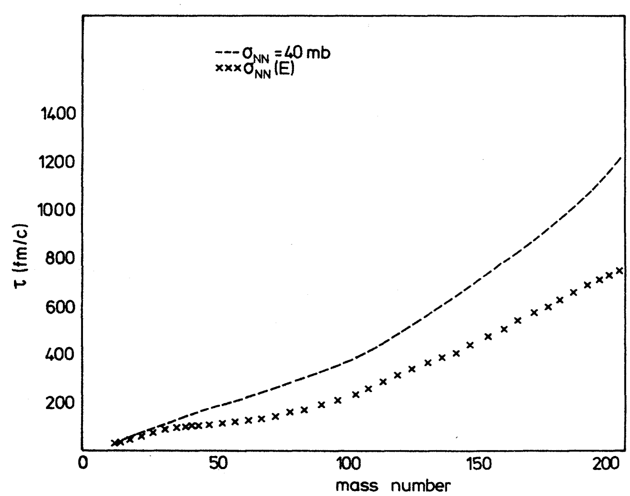

FIG. 2. Collisional relaxation times for a giant quadrupole mode as a function of the mass number. Dashed line: fixed $\overline{\sigma}_{NN}=40$ mb. ×'s: energy-dependent nucleon-nucleon cross section.

nucleon-nucleon cross section at about $2p_F$ relative momentum. With the symbol × we show the relaxation times obtained with an energy- and isospin-dependent cross section, parametrized according to the nucleon-nucleon experimental data.[11] Since we are taking into account the low-energy enhancement, we get shorter equilibration times. It is interesting to note that the change is less important for medium-light elements, i.e., in the presence of a larger amplitude of the deformation. In these cases the dominant collisions leading to equilibration are those between nucleons sitting on the extreme edges of the distribution, indeed at a relative momentum of the order of $2p_F$.

The quadrupole and octupole collisional widths ($\hbar/\tau$), evaluated with an energy-dependent cross section, are listed in Table I. We make the following remarks.

(i) We get a systematic mass dependence of the type $A^{-4/3}$ (for large mass numbers), for quadrupole as well as for octupole. However, surface effects are not explicitly accounted for, since we are using nuclear matter values for density and Fermi momentum.

(ii) The octupole collisional widths are systematically larger:

$$\frac{(\hbar/\tau)^{3-}}{(\hbar/\tau)^{2+}} \simeq 1.8 \ . \tag{2.9}$$

This clearly shows a strong multipolarity dependence of relaxation times, as expected from our procedure.

A very important point to stress is that the collisional widths are in general smaller, of about a factor 4, than the observed damping widths.[14] On the other hand, if we consider the effect of the Landau damping alone, i.e., the fragmentation of the strength in the RPA response function, we have also extremely small values for the corresponding widths of giant quadrupole and octupole resonances almost independent of the residual interaction used.[15–17] The conclusion is that we should expect to have a quite important interplay between long-range (one-body dissipation) and short-range (two-body collisions) correlations due to the self-consistency of the dynamics, which is finally able to reproduce the observed widths.

TABLE I. Collisional and final calculated spreading widths for giant quadrupole and octupole resonances for different nuclei.

Nucleus	Quadrupole (MeV) $\hbar/\tau_R$	Quadrupole (MeV) Γ	Octupole (MeV) $\hbar/\tau_R$	Octupole (MeV) Γ
^{40}Ca	1.83	7.05	3.79	10.55
^{110}Cd	0.664	1.96	1.103	4.2
^{144}Sm	0.425	1.30	0.715	3.1
^{208}Pb	0.259	1.12	0.463	1.5

III. RESPONSE FUNCTION WITH COLLISIONS

The interplay already discussed has been stressed in Ref. 17, where the Vlasov equation for small amplitude motions with a collision term of relaxation time structure has been solved with all the self-consistency effects, using separable residual interaction, as an extension of the method introduced in Ref. 18.

The correlated strength distributions given by the semiclassical approach have the structure (L multipolarity)

$$S_L(\omega) = \frac{S_L^0(\omega)}{[1-k_L\alpha_L^0(\omega)]^2 + k_L^2\pi^2 S_L^{0^2}(\omega)} \ , \tag{3.1}$$

where k_L is the coupling constant of the separable force (multipole-multipole type), and $S_L^0(\omega)$ is the uncorrelated strength function,

$$S_L^0(\omega) = -\frac{1}{\pi}\beta_L^0(\omega) \ , \tag{3.2}$$

with $\alpha_L^0(\omega)$ and $\beta_L^0(\omega)$ the real and imaginary parts of the free polarization propagator,

$$\Pi_L^0(\omega) = \frac{8\pi^2}{2L+1}\sum_{n,N}\int d\lambda\,\lambda \left| Y_{LN}\left[\frac{\pi}{2},\frac{\pi}{2}\right]\right|^2 T\frac{\tilde{\omega}_n(N)}{\omega-\tilde{\omega}_n(N)}|\tilde{Q}(n,N)|^2|_{E=E_F} \ , \tag{3.3}$$

with poles at

$$\tilde{\omega}_n(N) = n\frac{2\pi}{T} + N\frac{\Gamma}{T} - \frac{i}{\tau} \tag{3.4}$$

and residues

$$\tilde{Q}(n,N) = A + iB \ ,$$

$$A = \frac{2}{T}\int_{r_1}^{r_2} dr \frac{Q_L(r)}{v(r)} \cos[s_n(N,r)]\cosh\left[\frac{\tau(r)}{\tau}\right] \ , \tag{3.5}$$

$$B = \frac{2}{T}\int_{r_1}^{r_2} dr \frac{Q_L(r)}{v(r)} \sin[s_n(N,r)]\sinh\left[\frac{\tau(r)}{\tau}\right] \ . \tag{3.6}$$

Contributions are coming from single-particle orbits, in the mean field, with angular momentum λ and energy around the Fermi energy. Everything can be expressed in terms of radial and angular properties of these orbits, for a given multipole field $Q_L(r)$, where r_1, r_2 are the classical turning points, $v(r)$ is the radial velocity field, T is the period of the radial motion, Γ is the angular "period," and

$$s_n(N,r) = \left[n\frac{2\pi}{T} + N\frac{\Gamma}{T}\right]\tau(r) - N\psi(r) \ ,$$

with $\tau(r)$ and $\psi(r)$, respectively, time elapsed and angle spanned to reach the point r on the orbit (λ, E). n, N are integer numbers $-\infty < n < \infty$ and $-L < N < L$ $[(-1)^N = (-1)^L]$. Details can be found in Refs. 17 and 18.

In Fig. 3 we show 2^+ strength functions for ^{208}Pb,

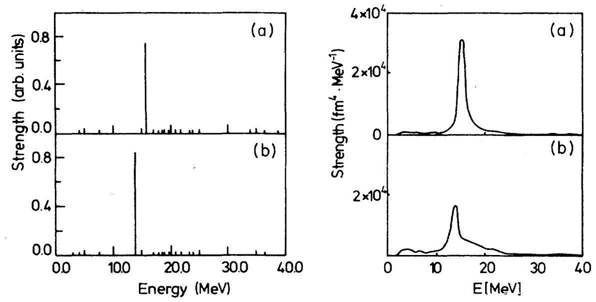

FIG. 3. Quadrupole strength distributions for ^{208}Pb, calculated without (a) and with (b) residual interaction. Left side: no collision term; right side: with collision terms. The different scales are discussed in the text.

without [part (a)] and with [part (b)] residual interactions, calculated with a collisional width $\Gamma = 0.259$ MeV (right) and without collisions (left). The residual interaction used is of quadrupole-quadrupole type with coupling constant fixed from a consistency condition for a Woods-Saxon mean field.[19] The strength units are different: On the right we plot the absolute value in $\mathrm{fm}^4\,\mathrm{MeV}^{-1}$, suitable to directly deduce the width; on the left we report the fraction of the EWSR. This is a typical behavior of the response function, valid also for the octupole. We can deduce the following.

(i) Without collisions the residual interaction produces a strong collectivity effect with almost no final fragmentation of the strength (no Landau damping).

(ii) If we go beyond RPA and include collisional widths on the uncorrelated eigenfrequencies, we still have a shift and change of the strength distribution, but the final width is much larger than the one related to the relaxation time. This means that we cannot simply sum up the two widths, Landau damping and collision, but there is a complicated interplay between the two sources of collective energy dissipation, due to the self-consistency of the dynamics: The Landau damping is enhancing the anticoherent effect of two-body collisions.

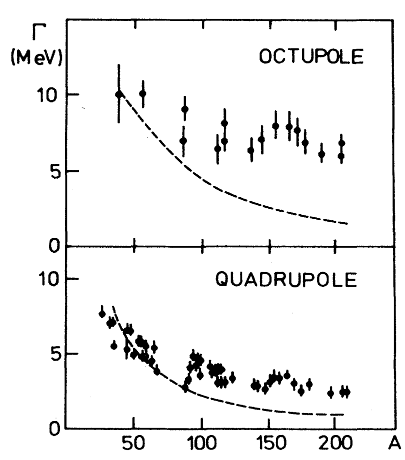

FIG. 4. Comparison of our results (dashed line) with the experimental giant quadrupole and giant octupole widths. Data are from Ref. 14.

In Table I we report also the final damping widths computed within our semiclassical framework for giant quadrupole and octupole resonances for different nuclei. We compare mainly with spherical nuclei in order to avoid effects coming from ground-state deformations. An overall comparison with experiments is shown in Fig. 4.

We are systematically below the observed values, particularly for heavy elements. However, we should mention two contributions which are missing here, more important for large mass numbers: (i) the escape widths are not included in our approach, since we consider only classical bound orbits in the mean field; (ii) the gain term, i.e., the rearrangement of the Pauli blocking at each time step, will shorten the relaxation time, more for smaller amplitude oscillations. Finally, a clear source of theoretical uncertainty comes from the nucleon-nucleon cross section. Here we use the free space values, although we should expect some energy-dependent medium effects.[28,29]

IV. CONCLUSIONS

The point we would like to stress is that using simple phase space arguments and a semiclassical self-consistent dynamics we are able to shed some light on the competition between one- and two-body dissipation in building up the damping widths of giant resonances.

This interplay has been already remarked upon in Ref. 20, starting from a quantum RPA approach with collision terms, where the contribution of Landau damping for quadrupole and octupole isoscalar modes is probably overestimated. The self-consistency enhancement of the elementary collisional width in the uncorrelated response function is also noticed in Refs. 21 and 22, where a constant phenomenological parameter ($\Gamma_{\mathrm{coll}} \simeq 1$ MeV) is used for the two-body width of unperturbed eigenfrequencies also at temperatures other than zero.

The observed width of giant resonances is obtained by adding the escape ($\Gamma\uparrow$) and spreading ($\Gamma\downarrow$), or statistical, widths. However, the latter is not just the sum of the Landau and collisional damping: We should consider a convolution of these elementary decays over the states which share the collective strength.

Applying our momentum space argument to the giant monopole resonance we expect to have a smaller spreading width than the quadrupole or octupole case. Indeed, in a breathing mode the momentum distribution remains spherical and therefore two-body collisions are strongly suppressed by Pauli blocking. The escape damping should now play a major role, and we expect, in general, for the ratio between escape and spreading widths

$$\left[\frac{\Gamma\uparrow}{\Gamma\uparrow+\Gamma\downarrow}\right]_{\mathrm{GMR}} > \left[\frac{\Gamma\uparrow}{\Gamma\uparrow+\Gamma\downarrow}\right]_{\mathrm{GQR}} . \tag{4.1}$$

Recent experiments on direct decays of giant monopole[23] and quadrupole[24] resonances have, however, shown that the escape probability is almost the same in the two cases, around 20%, actually somewhat smaller for the giant monopole in ^{208}Pb. Following our argument on the collisional part, these findings have a very interesting implication about the leading role played by the Landau damping for giant monopoles, in order to build up a large spreading width. Fully microscopic RPA calculations seem indeed to show a larger strength fragmentation in the monopole mode.[16,15]

The presence of a collision term also implies a redistribution of the strength of the collective motion (see Fig. 3). A similar effect has been observed in a quantum RPA calculation including particle-phonon couplings.[25] This could probably explain some anomalies in the giant dipole strength function recently seen at high temperatures,[26] where the relaxation time is strongly reduced. A nice feature of our method is that we can explicitly calculate the collisional widths for different nuclei and multipolarities and also we can simply introduce temperature effects.[8,17] Some results on isovector modes built on excited states are reported in Ref. 27 following the same approach but with a simplified geometrical method to compute the collisional relaxation time. For giant dipole resonances with increasing temperature one gets a moderate increase of the widths and a clear quenching of the strengths in the resonance region. A more microscopic analysis of temperature effects is in progress.

We conclude with a very general comment. The simple phase space scaling picture (Fig. 1) of giant resonances comes from a highly coherent combination of particle-hole excitations and leads, as shown here, to the enhancement of collisional damping. Coherence helps to destroy coherence.

ACKNOWLEDGMENTS

We are grateful to P. F. Bortignon, W. Nörenberg, P. Ring, and C. Toepffer for enlightening discussions. This work was supported in part by the Bundesministerium für Forschung und Technologie under Contract 06-ML177/8 and Deutscher Akademischer Austauschdient e.V. (DAAD), Federal Republic of Germany.

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