PHYSICAL REVIEW C VOLUME 39, NUMBER 6 JUNE 1989

Problems of describing spin-$\frac{3}{2}$ baryon resonances in the effective Lagrangian theory

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(Received 11 July 1988)

We investigate some important theoretical issues associated with the treatment of the spin-$\frac{3}{2}$ baryons in the effective Lagrangian theory. These are the form of the spin-$\frac{3}{2}$ particle propagator, off-shell parameters involving the spin-$\frac{3}{2}$ field; strategies of implementing gauge invariance, and unitarity. We comment on previous works by Peccei, Nath *et al.*, Williams, and Adelseck *et al.*, the last three works being in the context of recent revival of interest of baryon resonance structure in quantum chromodynamics. Our experience on the $\Delta(1232)$ resonance is invoked as a concrete example of dealing with these problems. Examples of some related problems in theories of massive vector and spin-$\frac{3}{2}$ particles, in pion decay and supersymmetry, respectively, are also discussed. These discussions should trigger new theoretical and experimental investigations in the study of baryon resonances excited from free hadrons and in complex nuclei.

I. INTRODUCTION

Thanks to the development of quantum chromodynamics (QCD), the color gauge theory for strong interaction, there has been a renewed interest in the structure of hadrons.[1] Building of new high-duty electron accelerators, such as the continuous electron beam acclerator facility (CEBAF), sets the stage for extensive electroweak studies of the excited baryons, many of which have spins greater than or equal to $\frac{3}{2}$, both in free state and in nuclei. Photo and electroproduction of these baryons require theoretical investigation of gauge-invariant γBB^* amplitudes (γ, real or virtual photon, B,B^*, nucleon and excited baryon, respectively), propagation and decay of the resonances B^*, ideally without violating unitarity. This last requirement is particularly difficult to implement for resonances beyond $\Delta(1232)$, due to the opening of many channels, to which B^* can decay by strong interaction. Another troublesome aspect is the ambiguity related to the background contribution.[2] Considerable progress has been made in the last decade in handling these difficulties for $\Delta(1232)$, but much remains to be done for other resonances.

The purpose of this paper is to examine some of these important theoretical issues for the spin-$\frac{3}{2}$ baryons, of which $\Delta(1232)$ is the most familiar example in the intermediate energy nuclear physics. While there is a long history[3] of this subject, starting with the classic papers by Dirac, Fierz and Pauli, Rarita, and Schwinger, among others, problems with massive spin-$\frac{3}{2}$ fields keep coming back. Among the most recent discussions that we shall address here, are those by Williams[4] and Adelseck *et al.*[5] on the propagator for these baryons, the handling of the gauge invariance and baryon decay width, by Adelseck *et al.*,[5] and constraints on the interaction Lagrangian for massive spin-$\frac{3}{2}$ fields, by Nath *et al.*[6] We shall also touch upon the recent discussion of the problem of interacting massive spin-$\frac{3}{2}$ fields, in supersymmetric theories.[7] Applications of our present discussions in the context of excitations of baryon resonances in complex nuclei are beyond the scope of this paper, and will be taken up elsewhere. An outline of the rest of this paper is given below.

In Sec. II we review the local relativistic wave equation and Lagrangian for a free massive spin-$\frac{3}{2}$ particle. We present the defining equation for the propagator and solve for the propagator in momentum space. Spin projection operators are introduced which ease the calculations in the remainder of the paper.

The propagator derived in Sec. II agrees with most previous works,[8,13] but not with the recent works of Williams[4] and Adelseck *et al.*[5] In Sec. III we examine the propagators used by Williams and Adelseck *et al.* It is shown that these proposed propagators do not have inverses, and corresponding wave equations for the spin-$\frac{3}{2}$ field cannot be defined. A brief discussion of the nonlocal wave equation is made and we show that the "nonlocal" propagator is also in disagreement with the propagators presented by Williams and Adelseck *et al.*

Since the effective Lagrangian approach remains very useful in medium energy physics, we consider, in Sec. IV, the ambiguities in constructing an effective Lagrangian, examining the example of photoproduction via the $\Delta(1232,\frac{3}{2}^+)$ resonance. At each vertex involving the delta, there is freedom related to the off-shell behavior of the delta. We review the theoretical attempts to fix the off-shell behavior of the delta and make comparisons with the data. As an additional example, we discuss how the off-shell behavior of the spin-$\frac{3}{2}$ particles is determined in the supersymmetric theories.[7] This is only a formal comparison, since supersymmetry has nothing to do directly with the baryon properties.

In Secs. V and VI we return to extant problems in the extraction of the γBB^* amplitudes. The first problem is related to a nongauge invariant vertex often found in literature.[4,5,9] The lack of gauge invariance may easily be overcome by coupling to the electromagnetic field tensor. The final problem we discuss is unitarity and the inconsistent[5] insertion of a width in the propagator at the level of the Lorentz invariant matrix element. Although the general problem of n-body, m-channel unitarity has been discussed to some extent in the literature,[10-12] our goal here is to illustrate the difficulties associated with unitarizing the amplitude obtained from evaluating the effective Lagrangian in the tree approximation. We finally summarize our conclusions.

In Appendix A, we demonstrate how an off-shell massive vector field can generate a spin-zero contribution in a transition amplitude. This demonstration is given in the context of pion decay in the intermediate vector boson theory. In Appendix B, we look at the πN scattering amplitude in the s-channel delta exchange approximation, and show that there is *always* a spin-$\frac{1}{2}$ piece of the amplitude that cannot be eliminated, disproving the anticipation of Peccei[13] that a pure spin-$\frac{3}{2}$ projection of such an amplitude might be possible with a suitable choice of off-shell parameters for the delta. These two appendices bring out an interesting common property of the massive vector and spinor fields for $J=1$, $\frac{3}{2}$, J being the angular momentum of the field, viz, such fields, "off-shell," would generate contributions involving the (J-1) sector of this field, in effective amplitudes.

II. FREE LAGRANGIAN AND PROPAGATOR FOR A MASSIVE SPIN-$\frac{3}{2}$ PARTICLE

To start, we recall the free Lagrangian[8] for the massive spin-$\frac{3}{2}$ field

$$L=\bar{\psi}^{\alpha}\Lambda_{\alpha\beta}\psi^{\beta}\ , \tag{1}$$

with

$$\begin{aligned}\Lambda_{\alpha\beta}=-[(-i\partial_\mu\gamma^\mu+M)g_{\alpha\beta}-iA(\gamma_\alpha\partial_\beta+\gamma_\beta\partial_\alpha)\\ -\frac{i}{2}(3A^2+2A+1)\gamma_\alpha\partial^\mu\gamma_\mu\gamma_\beta\\ -M[(3A^2+3A+1)\gamma_\alpha\gamma_\beta]\ ,\end{aligned} \tag{2}$$

where M is the mass of the spin-$\frac{3}{2}$ baryon and A is an arbitrary parameter subject to the restriction $A\neq-\frac{1}{2}$. In Eqs. (1) and (2), α, β, and μ are Lorentz indices. The notation and conventions are those of Bjorken and Drell.[31] Physical properties of the free field, such as energy-momentum tensor, do not depend on the parameter A, chosen here to be real. This is due to the fact that the free Lagrangian (1) is invariant under the "point transformation"[8]

$$\begin{aligned}&\psi^\mu\to\psi^\mu+a\gamma^\mu\gamma^\nu\psi_\nu\ ,\\ &A\to(A-2a)/(1+4a)\ ,\end{aligned} \tag{3}$$

where $a\neq-\frac{1}{4}$, but is otherwise arbitrary. We can derive, via the Euler-Lagrange equations starting with (1), the local wave equation for the spin-$\frac{3}{2}$ particle

$$(i\gamma_\mu\partial^\mu-M)\psi^\nu=0\ , \tag{4}$$

and the constraint equations

$$\gamma_\mu\psi^\mu=0\ , \tag{5a}$$

$$\partial_\mu\psi^\mu=0\ . \tag{5b}$$

Note that ψ^ν is a vector spinor[8] with a suppressed spinor index; thus Eq. (5a) implies a sum over the spinor index β

$$(\gamma_\mu)_{\alpha\beta}\psi^\mu_\beta=0\ . \tag{5c}$$

A "vector spinor" means it transforms, under Lorentz transformation, like a product of a four vector and a Dirac spinor. Eight constraint Eqs. (5a) and (5b) reduce the number of free components of ψ^ν_α to eight.

The propagator for the massive spin-$\frac{3}{2}$ particle satisfies the following equation in momentum space

$$\Lambda_{\alpha\beta}(p)G^\beta_\delta(p)=g_{\alpha\delta}\ , \tag{6}$$

where $g_{\alpha\delta}$ is the metric tensor, and

$$\begin{aligned}\Lambda_{\alpha\beta}=-[(-\gamma\cdot p+M)g_{\alpha\beta}-A(\gamma_\alpha p_\beta+\gamma_\beta p_\alpha)\\ -\tfrac{1}{2}(3A^2+2A+1)\gamma_\alpha\gamma\cdot p\gamma_\beta\\ -M(3A^2+3A+1)\gamma_\alpha\gamma_\beta]\ .\end{aligned} \tag{7}$$

Solving for G, we get

$$\begin{aligned}G_{\alpha\beta}(p)=\frac{\gamma\cdot p+M}{p^2-M^2}\left[g_{\alpha\beta}-\tfrac{1}{3}\gamma_\alpha\gamma_\beta-\frac{1}{3M}(\gamma_\alpha p_\beta-\gamma_\beta p_\alpha)\right.\\ \left.-\frac{2}{3M^2}p_\alpha p_\beta\right]-\frac{1}{3M^2}\frac{A+1}{2A+1}\\ \times\left[\gamma_\alpha p_\beta+\frac{A}{(2A+1)}\gamma_\alpha p_\beta\right.\\ \left.+\left[\frac{1}{2}\frac{A+1}{2A+1}\gamma\cdot p-\frac{AM}{2A+1}\right]\gamma_\alpha\gamma_\beta\right]\ .\end{aligned} \tag{8}$$

Since the physical properties of the free field are independent of the parameter A, we can make a particular choice in (8) by taking $A=-1$. This yields the expression often found in literature[7,8,13] for the spin-$\frac{3}{2}$ propagator

$$P^{\mu\nu}=\frac{M+\gamma\cdot p}{p^2-M^2}\left[g^{\mu\nu}-\tfrac{1}{3}\gamma^\mu\gamma^\nu-\frac{2p^\mu p^\nu}{3M^2}+\frac{p^\mu\gamma^\nu-p^\nu\gamma^\mu}{3M}\right]\ . \tag{9}$$

It is useful to introduce the spin projection operators[7]

$$\begin{aligned}&(P^{3/2})_{\mu\nu}=g_{\mu\nu}-\tfrac{1}{3}\gamma_\mu\gamma_\nu-\frac{1}{3p^2}(\gamma\cdot p\gamma_\mu p_\nu+p_\mu\gamma_\nu\gamma\cdot p)\ ,\\ &(P^{1/2}_{11})_{\mu\nu}=\tfrac{1}{3}\gamma_\mu\gamma_\nu-\frac{p_\mu p_\nu}{p^2}+\frac{1}{3p^2}(\gamma\cdot p\gamma_\mu p_\nu+p_\mu\gamma_\nu\gamma\cdot p)\ ,\\ &(P^{1/2}_{22})_{\mu\nu}=\frac{p_\mu p_\nu}{p^2}\ ,\\ &(P^{1/2}_{12})_{\mu\nu}=\frac{1}{\sqrt{3}p^2}(p_\mu p_\nu-\gamma\cdot pp_\nu\gamma_\mu)\ ,\\ &(P^{1/2}_{21})_{\mu\nu}=\frac{1}{\sqrt{3}p^2}(\gamma\cdot pp_\mu\gamma_\nu-p_\mu p_\nu)\ .\end{aligned} \tag{10}$$

These satisfy the orthonormality conditions

$$(P_{ij}^{I})_{\mu\nu}(P_{kl}^{J})^{\nu\delta}=\delta^{IJ}\delta_{jk}(P_{il}^{J})_{\mu}^{\delta}\ , \tag{11}$$

and the sum rule for the projection operators

$$P_{\mu\nu}^{3/2}+(P_{11}^{1/2})_{\mu\nu}+(P_{22}^{1/2})_{\mu\nu}=g_{\mu\nu}\ . \tag{12}$$

Following properties are also useful

$$\gamma\cdot pP_{ij}^{1/2}=\pm P_{ij}^{1/2}\gamma\cdot p,\quad +\text{for } i=j,\ -\text{for } i\neq j\ . \tag{12a}$$

$$\gamma\cdot pP^{3/2}=P^{3/2}\gamma\cdot p\ . \tag{12b}$$

In the next section, we examine arguments of Williams[4] and Adelseck *et al.*[5] for *their* versions of spin-$\frac{3}{2}$ propagators.

III. PROBLEMS WITH THE SPIN-$\frac{3}{2}$ PROPAGATORS PROPOSED BY WILLIAMS AND ADELSECK *et al.*

Hereafter we shall be dealing with an *interacting* spin-$\frac{3}{2}$ particle. If the interaction Lagrangian obeys the same point transformation (3) as the free Lagrangian, discussed earlier, the parameter A is again arbitrary,[14] and can be chosen to be -1. Thus, the spin-$\frac{3}{2}$ propagator remains the same as in Eq. (9).

We first examine Williams' suggestion[4] that Eq. (9) is not the correct propagator, and should be replaced by

$$P'^{\mu\nu}=\frac{M+\gamma\cdot p}{p^2-M^2}\times\left[g^{\mu\nu}-\tfrac{1}{3}\gamma^{\mu}\gamma^{\nu}-\frac{1}{3p^2}(\gamma\cdot p\gamma^{\mu}p^{\nu}+p^{\mu}\gamma^{\nu}\gamma\cdot p)\right]. \tag{13}$$

We have already shown that Eq. (9) *is* the correct spin-$\frac{3}{2}$ propagator. To see the problem with (13), we write it in the form

$$P'_{\mu\nu}=\frac{M+\gamma\cdot p}{p^2-M^2}P_{\mu\nu}^{3/2}\ . \tag{14}$$

As a propagator, it must satisfy the equation

$$\Lambda'_{\mu\nu}P'^{\nu}_{\lambda}=g_{\mu\lambda}\ . \tag{14'}$$

By Eq. (12), we require

$$\Lambda'_{\mu\nu}P'^{\nu}_{\lambda}=P_{\mu\lambda}^{3/2}+(P_{11}^{1/2})_{\mu\lambda}+(P_{22}^{1/2})_{\mu\lambda}\ . \tag{15}$$

Since $\Lambda'_{\mu\nu}$ can be expanded in terms of the basis of projection operators, and $P'_{\mu\lambda}$ involves only $P_{\mu\nu}^{3/2}$, the orthogonality conditions (11) along with relations (12′) guarantee that the left-hand side of (15) can *never* generate $(P_{11}^{1/2})_{\mu\lambda}$ and $(P_{22}^{1/2})_{\mu\lambda}$ terms on the right-hand side of (15). Thus, $P'_{\mu\nu}$ *does not have an inverse* and the operator Λ' satisfying Eq. (15) does not exist. Hence Williams' operator (13) *cannot* be the correct spin-$\frac{3}{2}$ propagator.

In the literature, there is a discussion of the nonlocal equation[7]

$$(P_{\mu\nu}^{3/2}\gamma\cdot p-Mg_{\mu\nu})\psi^{\nu}=0\ . \tag{15'a}$$

which also contains spin-$\frac{1}{2}$ contributions displayed explicitly in the following form

$$[P^{3/2}(\gamma\cdot p-M)-M(P_{11}^{1/2}+P_{22}^{1/2})]_{\mu\nu}\psi^{\nu}=0\ . \tag{15'b}$$

This defines a *nonlocal* propagator for spin-$\frac{3}{2}$ particle

$$P_{\text{nonlocal}}^{\mu\nu}=\left[\frac{\gamma\cdot p+M}{p^2-M^2}P^{3/2}-\frac{1}{M}(P_{11}^{1/2}+P_{22}^{1/2})\right]^{\mu\nu}. \tag{15'c}$$

Equation (15′a) has been rejected[7,8] in the past as a field equation because of the $1/p^2$ singularity in the $P_{\mu\nu}^{3/2}$. At the tree level, $p^2\neq 0$ in the s channel, and this does not cause any problem, and one might want to use the nonlocal equation (15′a) in the spirit of Williams. However, the u-channel Δ-exchange contribution could have kinematics for which $p^2=u=0$ (u, the appropriate Mandelstam variable), and (15′c) could be problematic. We have not used this in our phenomenological applications. The point we want to make here is that the correct propagator for the nonlocal equation (15′a) is (15′c) and *not* (14).

We now turn to the arguments of Adelseck *et al.*,[5] who in a recent paper on kaon photoproduction off nucleons in nuclei, have constructed γNN^* vertices and N^* propagators, where N^* represents the $\frac{3}{2}^-,D_{13}$ baryon at 1700 MeV and the $\frac{3}{2}^+,P_{13}$ baryon at 1720 MeV. We shall return to their vertex construction in the next section. Their propagator for the spin-$\frac{3}{2}$ particle of mass M, width Γ, and four-momentum p, is

$$P''_{\mu\nu}=\frac{\sqrt{s}+\gamma\cdot p}{3(p^2-M^2+iM\Gamma)}\times\left[3g_{\mu\nu}-\gamma_{\mu}\gamma_{\nu}-\frac{2}{s}p_{\mu}p_{\nu}-\frac{1}{\sqrt{s}}(\gamma_{\mu}p_{\nu}-\gamma_{\nu}p_{\mu})\right]. \tag{16}$$

Note that this expression would be identical to that in Eq. (13) in the limit of $\Gamma\to 0$, if we replaced $\sqrt{s}$ by M. Let us examine if this modified expression is acceptable as a spin-$\frac{3}{2}$ propagator in the limit $\Gamma\to 0$. Using our definitions of the projection operators in Eq. (10), we have

$$P''_{\mu\nu}(\Gamma\to 0)=\frac{\gamma\cdot p+\sqrt{s}}{(p^2-M^2)}\left[P_{\mu\nu}^{3/2}+\frac{1}{\sqrt{3s}}(\gamma\cdot p-\sqrt{s}\,)\times[(P_{12}^{1/2})_{\mu\nu}+(P_{21}^{1/2})_{\mu\nu}]\right], \tag{16'}$$

Using the identity

$$(\gamma\cdot p+\sqrt{s}\,)(\gamma\cdot p-\sqrt{s}\,)=0\ , \tag{17}$$

we get

$$P''_{\mu\nu}(\Gamma\to 0)=\frac{\gamma\cdot p+\sqrt{s}}{p^2-M^2}P_{\mu\nu}^{3/2}\ , \tag{18}$$

Thus, the propagator equation (7) written as

$$\Lambda_{\alpha\beta}P''^{\beta}_{\delta}=g_{\alpha\delta} \tag{18'}$$

cannot be satisfied, since the left-hand side of (18′) does not contain $P_{11}^{1/2}$ and $P_{22}^{1/2}$ terms, while the right-hand side does. Thus, $P''_{\mu\nu}$ of Adelseck *et al.*[5] cannot be a propagator for massive spin-$\frac{3}{2}$ particles.

IV. EFFECTIVE LAGRANGIAN FOR AN INTERACTING SPIN-$\frac{3}{2}$ FIELD: OFF-SHELL FREEDOM

A. Off-shell freedom of the masssive spin-$\frac{3}{2}$ field

This subject has a long history and we recapitulate here the essence of the results for strong and electromagnetic three-point functions. Taking the example of the delta resonance, the effective Lagrangian is given by[6,15]

$$L_\Delta = L_{\pi N\Delta} + L^1_{\gamma N\Delta} + L^2_{\gamma N\Delta} \ . \tag{19}$$

Here $L_{\pi N\Delta}$ is the strong Lagrangian

$$L_{\pi N\Delta} = \frac{g_{\pi N\Delta}}{m_\pi} \bar{\Delta}^{\nu} \tau \theta_{\nu\mu}(Z) N \partial^\mu \pi + \text{H.c.} \ , \tag{20}$$

and $L^1_{\gamma N\Delta}$, $L^2_{\gamma N\Delta}$ are the electromagnetic interactions

$$L^1_{\gamma N\Delta} = \frac{ieg_1}{2m} \bar{\Delta}^{\mu} \theta_{\mu\lambda}(Y) \gamma_\nu \gamma_5 \tau_3 F^{\nu\lambda} + \text{H.c.} \ , \tag{21}$$

$$L^2_{\gamma N\Delta} = -\frac{eg_2}{4m^2} \bar{\Delta}^{\mu} \theta_{\mu\nu}(X) \gamma_5 \tau_3 (\partial_\lambda N) F^{\nu\lambda} + \text{H.c.} \ , \tag{22}$$

where N and Δ_μ are nucleon spinor and delta vector sponsor respectively, τ's are the $\frac{1}{2} \to \frac{3}{2}$ isospin transition operators, and m_π and m are pion and nucleon masses. $\theta_{\mu\nu}(X)$ are given by

$$\theta_{\mu\nu}(X) = g_{\mu\nu} + [\tfrac{1}{2}(1+4X)A + X]\gamma_\mu \gamma_\nu \ . \tag{23}$$

$F^{\mu\nu}$ is the electromagnetic field tensor. The Lagrangian L_Δ has the same symmetry as the free one, under the point transformation (3), thus the parameter A drops out of the observables. $\theta_{\mu\nu}$ is the *most general tensor*[6] preserving this property of the free Lagrangian. The form of the strong Lagrangian (20), in which the derivative of the pion field appears, ensures the chiral invariance.[13,16] The interaction Lagrangian of the above form allows the freedom of choice for the value of A, exploited in the spin-$\frac{3}{2}$ propagator, given earlier [Eq. (8)].

B. Choices for "off-shell" parameters for delta

We shall now make a representative sampling of various choices of parameters available in the literature and compare rationales for such choices. We shall also give the values obtained by us from our fits to the available multipoles in the delta region.

Peccei[13] first made a theoretical effort to get a fix on the off-shell Δ parameters. He argued that $\theta_{\mu\nu}$ in Eq. (23) should be subject to the condition

$$\gamma_\mu \theta^{\mu\nu} = 0 \ . \tag{24}$$

He believed that the condition (24) was necessary for the independence of the hadronic transition amplitude from the parameter A in Eqs. (2) and (7). This constraint implies

$$\gamma^\nu + \lambda \gamma_\mu \gamma^\mu \gamma^\nu = 0 \ ,$$

where $\lambda = \frac{1}{2}(1+4Z)A + Z$, giving, with $A=-1$,

$$Z = -\tfrac{1}{4} \ . \tag{25}$$

Applying the same argument for the electromagnetic vertex, Peccei got for the g_1 coupling, with $A=-1$,

$$Y = -\tfrac{1}{4} \ . \tag{25'}$$

Nath *et al.*[6,8] have argued that Peccei's condition (24) is unnecessarily restrictive, and Eq. (23) provides the most general form of $\theta_{\mu\nu}$, consistent with the point transformation property. We cannot substantiate Peccei's claim[13] that (24) would result in a pure spin-$\frac{3}{2}$ coupling (See Appendix B). Thus, the s-channel Δ contribution to the πN elastic scattering always has a spin-$\frac{1}{2}$ contribution from the delta propagator for arbitrary value of Z.

Nath *et al.*[6] have concluded on the basis of field theoretic arguments originally formulated by Fierz and Pauli[3] that the second gauge coupling term [Eq. (22)] should be absent, and a special choice of values for the parameters Z and Y is required. Their choice is

$$Z = \tfrac{1}{2}, \quad Y = 0, \quad g_2 = 0 \ . \tag{26}$$

One curious consequence of (26), arising from the absence of the gauge coupling g_2, is that the dynamical freedom of two independent electromagnetic multipoles at the $\gamma N\Delta$ vertex is lost. Thus, the electric quadrupole ($E2$) to the magnetic dipole ($M1$) amplitude ratio (EMR) for the delta radiative decay is fixed kinematically, and is given by

$$\text{EMR} = -(M_\Delta - m)/(3M_\Delta + m) \simeq -6\% \ , \tag{27}$$

for the Nath *et al.*[6] choice of the absence of the g_2 coupling, with M_Δ, the Δ mass. Similar results would follow for other γNN^* ($J=\frac{3}{2}$) vertices, where the appropriate multipole amplitudes would be likewise constrained. Thus, for the $N \to N^*$ (1520, $\frac{3}{2}^-$, $T=\frac{1}{2}$) transition, $M2/E1 = (M_{N^*} - m)/(3M_{N^*} + m) \simeq 11\%$, M_{N^*} being the N^* mass.

There is an obvious objection to accepting the results like (27) as correct. Hadrons are composite particles, and the ratio of the $\gamma N\Delta$ transition amplitudes should not be fixed *a priori* by the vector-spin properties of delta alone. These should be determined by the interactions of the hadronic constituents. Thus, for the $\gamma N\Delta$ vertex, the SU_6 quark model, in its symmetry limit,[17] gives a vanishing $E2$ transition amplitude and the EMR. Realistic quark shell model wave functions[18] yield the EMR to be -0.4%, sensitive to the color magnetic interaction[19] between quarks as approximated by the one-gluon exchange model. In the soliton models,[20] the EMR tends to be -2.9%, while chiral bag models[21] yield a value of about -1% for the $N \leftrightarrow \Delta(1232)$ transition. Thus, the universality of the EMR value, independent of the quark-gluon dynamics of the hadron structure, implied by the arguments of Nath *et al.*,[6] is, on the face of it, unacceptable. We, therefore, conclude that the parameter choices of Nath *et al.*, given by (26), are invalid characterizations of the $\gamma N\Delta$ transition amplitude. This conclusion is strengthened by the relatively poor quality of fit[22] of the photoproduction multipoles using the parameter choices given in (26). Current analyses[22] seem to rule out EMR $\simeq -6\%$ and thus $g_2 \neq 0$ for the $\gamma N\Delta$ transition. The $M2/E1$ ratio for $\gamma N \leftrightarrow D_{13}(1535)$ is less well determined than the EMR, but available analyses[23] give $31\% \lesssim M2/E1 \lesssim 56\%$, inconsistent with $g_2=0$ for this transition. Finally, if $g_2=0$, then for all transitions $\gamma N \to \frac{3}{2}^+$ (or $\frac{3}{2}^-$) the ratio of $A_{1/2}$ to $A_{3/2}$ would be very nearly the same, which clearly[23] is not the case.

In work reported elsewhere, Davidson *et al.*[22] have taken the approach formulated by Olsson and collaborators[15,24] to its logical conclusion. In this approach, the effective Lagrangian (19) is assumed correct. The amplitudes are obtained at the tree level and unitarized by using the Watson theorem.[25] The strong and electromagnetic coupling parameters $g_{\pi N\Delta}$, g_1, and g_2, X, Y, Z, are assumed to be energy independent, and they are fitted to the multipole data. Including all data sets, the following ranges of parameters are obtained

$$-0.8 \le Z \le 0.28, \quad -0.75 \le Y \le 1.67, \quad -3.0 \le X \le 3.8 \ ,$$
$$3.94 \le g_1 \le 5.30, \quad 4.49 \le g_2 \le 9.24 \ . \tag{28}$$

The range of g_2 is clearly inconsistent with vanishing of g_2 predicted by Nath *et al.* (27). The EMR is consistent with realistic hadron models.

C. Choice for "off-shell" parameters for the massive spin-$\frac{3}{2}$ particles in supersymmetry

To extend the scope of our above discussion of the off-shell parameter of the massive spin-$\frac{3}{2}$ field, we shall discuss one example from another exciting area of the field theory; supersymmetry and supergravity.[7] The theory of supergravity describes the interaction of the massless graviton (of spin-2) with a massless spin-$\frac{3}{2}$ fermion, called gravitino, the gauge field arising from supersymmetry transformations. When the supersymmetry is coupled to supergravity, the massless spin-$\frac{3}{2}$ gravitino and the massless spin-$\frac{1}{2}$ goldstino combine to give the massive spin-$\frac{3}{2}$ gravitino. The supersymmetric theory provides us with one nice particular example of the off-shell parameters for the massive spin-$\frac{3}{2}$ sector. Consider the supersymmetric scalar multiplet which contains a scalar η, a pseudoscalar ϕ, and a Majorana spin-$\frac{1}{2}$ field χ. Then the supersymmetric interaction terms involving the gravitino-Majorana field—scalar and pseudoscalar are[26]

$$L = g\bar{\psi}_\mu(\gamma\cdot\partial\eta + i\gamma_5\gamma\cdot\partial\phi)\gamma^\mu\chi \ , \tag{29}$$

where g is the suitable coupling constant. To illustrate the off-shell parameters of the spin-$\frac{3}{2}$ sector, let us rewrite the pseudoscalar interaction term explicitly

$$L_\phi = 2ig\bar{\psi}^\mu\gamma_5(g_{\mu\nu} - \tfrac{1}{2}\gamma_\mu\gamma_\nu)\chi\partial^\nu\phi \ . \tag{30}$$

This is formally analogous to our $\pi N\Delta$ Lagrangain (20). Comparing with (23),

$$\tfrac{1}{2}(1+4Z)A + Z = -\tfrac{1}{2} \ . \tag{31}$$

Choosing $A=-1$ as before, we get

$$Z = 0 \ . \tag{31$'$}$$

For the electromagnetic coupling, the supersymmetric Lagrangian is

$$L = f\bar{\psi}_\mu\gamma_\alpha\gamma_\beta\gamma^\mu\chi F^{\alpha\beta} \ , \tag{32}$$

where $F^{\alpha\beta}$ is the electromagnetic field tensor, f is the appropriate coupling constant. It can be written as

$$L = 4f\bar{\psi}^\mu(g_{\mu\beta} - \tfrac{1}{4}\gamma_\mu\gamma_\eta)\gamma_\alpha\chi F^{\alpha\beta} \ . \tag{32$'$}$$

We thus have, in a formal comparison to the $\gamma N\Delta$ case,

$$\tfrac{1}{2}(1+4Y)A + Y = -\tfrac{1}{4} \ .$$

With $A=-1$, we get

$$Y = -\tfrac{1}{4} \ . \tag{33}$$

This, together with (32$'$), should be contrasted with (21) for the $\gamma N\Delta$ vertex. Such comparisons, we should stress, are purely formal and illustrative, as supersymmetric theories for a spin-$\frac{1}{2}$–spin-$\frac{3}{2}$-pseudoscalar vertex have *nothing* to do with the structure of the $\pi N\Delta$ vertex which involve particles involving internal structures. Effective Lagrangian theories such as the ones we have considered above for the $\pi N\Delta$ and $\gamma N\Delta$ interactions are only guided by Lorentz and gauge invariance for the most part, and other constraints such as the Fierz-Pauli ones for the point particles seem to be dynamically restrictive.

V. GAUGE INVARIANCE: SOME UNSATISFACTORY STRATEGIES OF ITS RESTORATION IN EFFECTIVE AMPLITUDES

Pilkuhn[9] and Williams[4] have described $\gamma N\Delta$ vertex of the form

$$\bar{\Delta}^\mu\left[\epsilon_\mu - \frac{\gamma\cdot\epsilon k_\mu}{M_\Delta + m}\right]\gamma_5 N \ . \tag{34}$$

This is gauge invariant only if both N and Δ are on mass shell. Thus, replacing ϵ_μ by $k_\mu = (p_\Delta - p_N)_\mu$, (34) becomes

$$\bar{\Delta}^\mu\left[k_\mu - \frac{\gamma\cdot p_\Delta k_\mu}{M_\Delta + m} + \frac{\gamma\cdot p_N k_\mu}{M_\Delta + m}\right]\gamma_5 N \ . \tag{34$'$}$$

This vanishes for on-shell hadrons, as is easily seen by the use of the Dirac equation

$$\bar{\Delta}^\mu\gamma\cdot p_\Delta = \bar{\Delta}^\mu M_\Delta, \quad \gamma\cdot p_N N = mN \ . \tag{34$''$}$$

However, (34$'$) does *not* vanish, if either of the particles is not on shell. Hence the vertex (34) is not theoretically satisfactory. Adelseck *et al.*[5] have devised a trick to solve this problem, by replacing $M_\Delta + m$ in (34) with $\sqrt{s} + m$. Reconstructing *their* expression for the amplitude $\gamma N \to \Delta \to N\pi$

$$A \propto \bar{N}q^\mu \frac{\gamma\cdot p + \sqrt{s}}{p_\Delta^2 - M_\Delta^2} P^{3/2}_{\mu\nu}\left[\epsilon^\nu - \frac{\gamma\cdot\epsilon k^\nu}{\sqrt{s}+m}\right]\gamma_5 N \ , \tag{35}$$

where q^μ is the pion four-momentum. Replacing ϵ^ν by k^ν, we get $A \rightarrow A'$

$$A' \propto \overline{N} q^\mu \frac{\gamma \cdot p_\Delta + \sqrt{s}}{p_\Delta^2 - M_\Delta^2} \times P_{\mu\nu}^{3/2} k^\nu \left[1 - \frac{\gamma \cdot P_\Delta}{\sqrt{s} + m} + \frac{\gamma \cdot p_N}{\sqrt{s} + m} \right] \gamma_5 N \ .$$

Using $\gamma \cdot p_N \gamma_5 = -\gamma_5 \gamma \cdot p_N$ and *assuming the nucleon to be on mass shell*, we get

$$A' \propto \overline{N} q^\mu \frac{\gamma \cdot p_\Delta + \sqrt{s}}{p_\Delta^2 - M_\Delta^2} k^\nu \left[\frac{\sqrt{s} - \gamma \cdot p_\Delta}{\sqrt{s} + m} \right] P_{\mu\nu}^{3/2} N \ , \tag{36}$$

making use of the identity

$$P_{\mu\nu}^{3/2} \gamma \cdot p_\Delta = \gamma \cdot p_\Delta P_{\mu\nu}^{3/2} \ . \tag{37}$$

Substituting $\gamma \cdot p_\Delta \gamma \cdot p_\Delta = s$ in (36), we verify that A' vanishes and A is indeed gauge invariant. However, Adelseck *et al.* have paid a price to achieve this: the propagator in (35) is incorrect as shown earlier, and the nucleon is required to be always on the mass shell. Therefore, it is an unsatisfactory theoretical strategy of construction of gauge-invariant three-point functions.

As we have discussed earlier, the effective Lagrangian (21) and (22) is manifestly gauge invariant without the trouble discussed above. Therein the direct coupling to the electromagnetic field tensor $\Gamma_{\mu\nu}$ guarantees the gauge invariance trivially. It also has the merit of preserving the point transformation property of the free field. It *does* treat interaction of hadrons whether or not they are on mass shell.

VI. UNITARITY CONSTRAINT

The problem of satisfying the requirements of unitarity is not specific to the spin-$\frac{3}{2}$ resonances, but it should be included in a general discussion of the treatment of all resonances, because many treatments in the literature often ignore the implementation of it due to technical reasons. We discuss the unitarity problem in several different contexts: (a) the constraints arising from different channels, (b) the ambiguity due to the interpretation of tree-level amplitude as T or K matrix, and (c) the treatment of widths of the resonances. Items (a) and (b) are, in the main, summary of existing results intended to stimulate future theoretical developments. Item (c) zeroes in on extant theoretical problems.

A. Multichannel S-matrix unitarity

For an $n \times n$ S-matrix, we have

$$S = \begin{bmatrix} \lambda_1 e^{2i\delta_1} & a_{12} & \cdots a_{1n} \\ a_{21} & \lambda_2 e^{2i\delta_2} & \cdots a_{2n} \\ a_{n1} & a_{n2} & \cdots \lambda_n e^{2i\delta_n} \end{bmatrix} . \tag{38}$$

Here λ_i and $\delta_i (i = 1, \ldots, n)$ are the elasticities and the phase shifts of the diagonal channels, respectively (λ_i's are real); a_{ij}'s are the $i \rightarrow j$ transition amplitudes. The time reversal invariance requires

$$a_{ij} = a_{ji} \ . \tag{39}$$

Thus, the S matrix is symmetric. Unitarity of S, i.e.,

$$SS^\dagger = I \ , \tag{40}$$

yields two sets of equations,

$$\lambda_i^2 + \sum_{\substack{j=1 \\ i \neq j}}^{n} |a_{ij}|^2 = 1 \ , \tag{41}$$

and

$$\lambda_i e^{2i\delta_i} |a_{ki}| e^{-i\phi_{ki}} + \lambda_k e^{-2i\delta_k} |a_{ik}| e^{i\phi_{ik}} + \sum_{\substack{j=1 \\ j \neq i,k}}^{n} |a_{ij}| |a_{ki}| e^{i\phi_{ij}} e^{-i\phi_{ki}} = 0 \ , \tag{42}$$

where

$$a_{ij} = |a_{ij}| e^{i\phi_{ij}} \ . \tag{43}$$

Thus, there are n equation in (41) and $n(n-1)/2$ equation in (42). If $|a_{ij}| \rightarrow 0$, the last term in Eq. (42) can be ignored. Using (39), we get, in the limit $\lambda_i, \lambda_k \rightarrow 1$, as $|a_{ij}| \rightarrow 0$,

$$e^{2i\phi_{ik}} = -e^{2i[\delta_i + \delta_k]} = e^{2i[\delta_i + \delta_k + (2l + 1/2)\pi]} \ ,$$

where l is an integer. This allows us to recover the result, originally due to Fermi,[10]

$$\phi_{ik} = \delta_i + \delta_k + l\pi + \frac{\pi}{2} \ . \tag{44}$$

In the two-channel case, this is exact and is the famous Watson's theorem,[25] which we could have proved starting with the unitarity of the 2×2 S matrix.

Equations (41) and (42) bring out a particular difficulty of baryon resonance physics: Most treatments in the literature are unable to put these constraints in the analysis of multipoles as soon as the number of channels exceed two. Even in the two-channel case, some treatments ignore unitarity for convenience. An example is the nonunitary version of the Blomqvist-Laget theory,[27] limitations of which we have discussed elsewhere.[28]

To give an example of the complex problem of the multichannel unitarity, consider the case of $\Delta(1700)$ $\frac{3}{2}$ resonance; its decay channels are $N\pi$ (10–20 %), $N\pi\pi$ (80–90 %), and $N\gamma$ (0.14–0.33 %), where the numbers in parentheses are the decay fractions. Clearly, the unitarity requires implementation of the constraints in Eqs. (41) and (42). Not enough experimental information is available on many of the primary strong interaction channels (e.g., $N\pi \rightarrow N\pi\pi$) to allow a unitarity-combined analysis of the photo- or electroproduction amplitudes. This situation could be helped enormously, if new experimental facilities became available in this high energy domain (e.g., proposed expansion of the Los Alamos Meson physics facility, called PILAC).[29] Attempts are still being made to treat the outgoing multiparticle channels in the two-body approximation,[12] such as treating the $N\pi\pi$ channels as $\Delta\pi$ or $N\rho$. Critical studies are needed to test the quality of these approaches.

B. Variety of unitarization strategies

The use of effective Lagrangians in the theory of photo- and electroproduction of mesons leads to *real* amplitudes at the tree level, if the widths of the unstable particles are neglected. This, of course, leads to the violation of unitarity conditions (40), if the tree-level amplitude is taken to be the T matrix. Davidson *et al.*[22] have extensively investigated the ambiguities due to the unitarization strategies of the $\gamma N\to\pi N$ in the $\Delta(1232)$ resonance region. Lessons learned from there are of general interest in the context of other resonances for future work and will be summarized here.

Consider the tree approximation for the πN scattering in the $J=T=\frac{3}{2}$ channel. The partial wave amplitude f_{1+} is given by

$$f_{1+}=f^B_{1+}+f^R_{1+}\ , \tag{45}$$

where f^B_{1+} contains all the background contributions and f^R_{1+} is the s-channel Δ contribution

$$qf^R_{1+}=\frac{M_\Delta\Gamma_\Delta}{M^2_\Delta-s}\ , \tag{46}$$

where q is the c.m. momentum, the numerator coming from the effective Lagrangian theory, and the denominator is the Δ-pole term. Clearly, Eq. (45) does not satisfy the Watson theorem, hence violating the unitarity condition (40).

Following Olsson and Noelle, we can unitarize (45) by writing qf_{1+} in the form

$$qf_{1+}=\sin\delta e^{i\delta}=\frac{qf^B_{1+}}{1-iqf^B_{1+}}+\frac{e^{i\alpha}}{\epsilon-i\gamma}\ , \tag{47}$$

δ being the πN phase shift in the 33-channel. This equation can be rewritten as

$$qf_{1+}=\sin\delta_B e^{i\delta_B}+\frac{e^{i\alpha}}{\epsilon-i\gamma}\ , \tag{48}$$

where

$$\tan\delta_B=qf^B_{1+},\quad \epsilon=\frac{M^2_\Delta-s}{M_\Delta\Gamma_\Delta}\ . \tag{49}$$

In Noelle's *Ansatz* for a single channel,

$$\gamma=1,\quad \alpha=2\delta_B\ , \tag{50}$$

producing a Breit-Wigner type resonance amplitude. For the multichannel case, Noelle has a trivially unitary S matrix for the resonance, S_R, and S is obtained by

$$S=e^{i\delta_B}S_R e^{i\delta_B}\ . \tag{51}$$

In the Olsson approach, the unitarization is done at the T-matrix level. In this approach,

$$\alpha=0,\quad \gamma=\frac{1+\epsilon\sin2\delta_B}{\cos2\delta_B}\ ,\qquad \tan\delta=\frac{1+\epsilon\tan\delta_B}{\epsilon+\tan\delta_B}\ , \tag{52}$$

the last two equations following from (47) and (48) setting $\alpha=0$. In this approach, the resonance is not of the Breit-Wigner form, since $\gamma\neq1$.

A third approach to unitarization is to interpret the tree-level amplitude as a K matrix. Here

$$T=K(1-iK)^{-1},\quad T=\frac{1}{2i}(S-1)\ . \tag{53}$$

From the unitarity of S matrix, the K matrix is real if all channels are open and is symmetric if time reversal invariance holds. In this approach, the πN scattering amplitude in the 33 resonance region takes the form

$$\tan\delta=\tan\delta_B+\frac{1}{\epsilon}\ . \tag{54}$$

The extension of the above discussion to photoproduction is straightforward. Taking the example of a three-channel case, where channels are indicated as $1=\gamma p$, $2=\pi^0p$, and $3=\pi^+n$, the T matrix element for $\gamma p\to\pi^0 p$ is given by[30]

$$T_{12}\approx\cos\delta e^{i\delta}\left[K_{12}+\frac{iK_{23}K_{13}}{1-iK_{33}}\right]\ , \tag{55}$$

where K_i are the K-matrix element for the ith channel. This explicitly indicates the unitarization procedure for the amplitude via the K-matrix method. In the Olsson and Noelle approaches, we can use the previous discussion considering the photoproduction amplitudes at the tree level.

The importance of the above discussion is that there is a variety of ways of unitarizing the amplitudes, resulting in a model dependence of the separation of the resonance contribution from the novel physical amplitude extracted from the experiment. It is crucial that this model dependence be explored in specific cases.

The model dependence is, in fact, the Achilles heel of the resonance physics. Feynman[2] put his finger on this problem by asking the following questions: "How much is resonances and how much is background? Can the background below a resonance be simply tails of other resonances? How big is the tail of a resonance?" Feynman's own answer[2] to the last question, "Impossible to answer except arbitrarily" is a warning that we should not forget to explore in every instance. Elsewhere we[22] have tried to do just that for the photoproduction in the delta (1232) region. Much remains to be done for other resonances. We hope this discussion will help support such a process.

C. Treatment of the widths of resonance in a unitary fashion

Another trouble with unitarity can be caused by an inconsistent insertion of the width of a baryon resonance, produced by the electromagnetic vertex, in the expression for the Lorentz invariant amplitude. We have detected this problem in the work of Adelseck *et al.*, where their tabulation (in their Table III) for the spin-$\frac{1}{2}$ and the spin-$\frac{3}{2}$ resonance propagators has a pole structure $(q^2-M^2+iM\Gamma)^{-1}$.

This problem is most easily demonstrated by the elastic πN scattering via a $\frac{1}{2}^+$ resonance. The amplitude for this process, following the method of Adeleseck *et al.* by inserting the width of the resonance, is of the form (with a *PS* coupling)

$$M_{fi}=g^2\overline{U}_f\gamma_5\frac{\gamma\cdot p_i+\gamma\cdot q_i+M_R}{s-M_R^2+iM_R\Gamma_R}\gamma_5U_i \ , \tag{56}$$

where the initial nucleon four-momentum is $p_i(E,\mathbf{p}_i)$, $s=(p_i+q_i)^2$, q_i, is the pion four-momentum.

Making partial wave projections for the amplitudes corresponding to the πN s wave and p wave, we get

$$qf_{0+}=\frac{g^2(E+m)(M_R-W)q}{8\pi W(s-M_R^2+iM_R\Gamma_R)} \ , \tag{57a}$$

and

$$qf_{1-}=-\frac{g^2(E-m)(M_R+W)q}{8\pi W(s-M_R^2+iM_R\Gamma_R)} \ , \tag{57b}$$

where $W=\sqrt{s}$. From the unitarity of the S matrix,

$$T^\dagger T=\mathrm{Im}T \ , \tag{58}$$

with $T=qf_{0+}$ or qf_{1-}. Thus, (58) yields for the respective cases

$$\Gamma_R=\frac{g^2q(E+m)(W-M_R)}{8\pi WM_R} \ , \tag{59a}$$

and

$$\Gamma_R=\frac{g^2q(E-m)(W+M_R)}{8\pi WM_R} \ . \tag{59b}$$

Thus, the widths obtained from the same resonance from the two different partial waves are *not* equal, indicating an inconsistency. Therefore, *inserting width at the level of the amplitude [Eq. (56)], as done by Adelseck et al., is incorrect.* At best it can be reasonable for one partial wave, but not the other. Thus, if the resonant partial wave has the correct width, the nonresonant partial wave violates unitarity with the *same* width.

The discussion for the $\pi N\to\pi N$ and $\gamma N\to\pi N$ processes in the Δ channel is more complicated, but the Adelseck *et al.* procedure again leads to inconsistency. For the πN scattering in the effective Lagrangian approach, using $L_{\pi N\Delta}$ of Eq. (20), we can compute the resonant and nonresonant partial wave amplitudes, ignoring the delta widths. These are

$$qf_{1+}=\frac{-g_{\pi N\Delta}^2(E+m)q^3(W+M_\Delta)}{24\pi Wm_\pi^2(s-M_\Delta^2)} \ , \tag{60a}$$

$$qf_{2-}=\frac{-g_{\pi N\Delta}^2(E-m)q^3(M_\Delta-W)}{24\pi Wm_\pi^2(s-M_\Delta^2)} \ . \tag{60b}$$

Suppose we had made the substitution $s-M_\Delta^2\to s-M_\Delta^2+iM_\Delta\Gamma_\Delta(s)$ in the manner of Adelseck *et al.* in above expressions. We can now calculate the K-matrix elements from then T-matrix elements, using $K=T(1+iT)^{-1}$. This yields for the above two amplitudes

$$M_\Delta\Gamma_\Delta=\frac{g_{\pi N\Delta}^2(E+m)q^3(W+M_\Delta)}{24\pi Wm_\pi^2} \ , \tag{61a}$$

$$M_\Delta\Gamma_\Delta=\frac{g_{\pi N\Delta}^2(E-m)q^3(W-M_\Delta)}{24\pi Wm_\pi^2} \ , \tag{61b}$$

again inconsistent with each other. For brevity, we shall omit demonstrating here the same phenomenon for the $\gamma N\to\pi N$ case.

To conclude, it is not appropriate to insert the width of the baryon resonance in the invariant amplitude by the prescription $s-M_\Delta^2\to s-M_\Delta^2+iM_\Delta\Gamma_\Delta(s)$, as Adelseck *et al.* have done. This is inconsistent with unitarity.

VII. SUMMARY AND CONCLUSION

We have investigated some of the theoretical issues associated with spin-$\frac{3}{2}$ particles. This is relevant to the production and decay process of spin-$\frac{3}{2}$ baryon resonances. We have reviewed the derivation of the relativistic spin-$\frac{3}{2}$ propagator and showed that the propagators presented by Williams[4] and Adelseck *et al.*[5] are inconsistent with both local and nonlocal wave equation of a massive spin-$\frac{3}{2}$ particle. Then we have discussed interacting the spin-$\frac{3}{2}$ field within the framework of the effective Lagrangian theory. We have used the most general interaction Lagrangian constructed in a way that it preserves the same symmetry as the free one, under the point transformation.[8] This introduces arbitrary parameters in the theory, which determines the off-shell contribution of the spin-$\frac{3}{2}$ particle. Peccei[13] first attempted to fix these off-shell parameters, but Nath *et al.*[8] showed that Peccei's condition is unnecessarily restrictive. However, Nath *et al.* have later[6] argued that the second gauge coupling term [Eq. (22)] should be excluded and specific values of the off-shell parameters are required. We have discussed why these new constraints of Nath *et al.* are theoretically poor characterizations of the $\gamma N\Delta$ transition amplitude, besides yielding a relatively poor quality of fit to the photoproduction multipoles. For a theoretical comparison, we have mentioned the choice of these off-shell parameters in supersymmetry which deals mainly with point spin-$\frac{3}{2}$ particles. This, of course, has nothing to do with particles, such as Δ, with composite structure, but is illustrative of the off-shell freedom of the massive spin-$\frac{3}{2}$ fields. We have also pointed to an analogous freedom in the massive vector fields, in Appendix A, discussing pion β decay in the intermediate vector boson theory.

The electromagnetic interaction Lagrangians ($L_{\gamma N\Delta}^1$, $L_{\gamma N\Delta}^2$), used by us [Eqs. (21) and (22)] are manifestly gauge invariant, irrespective of whether Δ and or N is on or off mass shell. Williams[4] and Pilkuhn[9] have used $\gamma N\Delta$ interactions which are not gauge invariant except when Δ is on mass shell. We have found these theoretically unsatisfactory. To overcome this problem, Adelseck *et al.*[5] have replaced $M_\Delta + m$ in (34) by $\sqrt{s}+m$. This has resulted in an incorrect spin-$\frac{3}{2}$ propagator, as we have mentioned earlier. We have stressed that these difficulties are avoidable.

Finally, we have discussed the multichannel S-matrix unitarity, variety of unitarization strategies in the two-channel case, such as photoproduction of pion below 2π threshold, and the treatment of the width of resonances in a unitary fashion. We have showed that the insertion of the width of resonance at the level of the Lorentz invariant amplitude for the πN scattering is inconsistent with unitarity; the same is true for the pion photoproduction.

The search for a unitary theory for baryon resonance production and decay above the two-pion threshold is, unfortunately, not yet complete. We hope our discussions here will set the stage for new theoretical efforts for a comprehensive treatment of final-state interactions for such cases, in a relativistic framework consistent with gauge invariance, as experimentalists prepare for new tools, both electroweak and strong, to explore these baryon resonances. This is an ambitious, yet entirely appropriate, objective for further tests of QCD in the nonperturbative domain. It would also help to explore the resonances beyond $\Delta(1232)$ in the nuclear many-hadron domain, a task that has hardly begun.

APPENDIX A

An analogous off-shell phenomenon can be demonstrated via the pion decay in the intermediate vector boson theory.[8] Consider the Lagrangian

$$L = w_\lambda \partial^\lambda \phi + \bar{\nu}\gamma_\lambda \left[\frac{1-\gamma_5}{2}\right]\mu w^\lambda , \tag{A1}$$

where w is the intermediate vector boson, ν,μ are lepton spinor for neutrino and muon respectively, and ϕ is the pion field. Then the matrix element for the decay $\pi^- \to \mu^- + \bar{\nu}_\mu$ is

$$M_{fi} = \bar{\nu}\gamma_\lambda \left[\frac{1-\gamma_5}{2}\right]\mu G^{\lambda\sigma} q_\sigma , \tag{A2}$$

$G^{\lambda\sigma}$ being the intermediate vector boson propagator

$$G^{\lambda\sigma} = \left[g^{\lambda\sigma} - \frac{p^\lambda p^\sigma}{M_w^2}\right](p^2 - M_w^2)^{-1} , \tag{A3}$$

M_w being the mass of the intermediate vector boson. We can decompose this propagator in terms of spin-one and spin-zero projection operators

$$(P_1)_{\lambda\sigma} = g_{\lambda\sigma} - \frac{p_\lambda p_\sigma}{p^2}, \quad (P_0)_{\lambda\sigma} = \frac{p_\lambda p_\sigma}{p^2} , \tag{A4}$$

which satisfy the following properties

$$(P_0 + P_1)_{\lambda\sigma} = g_{\lambda\sigma} , \tag{A5}$$

$$p_\lambda P_1^{\lambda\sigma} = 0 . \tag{A6}$$

Thus, the spin-one part of the intermediate vector boson propagator leads to vanishing amplitude. This confirms our intuitive expectation that a spin-zero system cannot decay via a spin-one vector boson, without the off-shell spin-zero piece of the latter coming into play. Indeed, the off-shell spin-zero piece of the intermediate vector boson propagator yields the resultant matrix element of the pion decay

$$M_{fi} = -\frac{\bar{\nu}\gamma\cdot q(1-\gamma_5)\mu}{2M_w^2} , \tag{A7}$$

as in the ordinary Fermi theory.

APPENDIX B

Using the Lagrangian defined in Eq. (23), the πN transition amplitude, via the intermediate s-channel exchange can be evaluated in the tree approximation. We find

$$M_{fi} = -\frac{g_{\pi N\Delta}^2}{m_\pi^2}\bar{U}_f q_f^\mu \theta_{\mu\nu}(Z) P^{\nu\alpha}(p)\theta_{\alpha\beta}(Z) q_i^\beta U_i , \tag{B1}$$

U_i, U_f are the initial and final state nucleon spinors, q_i, q_f are the incoming and outgoing pion momenta, and $p = p_i + q_i$, where p_i is the nucleon momentum in the initial state.

In terms of the projection operators, $\theta_{\mu\nu}(Z)$ is written as

$$\theta_{\mu\nu} = [P^{3/2} + (1+3a)P_{11}^{1/2} + (1+a)P_{22}^{1/2} + \sqrt{3}a(P_{12}^{1/2} + P_{21}^{1/2})]_{\mu\nu} , \tag{B2}$$

where $a = -\frac{1}{2}(1+2Z)$. The transition matrix M_{fi} can be rewritten as

$$M_{fi} = \bar{U}_f[A + \tfrac{1}{2}(\gamma\cdot q_i + \gamma\cdot q_f)B]U_i . \tag{B3}$$

For the spin-$\frac{1}{2}$ part of the amplitude, we obtain

$$M_\Delta A_{1/2} = -\tfrac{2}{3}(W^2 - m^2)\left[\frac{m+2M_\Delta}{M_\Delta}\right]a^2 - \tfrac{2}{3}(W^2-m^2)a + \frac{2\omega}{3}\left[\frac{m\omega}{M_\Delta} + \frac{m_\pi^2}{W}\right] , \tag{B4}$$

$$M_\Delta B_{1/2} = \frac{2}{3}\left[4m + \frac{W^2+m^2}{M_\Delta}\right]a^2 + \frac{4}{3}\left[m + \frac{W\omega}{M_\Delta}\right]a + \frac{2\omega}{3}\left[\frac{\omega}{M_\Delta} + \frac{m}{W}\right] ,$$

with (B5)

$$\omega = \frac{W^2 + m_\pi^2 - m^2}{2W} ;$$

the corresponding partial wave amplitudes are:

$$f_{0+} = \frac{E+m}{8\pi W}[A_{1/2} + (W-m)B_{1/2}] , \tag{B6}$$

$$f_{1-} = \frac{E-m}{8\pi W}[-A_{1/2} + (W+m)B_{1/2}] . \tag{B7}$$

In order to have pure coupling to spin-$\frac{3}{2}$ part of the amplitude (B1), f_{0+} and f_{1-} must vanish simultaneously. We can now set f_{0+} and f_{1-} equal to zero and try to solve for the parameter a. We find that these partial waves cannot vanish for an arbitrary or specific value of the parameter a. Therefore, the spin-$\frac{1}{2}$ part of the amplitude cannot be eliminated, no matter how we choose the off-shell parameter Z, contradicting the anticipated result of Peccei[13] that this should be possible.

This work was supported by the U.S. Dept. of Energy under Contract No. DE-FG02-88ER40448.

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