Nuclear interaction currents

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A procedure for extracting electromagnetic exchange-current operators from a given nucleon-nucleon potential is developed on the basis of the continuity-equation constraint. The present method is applied to the one-boson-exchange nucleon-nucleon potential to yield exchange-current operators. It is shown that the proposed formulas reproduce, up to divergence-free four-currents, both the contact current and the mesonic current derived consistently within the one-boson-exchange model including meson retardation and nucleon recoil.

I. INTRODUCTION

It has long been known that, in the nonrelativistic nucleon-nucleon potential model, the velocity-dependent term introduces contributions to nuclear electromagnetic transitions.[1-13] The "interaction currents" arise because the momentum operator of the ith nucleon is modified by the substitution

$$\mathbf{p}_i \rightarrow \mathbf{p}_i - e_i\,\mathbf{A}(\mathbf{r}_i)\ ,$$

where $e_i=\frac{1}{2}e(1+\tau_{iz})$ is the charge operator and $\mathbf{A}(\mathbf{r}_i)$ is the electromagnetic vector potential evaluated at the position $\mathbf{r}_i$. Gauge invariance alone cannot determine the whole electromagnetic interactions but it gives us a very stringent constraint on their forms. For the two-nucleon system the electromagnetic current is the sum of one-body and two-body operators,

$$\mathbf{J}(\mathbf{x})=\mathbf{J}^{(1)}(\mathbf{x})+\mathbf{J}^{(2)}(\mathbf{x})\ . \tag{1.1}$$

It is well known that in the nonrelativistic nuclear theory the exchange current $\mathbf{J}^{(2)}(\mathbf{x})$ should satisfy the condition,

$$\nabla_x\cdot\mathbf{J}^{(2)}(\mathbf{x})=-i\left[V,\sum_i e_i\delta(\mathbf{x}-\mathbf{r}_i)\right]\ , \tag{1.2}$$

where V is the nuclear potential and the sum is over $i=1,2$.

The velocity dependence is not the only source of the interaction currents. The isospin dependence of the nuclear force,

$$V=V^0+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2V^\tau\ , \tag{1.3}$$

should also produce exchange currents. Forty years ago Sachs[14] derived from V^τ an exchange current

$$\mathbf{J}^{\mathrm{Sachs}}(\mathbf{x})=-e(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z\mathbf{r}V^\tau \times\int_0^1 ds\,\delta(\mathbf{x}-s\mathbf{r}_1-(1-s)\mathbf{r}_2)\ , \tag{1.4}$$

where $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$ is the distance between two nucleons. The difficulty of this current is immediately seen by taking the one-pion-exchange potential

$$V^\tau_{\mathrm{OPE}}=-\frac{f^2}{\mu^2}\boldsymbol{\sigma}_1\cdot\nabla_1\boldsymbol{\sigma}_2\cdot\nabla_2\frac{e^{-\mu r}}{4\pi r}\ , \tag{1.5}$$

where f is the pion-nucleon coupling constant, μ is the pion mass, and $\nabla_i=\partial/\partial\mathbf{r}_i$. With (1.5) inserted into (1.4), the Sachs current has no resemblance to the one-pion-exchange current,[15]

$$\mathbf{J}^\tau_{\mathrm{OPE}}(\mathbf{x})=\mathbf{J}^{\tau,\mathrm{I}}_{\mathrm{OPE}}(\mathbf{x})+\mathbf{J}^{\tau,\mathrm{II}}_{\mathrm{OPE}}(\mathbf{x})\ , \tag{1.6}$$

where

$$\mathbf{J}^{\tau,\mathrm{I}}_{\mathrm{OPE}}(\mathbf{x})=e\frac{f^2}{\mu^2}(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z\left[-\delta(\mathbf{x}-\mathbf{r}_1)\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2\cdot\nabla_2\frac{e^{-\mu|\mathbf{x}-\mathbf{r}_2|}}{4\pi|\mathbf{x}-\mathbf{r}_2|}+\delta(\mathbf{x}-\mathbf{r}_2)\boldsymbol{\sigma}_2\boldsymbol{\sigma}_1\cdot\nabla_1\frac{e^{-\mu|\mathbf{x}-\mathbf{r}_1|}}{4\pi|\mathbf{x}-\mathbf{r}_1|}\right] \tag{1.7}$$

is the contact current and

$$\mathbf{J}^{\tau,\mathrm{II}}_{\mathrm{OPE}}(\mathbf{x})=-e\frac{f^2}{\mu^2}(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z\boldsymbol{\sigma}_1\cdot\nabla_1\boldsymbol{\sigma}_2\cdot\nabla_2\left[\frac{e^{-\mu|\mathbf{x}-\mathbf{r}_1|}}{4\pi|\mathbf{x}-\mathbf{r}_1|}\nabla_x\frac{e^{-\mu|\mathbf{x}-\mathbf{r}_2|}}{4\pi|\mathbf{x}-\mathbf{r}_2|}-\frac{e^{-\mu|\mathbf{x}-\mathbf{r}_2|}}{4\pi|\mathbf{x}-\mathbf{r}_2|}\nabla_x\frac{e^{-\mu|\mathbf{x}-\mathbf{r}_1|}}{4\pi|\mathbf{x}-\mathbf{r}_1|}\right] \tag{1.8}$$

is the pionic current.

Riska[16] tackled the problem from a different point of view. He projected out the tensor and spin-spin components of the nuclear interaction, and wrote down corresponding exchange-current operators by invoking the known form of the single-pion exchange-current operators which satisfy the condition (1.2). Buchmann, Leidemann, and Arenhövel[13] took a similar approach on the basis of the parametrized form of the Paris potential. Riska's idea is further extended to a relativistic formalism.[17] Recently the present author has given a rigorous proof that the exchange-current operators proposed by Riska indeed result from the minimal-substitution prescription applied to a momentum-dependent representation of local potentials.[18]

The isovector nucleon-nucleon potential implies the existence of currents flowing over its range. It is desired that a general rule for deducing such currents should be given. The interaction currents appear not only in the nucleon-nucleon interaction but also in meson-nucleon[19] and any other hadronic interactions. Since interactions are usually introduced phenomenologically, model-independent determination of the interaction currents are essentially required.

The purpose of this paper is to propose a general prescription for calculating nuclear interaction currents from an arbitrary nucleon-nucleon potential. We shall show that our method can reproduce both of the one-pion-exchange currents (1.7) and (1.8) rigorously from the one-pion-exchange potential (1.5). In Sec. II we give a rigorous derivation of the gauge-invariance condition (1.2). In Sec. III we propose exchange-current operators which are written in terms of nuclear potentials. In Sec. IV we consider a one-boson-exchange model and calculate the nucleon-nucleon interaction and the exchange currents to illustrate the usefulness of our approach. A discussion is given in Sec. V.

II. GAUGE-INVARIANCE CONDITION

A. Gauge invariance of the equations of motion

The gauge-invariance condition (1.2) is usually derived from the continuity equation for the total current operator,

$$\partial_\mu[e^{-iH_0t}J_\mu(\mathbf{x})e^{iH_0t}] = e^{-iH_0t}\{\nabla_x\cdot\mathbf{J}(\mathbf{x})+i[H_0,\rho(\mathbf{x})]\}e^{iH_0t}=0 \ . \qquad (2.1)$$

The time evolution of physical quantities in the nuclear configuration space is dictated by the nonradiative total Hamiltonian

$$H_0=T+V \ , \qquad (2.2)$$

where T is the kinetic-energy operator. Thus the time derivative of any operator is evaluated by taking its commutator with H_0. By separating one-body and two-body parts and assuming

$$\rho^{(1)}(\mathbf{x})\cong\sum_i e_i\delta(\mathbf{x}-\mathbf{r}_i) \qquad (2.3)$$

and

$$\rho^{(2)}(\mathbf{x})\cong 0 \ , \qquad (2.4)$$

we obtain Eq. (1.2). On this account, it is sometimes argued that Eq. (1.2) is valid only in the point-source limit. However, this is not true. Here we give a rigorous derivation of the gauge-invariance constraint on the two-body current operators and make clear the meaning of Eq. (1.2).

According to Sachs and Austern,[20] the gauge invariance of the equations of motion requires that the total Hamiltonian

$$H(A_\mu)=H_0+H_{\text{em}}(A_\mu) \qquad (2.5)$$

should obey the condition

$$H'(A_\mu)=H(A'_\mu) \ . \qquad (2.6)$$

$H'(A_\mu)$ is the new Hamiltonian transformed as

$$H'(A_\mu)=U\left[H(A_\mu)-i\frac{\partial}{\partial t}\right]U^{-1} \qquad (2.7)$$

under a time-dependent unitary transformation

$$U=\exp\left[i\sum_i e_i G(\mathbf{r}_i)\right] \qquad (2.8)$$

with $G(\mathbf{r}_i)$ being the gauge function. The two-nucleon wave function ψ and the electromagnetic vector potential $A_\mu=(\mathbf{A},iA_0)$ are transformed simultaneously as

$$\psi'=U\psi \ , \qquad (2.9)$$

$$A'_\mu=A_\mu+\partial_\mu G \ . \qquad (2.10)$$

The electromagnetic interaction Hamiltonian is decomposed into one-body and two-body interactions,

$$H_{\text{em}}(A_\mu)=H^{(1)}_{\text{em}}(A_\mu)+H^{(2)}_{\text{em}}(A_\mu) \ . \qquad (2.11)$$

The one-body and two-body current operators are defined by

$$H^{(1)}_{\text{em}}(A_\mu)=-\int d^3x\, J^{(1)}_\mu(\mathbf{x})A_\mu(\mathbf{x}) \ , \qquad (2.12)$$

$$H^{(2)}_{\text{em}}(A_\mu)=-\int d^3x\, J^{(2)}_\mu(\mathbf{x})A_\mu(\mathbf{x}) \ . \qquad (2.13)$$

The separation of the total current into one-body and two-body currents does not necessarily preserve the gauge invariance of the one-body current. This is because the one-body current has a freedom of being transformed under an arbitrary unitary transformation which, in general, violates gauge invariance.[21] Among these unitary-equivalent one-body currents, we choose the one that is manifestly gauge invariant. Then it obeys the requirement of gauge invariance independently,

$$U\left[T+H^{(1)}_{\text{em}}(A_\mu)-i\frac{\partial}{\partial t}\right]U^{-1}=T+H^{(1)}_{\text{em}}(A'_\mu) \ . \qquad (2.14)$$

We expand the left-hand side of this equation in a power series of e. To order e we obtain

$$U\left[T+H_{\rm em}^{(1)}(A_\mu)-i\frac{\partial}{\partial t}\right]U^{-1}=T+H_{\rm em}^{(1)}(A_\mu)-i\left[T,\sum_i e_i G(\mathbf{r}_i)\right]+i\left[H_0,\sum_i e_i G(\mathbf{r}_i)\right] . \quad (2.15)$$

The gauge-invariance condition Eq. (2.14) with Eq. (2.10) leads us to

$$H_{\rm em}^{(1)}(\partial_\mu G)=-i\left[T,\sum_i e_i G(\mathbf{r}_i)\right]+i\left[H_0,\sum_i e_i G(\mathbf{r}_i)\right] . \quad (2.16)$$

On the other hand, from Eq. (2.12),

$$H_{\rm em}^{(1)}(\partial_\mu G)=-\int d^3x\, J_\mu^{(1)}(\mathbf{x})\partial_\mu G(\mathbf{x})=\int d^3x\, \nabla_x\cdot\mathbf{J}^{(1)}(\mathbf{x})G(\mathbf{x})+i\left[H_0,\int d^3x\,\rho^{(1)}(\mathbf{x})G(\mathbf{x})\right] . \quad (2.17)$$

Comparing this with Eq. (2.16) we find

$$\nabla_x\cdot\mathbf{J}^{(1)}(\mathbf{x})+i[H_0,\rho^{(1)}(\mathbf{x})]=-i\left[T,\sum_i e_i\delta(\mathbf{x}-\mathbf{r}_i)\right]+i\left[H_0,\sum_i e_i\delta(\mathbf{x}-\mathbf{r}_i)\right] . \quad (2.18)$$

In the point-source limit (2.3) we gain

$$\nabla_x\cdot\mathbf{J}^{(1)}(\mathbf{x})\cong -i[T,\rho^{(1)}(\mathbf{x})] , \quad (2.19)$$

It is important to note that, in the presence of the potential V, Eq. (2.19) is not the exact relation in contrast to the usual argument in decomposing the continuity equation into one-body and two-body components. Equation (2.19) holds true in free space but loses its validity in the nucleus because the interaction potential V enters through the time derivative of one-body quantities.

We turn to the two-body current. From Eqs. (2.6) and (2.14) we have

$$U(V+H_{\rm em}^{(2)}(A_\mu))U^{-1}=V+H_{\rm em}^{(2)}(A'_\mu) . \quad (2.20)$$

By expanding in powers of e and retaining the first-order terms in e we obtain

$$U(V+H_{\rm em}^{(2)}(A_\mu))U^{-1}=V+H_{\rm em}^{(2)}(A_\mu)-i\left[V,\sum_i e_i G(\mathbf{r}_i)\right] . \quad (2.21)$$

As a consequence we get

$$H_{\rm em}^{(2)}(\partial_\mu G)=-i\left[V,\sum_i e_i G(\mathbf{r}_i)\right]=-i\int d^3x\left[V,\sum_i e_i\delta(\mathbf{x}-\mathbf{r}_i)\right]G(\mathbf{x}) . \quad (2.22)$$

We compare this with

$$H_{\rm em}^{(2)}(\partial_\mu G)=\int d^3x\,\nabla_x\cdot\mathbf{J}^{(2)}(\mathbf{x})G(\mathbf{x})+i\left[H_0,\int d^3x\,\rho^{(2)}(\mathbf{x})G(\mathbf{x})\right] , \quad (2.23)$$

and we find

$$\nabla_x\cdot\mathbf{J}^{(2)}(\mathbf{x})+i[H_0,\rho^{(2)}(\mathbf{x})]=-i\left[V,\sum_i e_i\delta(\mathbf{x}-\mathbf{r}_i)\right] . \quad (2.24)$$

This is the gauge-invariance condition on the exchange currents. One easily sees that the sum of (2.18) and (2.24) indeed leads to conservation of the total current. We divide the two-body current $\mathbf{J}^{(2)}(\mathbf{x})$ into two pieces,

$$\mathbf{J}^{(2)}(\mathbf{x})=\mathbf{J}^V(\mathbf{x})+\delta\mathbf{J}(\mathbf{x}) , \quad (2.25)$$

in such a way that $\mathbf{J}^V(\mathbf{x})$ satisfies

$$\nabla_x\cdot\mathbf{J}^V(\mathbf{x})=-i\left[V,\sum_i e_i\delta(\mathbf{x}-\mathbf{r}_i)\right] . \quad (2.26)$$

Then the remaining current $\delta\mathbf{J}(\mathbf{x})$ and the exchange charge density $\delta\rho(\mathbf{x})\equiv\rho^{(2)}(\mathbf{x})$ form a divergence-free four-current

$$\nabla_x\cdot\delta\mathbf{J}(\mathbf{x})+i[H_0,\delta\rho(\mathbf{x})]=0 . \quad (2.27)$$

It should be stressed that the condition on the interaction current, Eq. (2.26), has exactly the same form as Eq. (1.2) in the nonrelativistic theory. Now it is clear that the condition, Eq. (2.26), is derived from the assumption that the one-body current satisfies the gauge-invariance condition separately. We did not take the nonrelativistic limit nor the point-source limit. The δ-function in (2.26) emerges not because we made the point-change approximation (2.3) but because the local gauge invariance is imposed.

B. Nonlocal current operators

It is well known that a nonlocal potential is represented by a momentum-dependent potential. In this section we give a general prescription for deriving a momentum-dependent representation of a nonlocal current operator. First we transform the two-body current operator from the photon position space into the photon momentum space

$$\mathbf{J}^{(2)}(\mathbf{k})=\int d^3x\, e^{i\mathbf{k}\cdot\mathbf{x}}\mathbf{J}^{(2)}(\mathbf{x}) . \quad (2.28)$$

The current operator in the nuclear configuration space is given by

$$\langle \mathbf{r}_1'\mathbf{r}_2'|\mathbf{J}^{(2)}(\mathbf{k})|\mathbf{r}_1\mathbf{r}_2\rangle=(2\pi)^{-12}\int d^3k_1d^3k_2d^3k_1'd^3k_2'e^{i(\mathbf{k}_1'\cdot\mathbf{r}_1'+\mathbf{k}_2'\cdot\mathbf{r}_2'-\mathbf{k}_1\cdot\mathbf{r}_1-\mathbf{k}_2\cdot\mathbf{r}_2)}\langle \mathbf{k}_1'\mathbf{k}_2'|\mathbf{J}^{(2)}(\mathbf{k})|\mathbf{k}_1\mathbf{k}_2\rangle \ . \tag{2.29}$$

Because of the conservation of total momentum, the current operator in momentum space is written in the form

$$\langle \mathbf{k}_1'\mathbf{k}_2'|\mathbf{J}^{(2)}(\mathbf{k})|\mathbf{k}_1\mathbf{k}_2\rangle=(2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}-\mathbf{k}_1'-\mathbf{k}_2')\mathbf{J}^{(2)}(\bar{\mathbf{k}}_1,\bar{\mathbf{k}}_2,\mathbf{q},\mathbf{k}) \ , \tag{2.30}$$

where we have introduced momentum variables

$$\bar{\mathbf{k}}_1=\tfrac{1}{2}(\mathbf{k}_1+\mathbf{k}_1') \ , \tag{2.31}$$

$$\bar{\mathbf{k}}_2=\tfrac{1}{2}(\mathbf{k}_2+\mathbf{k}_2') \ , \tag{2.32}$$

$$\mathbf{q}=\tfrac{1}{2}(\mathbf{k}_1-\mathbf{k}_1'-\mathbf{k}_2+\mathbf{k}_2') \ . \tag{2.33}$$

Inserting (2.30) into (2.29) we get

$$\begin{aligned}\langle \mathbf{r}_1'\mathbf{r}_2'|\mathbf{J}^{(2)}(\mathbf{k})|\mathbf{r}_1\mathbf{r}_2\rangle&=(2\pi)^{-9}\int d^3\bar{k}_1d^3\bar{k}_2d^3q\ e^{-i(\bar{\mathbf{k}}_1\cdot\mathbf{s}_1+\bar{\mathbf{k}}_2\cdot\mathbf{s}_2+\mathbf{q}\cdot\bar{\mathbf{r}}-\mathbf{k}\cdot\bar{\mathbf{R}})}\mathbf{J}^{(2)}(\bar{\mathbf{k}}_1,\bar{\mathbf{k}}_2,\mathbf{q},\mathbf{k})\\&=e^{i\mathbf{k}\cdot\bar{\mathbf{R}}}\mathbf{J}^{(2)}\left[i\frac{\partial}{\partial\mathbf{s}_1},i\frac{\partial}{\partial\mathbf{s}_2},\bar{\mathbf{r}},\mathbf{k}\right]\delta(\mathbf{s}_1)\delta(\mathbf{s}_2) \ .\end{aligned} \tag{2.34}$$

We have made a change of variables,

$$\mathbf{s}_1=\mathbf{r}_1-\mathbf{r}_1' \ , \tag{2.35}$$

$$\mathbf{s}_2=\mathbf{r}_2-\mathbf{r}_2' \ , \tag{2.36}$$

$$\bar{\mathbf{r}}=\tfrac{1}{2}(\mathbf{r}_1+\mathbf{r}_1'-\mathbf{r}_2-\mathbf{r}_2') \ , \tag{2.37}$$

$$\bar{\mathbf{R}}=\tfrac{1}{4}(\mathbf{r}_1+\mathbf{r}_1'+\mathbf{r}_2+\mathbf{r}_2') \ , \tag{2.38}$$

and integrated over $\bar{\mathbf{k}}_1$ and $\bar{\mathbf{k}}_2$ by replacing them with differential operators acting on $\mathbf{s}_1$ and $\mathbf{s}_2$. We have defined

$$\mathbf{J}^{(2)}\left[i\frac{\partial}{\partial\mathbf{s}_1},i\frac{\partial}{\partial\mathbf{s}_2},\bar{\mathbf{r}},\mathbf{k}\right]=\int\frac{d^3q}{(2\pi)^3}e^{-i\mathbf{q}\cdot\bar{\mathbf{r}}}\mathbf{J}^{(2)}\left[i\frac{\partial}{\partial\mathbf{s}_1},i\frac{\partial}{\partial\mathbf{s}_2},\mathbf{q},\mathbf{k}\right] \ . \tag{2.39}$$

We consider a matrix element of the nonlocal current operator between the two-nucleon wave functions,

$$\begin{aligned}M_{fi}&=\int d^3r_1d^3r_2d^3r_1'd^3r_2'\psi_i^*(\mathbf{r}_1',\mathbf{r}_2')\langle\mathbf{r}_1'\mathbf{r}_2'|\mathbf{J}^{(2)}(\mathbf{k})|\mathbf{r}_1\mathbf{r}_2\rangle\psi_i(\mathbf{r}_1,\mathbf{r}_2)\\&=\int d^3s_1d^3s_2d^3\bar{r}\,d^3\bar{R}\,\psi_i^*(\bar{\mathbf{R}}+\tfrac{1}{2}\bar{\mathbf{r}}-\tfrac{1}{2}\mathbf{s}_1,\bar{\mathbf{R}}-\tfrac{1}{2}\bar{\mathbf{r}}-\tfrac{1}{2}\mathbf{s}_2)\\&\quad\times\left[e^{i\mathbf{k}\cdot\bar{\mathbf{R}}}\mathbf{J}^{(2)}\left[i\frac{\partial}{\partial\mathbf{s}_1},i\frac{\partial}{\partial\mathbf{s}_2},\bar{\mathbf{r}},\mathbf{k}\right]\delta(\mathbf{s}_1)\delta(\mathbf{s}_2)\right]\psi_i(\bar{\mathbf{R}}+\tfrac{1}{2}\bar{\mathbf{r}}+\tfrac{1}{2}\mathbf{s}_1,\bar{\mathbf{R}}-\tfrac{1}{2}\bar{\mathbf{r}}+\tfrac{1}{2}\mathbf{s}_2) \ .\end{aligned} \tag{2.40}$$

We integrate by parts to make the differential operators act on the initial and final wave functions. Then we can carry out integrations over $\mathbf{s}_1$ and $\mathbf{s}_2$ to yield

$$\begin{aligned}M_{fi}=\int d^3r_1d^3r_2\psi_f^*(\mathbf{r}_1,\mathbf{r}_2)e^{i\mathbf{k}\cdot\mathbf{R}}\\ \times\mathbf{J}^{(2)}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{r},\mathbf{k})\psi_i(\mathbf{r}_1,\mathbf{r}_2) \ ,\end{aligned} \tag{2.41}$$

where

$$\mathbf{R}=\tfrac{1}{2}(\mathbf{r}_1+\mathbf{r}_2) \tag{2.42}$$

is the overall c.m. position of the two nucleons and $\mathbf{p}_1$ and $\mathbf{p}_2$ are the operators

$$\mathbf{p}_i=-\tfrac{1}{2}i(\vec{\nabla}_i-\overleftarrow{\nabla}_i) \ . \tag{2.43}$$

Equation (2.41) defines the momentum-dependent current operator

$$\begin{aligned}\mathbf{J}^{(2)}(\mathbf{k})&=e^{i\mathbf{k}\cdot\mathbf{R}}\mathbf{J}^{(2)}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{r},\mathbf{k})\\&=e^{i\mathbf{k}\cdot\mathbf{R}}\int\frac{d^3q}{(2\pi)^3}e^{-i\mathbf{q}\cdot\mathbf{r}}\mathbf{J}^{(2)}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{q},\mathbf{k}) \ .\end{aligned} \tag{2.44}$$

In Eq. (2.43) $\vec{\nabla}_i$ differentiates the right-hand side wave function and $\overleftarrow{\nabla}_i$ the left-hand side wave function. It is to be noted that the momenta defined in Eq. (2.43) never act on $\mathbf{r}$ or $\mathbf{R}$ inside the current operator.

C. Minimal substitution

In the Introduction we observed that the minimal substitution of the momentum operator in the potential yields an electromagnetic interaction. However, for the isospin-dependent potential there occurs an ambiguity in this procedure. To illustrate this we consider the problem of one nucleon embedded in the potential

$$V(\mathbf{p},\mathbf{r})=\mathbf{p}^2\boldsymbol{\tau}\cdot\boldsymbol{\phi}(\mathbf{r}) \ . \tag{2.45}$$

The momentum operator $\mathbf{p}$ is defined as above,

$$\mathbf{p}=-\tfrac{1}{2}i(\vec{\nabla}-\overleftarrow{\nabla}) . \qquad (2.46)$$

Partial integration without surface terms allows us to replace $\overleftarrow{\nabla}$ with the operator acting on its right-hand side, thus generating ordered operator products (Weyl-ordered products), e.g.,

$$\mathbf{p}\varphi(\mathbf{r})=\tfrac{1}{2}[\mathbf{p}_u\varphi(\mathbf{r})+\varphi(\mathbf{r})\mathbf{p}_u] , \qquad (2.47)$$

$$\mathbf{p}^2\varphi(\mathbf{r})=\tfrac{1}{4}[\mathbf{p}_u^2\varphi(\mathbf{r})+2\mathbf{p}_u\varphi(\mathbf{r})\cdot\mathbf{p}_u+\varphi(\mathbf{r})\mathbf{p}_u^2] , \qquad (2.48)$$

where $\varphi(\mathbf{r})=\boldsymbol{\tau}\cdot\boldsymbol{\phi}(\mathbf{r})$ and $\mathbf{p}_u$ is the unsymmetrized momentum operator (the momentum operator in the usual sense). The symmetrized momentum operator is very convenient because it commutes with $\varphi(\mathbf{r})$ so that we can place it on either side of $\varphi(\mathbf{r})$, i.e.,

$$\mathbf{p}\varphi(\mathbf{r})=\varphi(\mathbf{r})\mathbf{p} , \qquad (2.49)$$

$$\mathbf{p}^2\varphi(\mathbf{r})=\varphi(\mathbf{r})\mathbf{p}^2=\mathbf{p}\cdot\varphi(\mathbf{r})\mathbf{p} . \qquad (2.50)$$

If, however, one is considering a commutator of a symmetrized ordered product with some function $g(\mathbf{r})$, $\mathbf{p}$ acts on it. For (2.47) or (2.48), for example, we have

$$[\mathbf{p}\varphi(\mathbf{r}),g(\mathbf{r})]=-i\varphi(\mathbf{r})\nabla g(\mathbf{r}) , \qquad (2.51)$$

$$[\mathbf{p}^2\varphi(\mathbf{r}),g(\mathbf{r})]=-2i\mathbf{p}\cdot[\varphi(\mathbf{r})\nabla g(\mathbf{r})] . \qquad (2.52)$$

In the left-hand side the momentum operator is symmetrized with respect to $\varphi(\mathbf{r})$, whereas in the right-hand side the momentum is symmetrized with respect to $\varphi(\mathbf{r})g(\mathbf{r})$. For later applications, it is important to note that the anticommutators of symmetrized products with $g(\mathbf{r})$ become

$$\{\mathbf{p}\varphi(\mathbf{r}),g(\mathbf{r})\}=2\mathbf{p}[\varphi(\mathbf{r})g(\mathbf{r})] , \qquad (2.53)$$

$$\{\mathbf{p}^2\varphi(\mathbf{r}),g(\mathbf{r})\}=2\mathbf{p}^2[\varphi(\mathbf{r})g(\mathbf{r})] -\tfrac{1}{2}\varphi(\mathbf{r})\nabla^2 g(\mathbf{r}) . \qquad (2.54)$$

This implies that the anticommutator of $V(\mathbf{p},\mathbf{r})$ with $g(\mathbf{r})$ defines an effective potential

$$\hat{V}(\mathbf{p},\mathbf{r})=V(\mathbf{p},\mathbf{r})-\tfrac{1}{4}\varphi(\mathbf{r})\nabla^2 , \qquad (2.55)$$

where ∇ is to act only on $g(\mathbf{r})$, while $\mathbf{p}$ is to be ordered according to the Weyl rule.

In (2.48) we follow the standard procedure and make a replacement

$$\mathbf{p}_u\rightarrow\mathbf{p}_u-e\,\mathbf{A}(\mathbf{r}) \qquad (2.56)$$

to get the electromagnetic interaction

$$-\tfrac{1}{4}\{(\mathbf{p}_u\cdot\mathbf{A}+\mathbf{A}\cdot\mathbf{p}_u),\{e,\varphi\}\}-\tfrac{1}{4}i\nabla\cdot\mathbf{A}[e,\varphi] =-\mathbf{p}\cdot\mathbf{A}\{e,\varphi\}-\tfrac{1}{4}i\nabla\cdot\mathbf{A}[e,\varphi] . \qquad (2.57)$$

The last term is present due to the uncommutativity of e with φ.

Next let us rewrite (2.48) as

$$\mathbf{p}^2\varphi(\mathbf{r})=\tfrac{1}{2}[\mathbf{p}_u^2\varphi(\mathbf{r})+\varphi(\mathbf{r})\mathbf{p}_u^2]+\tfrac{1}{4}\nabla^2\varphi(\mathbf{r}) . \qquad (2.58)$$

The minimal replacement of $\mathbf{p}_u$ in this potential produces

$$-\tfrac{1}{4}\{(\mathbf{p}_u\cdot\mathbf{A}+\mathbf{A}\cdot\mathbf{p}_u),\{e,\varphi\}\}=-\mathbf{p}\cdot\mathbf{A}\{e,\varphi\} \qquad (2.59)$$

which lacks the term proportional to $[e,\varphi]$. The reason for this discrepancy is obvious: The ∇ term in (2.58) is written as

$$\nabla^2\varphi(\mathbf{r})=-\mathbf{p}_u^2\varphi(\mathbf{r})+2\mathbf{p}_u\varphi(\mathbf{r})\cdot\mathbf{p}_u-\varphi(\mathbf{r})\mathbf{p}_u^2 . \qquad (2.60)$$

Thus this can be interpreted as a momentum-dependent term, although it commutes with any function of $\mathbf{r}$. The minimal substitution indeed generates the missing $[e,\varphi]$ term. From this exercise we learned that for isospin-dependent potentials the electromagnetic interaction from the minimal substitution is not uniquely determined. This nonuniqueness is attributed to the fact that the momentum representation of the potential has arbitrariness. We also learned that for isospin-dependent potentials the electromagnetic field couples to ∇.

D. Constraint on the exchange currents

Following exactly the same procedure as in Sec. II B, we represent the nuclear interaction in the form of the momentum-dependent potential

$$V=\int\frac{d^3q}{(2\pi)^3}e^{-i\mathbf{q}\cdot\mathbf{r}}V(\mathbf{p}_1,\mathbf{p}_2,\mathbf{q}) =V(\mathbf{p}_1,\mathbf{p}_2,\mathbf{r}) . \qquad (2.61)$$

The operator $\mathbf{p}_i$ is again symmetrized within the potential and does not differentiate $\mathbf{r}$ in the potential. In the presence of the external electromagnetic field the differential operators acting on the wave functions should be replaced by

$$\vec{\nabla}_i\rightarrow\vec{\nabla}_i-i\vec{e}_i\,\mathbf{A}(\mathbf{r}_i) , \qquad (2.62)$$

$$\overleftarrow{\nabla}_i\rightarrow\overleftarrow{\nabla}_i+i\overleftarrow{e}_i\,\mathbf{A}(\mathbf{r}_i) , \qquad (2.63)$$

where $\vec{e}_i$ and $\overleftarrow{e}_i$ are the charge operators which should be placed at the right-hand side and left-hand side of $\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$, respectively. The minimal replacement of the symmetrized momentum operators becomes

$$\mathbf{p}_i\rightarrow\mathbf{p}_i-\hat{e}_i\,\mathbf{A}(\mathbf{r}_i) , \qquad (2.64)$$

where $\hat{e}_i$ is the symmetrized charge operator

$$\hat{e}_i=\tfrac{1}{2}(\vec{e}_i+\overleftarrow{e}_i) . \qquad (2.65)$$

The minimal replacement in the momentum-dependent potential produces the electromagnetic interaction

$$H_{\rm em}^{\rm min}=V(\mathbf{p}_1-\hat{e}_1\,\mathbf{A}(\mathbf{r}_1),\mathbf{p}_2-\hat{e}_2\,\mathbf{A}(\mathbf{r}_2),\mathbf{r}) -V(\mathbf{p}_1,\mathbf{p}_2,\mathbf{r}) . \qquad (2.66)$$

As emphasized in the foregoing subsection, there exists an ambiguity in the minimal replacement. However, it is also true that once we make minimal replacement everywhere including ∇, the total electromagnetic interaction is the same irrespectively of how we express the potential. It is desirable to distinguish two types, $\mathbf{p}$ dependence and ∇ dependence, by imposing some criterion. To find this we rewrite the divergence, (2.26), as

$$\nabla_x \cdot \mathbf{J}^V(\mathbf{x}) = -\tfrac{1}{2}i \sum_i [\{e_i, V\}, \delta(\mathbf{x}-\mathbf{r}_i)] + \tfrac{1}{2}i \sum_i \{[e_i, V], \delta(\mathbf{x}-\mathbf{r}_i)\} . \tag{2.67}$$

We readily find that currents coming from V^τ can be classified according to their isospin structure. The first term of (2.67) has the isospin dependence

$$\{e_i, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2\}$$

and the second term

$$[e_i, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2] .$$

Since the first term takes the form

$$-\tfrac{1}{2}i \sum_i \{e_i, [V, \delta(\mathbf{x}-\mathbf{r}_i)]\} , \tag{2.68}$$

it is clear that only the explicit momentum dependence of V is responsible for the nonvanishing divergence. On account of this fact, we demand that the divergence of the minimal current from the explicit $\mathbf{p}$ dependence should be

$$\nabla_x \cdot \mathbf{J}^{\min}(\mathbf{x}) = -\tfrac{1}{2}i \sum_i [\{e_i, V\}, \delta(\mathbf{x}-\mathbf{r}_i)] , \tag{2.69}$$

and split $\mathbf{J}^V(\mathbf{x})$ as

$$\mathbf{J}^V(\mathbf{x}) = \mathbf{J}^{\min}(\mathbf{x}) + \mathbf{J}^\tau(\mathbf{x}) . \tag{2.70}$$

Then the remaining current $\mathbf{J}^\tau$ must satisfy

$$\nabla_x \cdot \mathbf{J}^\tau(\mathbf{x}) = \tfrac{1}{2}e(\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2)_z \times \{V^\tau, \delta(\mathbf{x}-\mathbf{r}_1) - \delta(\mathbf{x}-\mathbf{r}_2)\} . \tag{2.71}$$

For a local charge-exchange potential V^τ we assume

$$\mathbf{J}^\tau(\mathbf{x}) = e(\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2)_z V^\tau \boldsymbol{\xi}(\mathbf{x}) \tag{2.72}$$

and obtain the condition

$$\nabla_x \cdot \boldsymbol{\xi}(\mathbf{x}) = \delta(\mathbf{x}-\mathbf{r}_1) - \delta(\mathbf{x}-\mathbf{r}_2) . \tag{2.73}$$

This is given by Osborn and Foldy[22] and discussed by Heller.[23] The Sachs current (1.4) is an example which satisfies this condition. In fact, with the aid of the formula

$$\mathbf{r} \cdot \nabla_x \int_0^1 ds\, \delta(\mathbf{x} - s\mathbf{r}_1 - (1-s)\mathbf{r}_2) = -\delta(\mathbf{x}-\mathbf{r}_1) + \delta(\mathbf{x}-\mathbf{r}_2) , \tag{2.74}$$

we can verify that $\mathbf{J}^{\rm Sachs}(\mathbf{x})$ satisfies the required condition. The constraint (2.71) is a generalization of that of Osborn and Foldy. When the potential has momentum dependence, the symmetrization in Eq. (2.71) is crucial, as we shall soon see.

In the photon momentum space, Eq. (2.71) becomes

$$\mathbf{k} \cdot \mathbf{J}^\tau(\mathbf{k}) = \tfrac{1}{2}ie(\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2)_z \{V^\tau, e^{i\mathbf{k}\cdot\mathbf{r}_1} - e^{i\mathbf{k}\cdot\mathbf{r}_2}\} . \tag{2.75}$$

Further we go to the $\mathbf{q}$ space,

$$e^{i\mathbf{k}\cdot\mathbf{R}} \mathbf{k} \cdot \mathbf{J}^\tau(\mathbf{p}_1, \mathbf{p}_2, \mathbf{q}, \mathbf{k}) = \int d^3r\, e^{i\mathbf{q}\cdot\mathbf{r}} \mathbf{k} \cdot \mathbf{J}^\tau(\mathbf{k}) . \tag{2.76}$$

In Fourier-transforming the right-hand side of Eq. (2.75), a caution is necessary because V^τ and exponentials do not commute. As is shown in the preceding subsection, we have to rearrange operator ordering and define V^τ such that

$$\tfrac{1}{2}\{V^\tau(\mathbf{p}_1, \mathbf{p}_2, \mathbf{r}), e^{i\mathbf{k}\cdot\mathbf{r}_1} - e^{i\mathbf{k}\cdot\mathbf{r}_2}\} = \hat{V}^\tau(\mathbf{p}_1, \mathbf{p}_2, \mathbf{r})(e^{i\mathbf{k}\cdot\mathbf{r}_1} - e^{i\mathbf{k}\cdot\mathbf{r}_2}) . \tag{2.77}$$

The momenta in the left-hand side, which are symmetrized within the V^τ and are acting on the exponentials as well as on the wave functions, are reshuffled to give a new potential $\hat{V}^\tau$ in which the momenta operate only on the wave functions. After this procedure, we define the Fourier transform of the potential as

$$\hat{V}^\tau(\mathbf{p}_1, \mathbf{p}_2, \mathbf{q}) = \int d^3r\, e^{i\mathbf{q}\cdot\mathbf{r}} \hat{V}^\tau(\mathbf{p}_1, \mathbf{p}_2, \mathbf{r}) , \tag{2.78}$$

and compute

$$\tfrac{1}{2}\int d^3r\, e^{i\mathbf{q}\cdot\mathbf{r}} \{V^\tau, e^{i\mathbf{k}\cdot\mathbf{r}_1}\} = e^{i\mathbf{k}\cdot\mathbf{R}} \hat{V}^\tau(\mathbf{p}_1, \mathbf{p}_2, \mathbf{q} + \tfrac{1}{2}\mathbf{k}) , \tag{2.79}$$

$$\tfrac{1}{2}\int d^3r\, e^{i\mathbf{q}\cdot\mathbf{r}} \{V^\tau, e^{i\mathbf{k}\cdot\mathbf{r}_2}\} = e^{i\mathbf{k}\cdot\mathbf{R}} \hat{V}^\tau(\mathbf{p}_1, \mathbf{p}_2, \mathbf{q} - \tfrac{1}{2}\mathbf{k}) . \tag{2.80}$$

It should be emphasized that $\hat{V}^\tau$'s cannot be obtained from the Fourier transform

$$V^\tau(\mathbf{p}_1, \mathbf{p}_2, \mathbf{q}) = \int d^3r\, e^{i\mathbf{q}\cdot\mathbf{r}} V^\tau(\mathbf{p}_1, \mathbf{p}_2, \mathbf{r}) \tag{2.81}$$

by shifting the momentum $\mathbf{q}$ to $\mathbf{q} \pm \frac{1}{2}\mathbf{k}$. Equation (2.75) is cast in the form

$$\mathbf{k} \cdot \mathbf{J}^\tau(\mathbf{p}_1, \mathbf{p}_2, \mathbf{q}, \mathbf{k}) = ie(\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2)_z [\hat{V}^\tau(\mathbf{p}_1, \mathbf{p}_2, \mathbf{q} + \tfrac{1}{2}\mathbf{k}) - \hat{V}^\tau(\mathbf{p}_1, \mathbf{p}_2, \mathbf{q} - \tfrac{1}{2}\mathbf{k})] . \tag{2.82}$$

In the following sections we do not use the notation $\hat{V}^\tau$ but retain V^τ for simplicity. However, the precise definition of the Fourier transform should be recalled whenever we encounter potentials which contain momentum operators in quadratic and higher powers. We shall also suppress the momentum operators in the argument to simplify the notation.

III. EXCHANGE CURRENTS FROM ISOVECTOR POTENTIALS

As mentioned in the Introduction, the Sachs current is not successful because it does not reproduce the well-established one-pion-exchange current. In this section we propose a new method of constructing the exchange-current operator from a given charge-exchange potential. Our basic tool is the gauge-invariance constraint on the current $\mathbf{J}^\tau$, Eq. (2.82). Its solution is with great arbitrariness and is dependent on the particular form of V^τ. First we consider the central local potential $V_C^\tau(q)$. Equation (2.82) gives us the condition

$$\mathbf{k} \cdot \mathbf{J}_C^\tau(\mathbf{q}, \mathbf{k}) = ie(\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2)_z [V_C^\tau(|\mathbf{q} + \tfrac{1}{2}\mathbf{k}|) - V_C^\tau(|\mathbf{q} - \tfrac{1}{2}\mathbf{k}|)] . \tag{3.1}$$

The quantity in the square brackets has the structure

$$V_C^\tau(|\mathbf{q}+\tfrac{1}{2}\mathbf{k}|)-V_C^\tau(|\mathbf{q}-\tfrac{1}{2}\mathbf{k}|)=\mathbf{q}\cdot\mathbf{k}F(q,k,(\hat{q}\cdot\hat{k})^2) \quad (3.2)$$

so that the current

$$\mathbf{J}_C^\tau(\mathbf{q},\mathbf{k})=ie(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z\mathbf{q}F(q,k,(\hat{q}\cdot\hat{k})^2)$$
$$=ie(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z\frac{\mathbf{q}}{\mathbf{q}\cdot\mathbf{k}}\times[V_C^\tau(|\mathbf{q}+\tfrac{1}{2}\mathbf{k}|)-V_C^\tau(|\mathbf{q}-\tfrac{1}{2}\mathbf{k}|)] \quad (3.3)$$

indeed satisfies the required condition. Its physical meaning is elucidated by the fact that the obtained current is also deduced from $V_C^\tau(q)$ by using its nonlocal representation and the minimal-substitution prescription as is proved in a previous paper.[18]

For noncentral potentials a modification of the above prescription is necessary. To see this we consider the tensor-type interaction

$$V_T^\tau(\mathbf{q})=\boldsymbol{\sigma}_1\cdot\mathbf{q}\boldsymbol{\sigma}_2\cdot\mathbf{q}V_T^\tau(q) \ . \quad (3.4)$$

In the position space it is written in the form

$$V_T^\tau(\mathbf{r})=\boldsymbol{\sigma}_1\cdot\nabla_1\boldsymbol{\sigma}_2\cdot\nabla_2V_T^\tau(r)$$
$$=\boldsymbol{\sigma}_1\cdot(\vec{\nabla}_1+\overleftarrow{\nabla}_1)\boldsymbol{\sigma}_2\cdot(\vec{\nabla}_2+\overleftarrow{\nabla}_2)V_T^\tau(r) \ . \quad (3.5)$$

In the external electromagnetic field we should make the replacement

$$\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2(\vec{\nabla}_1+\overleftarrow{\nabla}_1)\rightarrow\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2(\vec{\nabla}_1+\overleftarrow{\nabla}_1)+e(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z\mathbf{A}(\mathbf{r}_1) \ , \quad (3.6)$$

$$\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2(\vec{\nabla}_2+\overleftarrow{\nabla}_2)\rightarrow\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2(\vec{\nabla}_2+\overleftarrow{\nabla}_2)-e(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z\mathbf{A}(\mathbf{r}_2) \ . \quad (3.7)$$

We immediately obtain the current

$$\mathbf{J}_T^{\tau,\mathrm{I}}(\mathbf{x})=-e(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z[\delta(\mathbf{x}-\mathbf{r}_1)\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2\cdot\nabla V_T^\tau(r)+\delta(\mathbf{x}-\mathbf{r}_2)\boldsymbol{\sigma}_1\cdot\nabla V_T^\tau(r)\boldsymbol{\sigma}_2] \ . \quad (3.8)$$

In the $\mathbf{q}$ space, it becomes

$$\mathbf{J}_T^{\tau,\mathrm{I}}(\mathbf{q},\mathbf{k})=ie(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z\times[\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2\cdot(\mathbf{q}+\tfrac{1}{2}\mathbf{k})V_T^\tau(|\mathbf{q}+\tfrac{1}{2}\mathbf{k}|)+\boldsymbol{\sigma}_1\cdot(\mathbf{q}-\tfrac{1}{2}\mathbf{k})V_T^\tau(|\mathbf{q}-\tfrac{1}{2}\mathbf{k}|)\boldsymbol{\sigma}_2] \ . \quad (3.9)$$

The current obtained in this way does not satisfy the condition (2.82). The current $\mathbf{J}_T^\tau(\mathbf{q},\mathbf{k})$ must satisfy

$$\mathbf{k}\cdot\mathbf{J}_T^\tau(\mathbf{q},\mathbf{k})=ie(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z[V_T^\tau(\mathbf{q}+\tfrac{1}{2}\mathbf{k})-V_T^\tau(\mathbf{q}-\tfrac{1}{2}\mathbf{k})]$$
$$=ie(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z[\boldsymbol{\sigma}_1\cdot(\mathbf{q}+\tfrac{1}{2}\mathbf{k})\boldsymbol{\sigma}_2\cdot(\mathbf{q}+\tfrac{1}{2}\mathbf{k})V_T^\tau(|\mathbf{q}+\tfrac{1}{2}\mathbf{k}|)-\boldsymbol{\sigma}_1\cdot(\mathbf{q}-\tfrac{1}{2}\mathbf{k})\boldsymbol{\sigma}_2\cdot(\mathbf{q}-\tfrac{1}{2}\mathbf{k})V_T^\tau(|\mathbf{q}-\tfrac{1}{2}\mathbf{k}|)] \ . \quad (3.10)$$

We split the total $\mathbf{J}_T^\tau(\mathbf{q},\mathbf{k})$ into two parts:

$$\mathbf{J}_T^\tau(\mathbf{q},\mathbf{k})=\mathbf{J}_T^{\tau,\mathrm{I}}(\mathbf{q},\mathbf{k})+\mathbf{J}_T^{\tau,\mathrm{II}}(\mathbf{q},\mathbf{k}) \ . \quad (3.11)$$

The gauge-invariance condition then turns out to be

$$\mathbf{k}\cdot\mathbf{J}_T^{\tau,\mathrm{II}}(\mathbf{q},\mathbf{k})=ie(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z\boldsymbol{\sigma}_1\cdot(\mathbf{q}-\tfrac{1}{2}\mathbf{k})\boldsymbol{\sigma}_2\cdot(\mathbf{q}+\tfrac{1}{2}\mathbf{k})[V_T^\tau(|\mathbf{q}+\tfrac{1}{2}\mathbf{k}|)-V_T^\tau(|\mathbf{q}-\tfrac{1}{2}\mathbf{k}|)] \ . \quad (3.12)$$

Now one can apply the same procedure as the central potential to give

$$\mathbf{J}_T^{\tau,\mathrm{II}}(\mathbf{q},\mathbf{k})=ie(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z\boldsymbol{\sigma}_1\cdot(\mathbf{q}-\tfrac{1}{2}\mathbf{k})\boldsymbol{\sigma}_2\cdot(\mathbf{q}+\tfrac{1}{2}\mathbf{k})\frac{\mathbf{q}}{\mathbf{k}\cdot\mathbf{q}}[V_T^\tau(|\mathbf{q}+\tfrac{1}{2}\mathbf{k}|)-V_T^\tau(|\mathbf{q}-\tfrac{1}{2}\mathbf{k}|)] \ . \quad (3.13)$$

As the third example we consider the spin-orbit potential

$$V_{LS}^\tau(\mathbf{q})=i\mathbf{S}\cdot\mathbf{q}\times\mathbf{p}_{12}V_{LS}^\tau(q) \ , \quad (3.14)$$

where

$$\mathbf{S}=\tfrac{1}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)$$

is the total spin operator and

$$\mathbf{p}_{12}=\tfrac{1}{2}(\mathbf{p}_1-\mathbf{p}_2)$$

is the relative momentum. In the $\mathbf{r}$ space, the spin-orbit potential becomes

$$V_{LS}^\tau(\mathbf{r})=-\mathbf{S}\cdot\nabla V_{LS}^\tau(r)\times\mathbf{p}_{12} \ . \quad (3.15)$$

In the explicit $\mathbf{p}$ dependence we make the substitution

$$\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\mathbf{p}_i\rightarrow\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\mathbf{p}_i-\tfrac{1}{2}\{e_i,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\}\mathbf{A}(\mathbf{r}_i) \quad (3.16)$$

and get the minimal current

$$\mathbf{J}_{LS}^{\min}(\mathbf{x})=-\tfrac{1}{4}\{e_1,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\}\mathbf{S}\times\nabla V_{LS}^\tau(r)\delta(\mathbf{x}-\mathbf{r}_1)+(1\leftrightarrow2) \ . \quad (3.17)$$

Next we rewrite (3.15) as

$$V_{LS}^\tau(\mathbf{r})=-\tfrac{1}{2}\mathbf{S}\cdot(-\vec{\nabla}_1-\overleftarrow{\nabla}_1+\vec{\nabla}_2+\overleftarrow{\nabla}_2)V_{LS}^\tau(r)\times\mathbf{p}_{12} \ . \quad (3.18)$$

In this form the minimal coupling to ∇ is induced with the result

$$\mathbf{J}_{LS}^{\tau,\mathrm{I}}(\mathbf{x})=\tfrac{1}{2}e(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z\mathbf{S}\times\mathbf{p}_{12}V_{LS}^{\tau}(r)[\delta(\mathbf{x}-\mathbf{r}_1)+\delta(\mathbf{x}-\mathbf{r}_2)]\ . \tag{3.19}$$

We again find that the current $\mathbf{J}_{LS}^{\tau}(\mathbf{q},\mathbf{k})$ is the sum of two terms,

$$\mathbf{J}_{LS}^{\tau}(\mathbf{q},\mathbf{k})=\mathbf{J}_{LS}^{\tau,\mathrm{I}}(\mathbf{q},\mathbf{k})+\mathbf{J}_{LS}^{\tau,\mathrm{II}}(\mathbf{q},\mathbf{k})\ , \tag{3.20}$$

where the first term,

$$\mathbf{J}_{LS}^{\tau,\mathrm{I}}(\mathbf{q},\mathbf{k})=\tfrac{1}{2}e(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z\mathbf{S}\times\mathbf{p}_{12}[V_{LS}^{\tau}(|\mathbf{q}+\tfrac{1}{2}\mathbf{k}|)+V_{LS}^{\tau}(|\mathbf{q}-\tfrac{1}{2}\mathbf{k}|)]\ , \tag{3.21}$$

is the Fourier transform of (3.19). Invoking Eq. (2.82) we have the gauge-invariance condition

$$\begin{aligned}\mathbf{k}\cdot\mathbf{J}_{LS}^{\tau}(\mathbf{q},\mathbf{k})&=ie(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z[V_{LS}^{\tau}(\mathbf{q}+\tfrac{1}{2}\mathbf{k})-V_{LS}^{\tau}(\mathbf{q}-\tfrac{1}{2}\mathbf{k})]\\&=-e(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z[\mathbf{S}\cdot(\mathbf{q}+\tfrac{1}{2}\mathbf{k})\times\mathbf{p}_{12}V_{LS}^{\tau}(|\mathbf{q}+\tfrac{1}{2}\mathbf{k}|)-\mathbf{S}\cdot(\mathbf{q}-\tfrac{1}{2}\mathbf{k})\times\mathbf{p}_{12}V_{LS}^{\tau}(|\mathbf{q}-\tfrac{1}{2}\mathbf{k}|)]\ .\end{aligned} \tag{3.22}$$

Consequently, $\mathbf{J}_{LS}^{\tau,\mathrm{II}}$ is constrained by

$$\mathbf{k}\cdot\mathbf{J}_{LS}^{\tau,\mathrm{II}}(\mathbf{q},\mathbf{k})=-e(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z\mathbf{S}\cdot\mathbf{q}\times\mathbf{p}_{12}[V_{LS}^{\tau}(|\mathbf{q}+\tfrac{1}{2}\mathbf{k}|)-V_{LS}^{\tau}(|\mathbf{q}-\tfrac{1}{2}\mathbf{k}|)]\ . \tag{3.23}$$

We can solve this to obtain

$$\mathbf{J}_{LS}^{\tau,\mathrm{II}}(\mathbf{q},\mathbf{k})=-e(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z\mathbf{S}\cdot\mathbf{q}\times\mathbf{p}_{12}\frac{\mathbf{q}}{\mathbf{k}\cdot\mathbf{q}}[V_{LS}^{\tau}(|\mathbf{q}+\tfrac{1}{2}\mathbf{k}|)-V_{LS}^{\tau}(|\mathbf{q}-\tfrac{1}{2}\mathbf{k}|)]\ . \tag{3.24}$$

From these three examples a general prescription to derive exchange-current operators from a given nucleon-nucleon potential emerges.

(i) First we rewrite the potential in terms of the symmetrized momenta:

$$\mathbf{p}_1=-\tfrac{1}{2}i(\vec{\nabla}_1-\overleftarrow{\nabla}_1)\ , \tag{3.25}$$

$$\mathbf{p}_2=-\tfrac{1}{2}i(\vec{\nabla}_2-\overleftarrow{\nabla}_2)\ , \tag{3.26}$$

$$\nabla=\begin{cases}-\vec{\nabla}_1-\overleftarrow{\nabla}_1\ ,\\ \vec{\nabla}_2+\overleftarrow{\nabla}_2\ ,\\ \tfrac{1}{2}(-\vec{\nabla}_1-\overleftarrow{\nabla}_1+\vec{\nabla}_2+\overleftarrow{\nabla}_2)\ .\end{cases} \tag{3.27}$$

(ii) Next we make a minimal replacement everywhere:

$$\vec{\nabla}_i\rightarrow\vec{\nabla}_i-ie_i\,\mathbf{A}(\mathbf{r}_i)\ ,$$

$$\overleftarrow{\nabla}_1\rightarrow\overleftarrow{\nabla}_i+ie_i\,\mathbf{A}(\mathbf{r}_i)\ .$$

We classify the obtained currents into $\mathbf{J}^{\min}$ and $\mathbf{J}^{\tau,\mathrm{I}}$ according to their isospin structure: $\mathbf{J}^{\min}$ is proportional to $\{e_i,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\}$ and $\mathbf{J}^{\tau,\mathrm{I}}$ is proportional to $e(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z$.

(iii) We calculate the divergence of $\mathbf{J}^{\tau,\mathrm{I}}$ and subtract it from the divergence of $\mathbf{J}^{\tau}$, Eq. (2.82), thereby giving

$$\mathbf{k}\cdot\mathbf{J}^{\tau,\mathrm{II}}(\mathbf{q},\mathbf{k})=ie(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_zG(\mathbf{q},\mathbf{k})\times[V^{\tau}(|\mathbf{q}+\tfrac{1}{2}\mathbf{k}|)-V^{\tau}(|\mathbf{q}-\tfrac{1}{2}\mathbf{k}|)]\ , \tag{3.28}$$

where $V^{\tau}(q)$ is some remaining radial function. In general, $G(\mathbf{q},\mathbf{k})$, which characterizes the given potential, depends on spin and momentum operators of the nucleons. This constraint is solved for

$$\mathbf{J}^{\tau,\mathrm{II}}(\mathbf{q},\mathbf{k})=ie(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_zG(\mathbf{q},\mathbf{k})\frac{\mathbf{q}}{\mathbf{k}\cdot\mathbf{q}}\times[V^{\tau}(|\mathbf{q}+\tfrac{1}{2}\mathbf{k}|)-V^{\tau}(|\mathbf{q}-\tfrac{1}{2}\mathbf{k}|)]\ . \tag{3.29}$$

The sum

$$\mathbf{J}^{V}=\mathbf{J}^{\min}+\mathbf{J}^{\tau,\mathrm{I}}+\mathbf{J}^{\tau,\mathrm{II}} \tag{3.30}$$

is our result. The isospin-independent potential V^0 contributes only to $\mathbf{J}^{\min}$.

IV. ONE-BOSON-EXCHANGE MODEL

A. One-pion exchange

Our method of constructing the interaction current devised in the previous section is quite general and is applicable to any nuclear interaction models. In this section we illustrate its usefulness by employing the particular field-theoretic model, the one-boson exchange model. Among other things, it is essentially important to show that our approach is consistent with one-pion exchange, the hallmark of the nuclear theory. Since the one-pion exchange potential is of tensor type, we can substitute

$$V_{\mathrm{OPE}}^{\tau}(q)=-\frac{f^2}{\mu^2}\frac{1}{\mathbf{q}^2+\mu^2} \tag{4.1}$$

into Eqs. (3.9) and (3.13). We can easily see that

$$\mathbf{J}^{\tau,\mathrm{I}}_{\mathrm{OPE}}(\mathbf{q},\mathbf{k})=-ie\frac{f^2}{\mu^2}(\tau_1\times\tau_2)_z\left[\sigma_1\sigma_2\cdot(\mathbf{q}+\tfrac{1}{2}\mathbf{k})\frac{1}{(\mathbf{q}+\frac{1}{2}\mathbf{k})^2+\mu^2}+\sigma_1\cdot(\mathbf{q}-\tfrac{1}{2}\mathbf{k})\sigma_2\frac{1}{(\mathbf{q}-\frac{1}{2}\mathbf{k})^2+\mu^2}\right], \tag{4.2}$$

$$\mathbf{J}^{\tau,\mathrm{II}}_{\mathrm{OPE}}(\mathbf{q},\mathbf{k})=ie\frac{f^2}{\mu^2}(\tau_1\times\tau_2)_z\sigma_1\cdot(\mathbf{q}-\tfrac{1}{2}\mathbf{k})\sigma_2\cdot(\mathbf{q}+\tfrac{1}{2}\mathbf{k})\frac{2\mathbf{q}}{[(\mathbf{q}-\frac{1}{2}\mathbf{k})^2+\mu^2][(\mathbf{q}+\frac{1}{2}\mathbf{k})^2+\mu^2]}\ . \tag{4.3}$$

In deriving Eq. (4.3) we have used the relation

$$\frac{\mathbf{q}}{\mathbf{k}\cdot\mathbf{q}}\left[\frac{1}{(\mathbf{q}+\frac{1}{2}\mathbf{k})^2+\mu^2}-\frac{1}{(\mathbf{q}-\frac{1}{2}\mathbf{k})^2+\mu^2}\right]=-\frac{2\mathbf{q}}{[(\mathbf{q}-\frac{1}{2}\mathbf{k})^2+\mu^2][(\mathbf{q}+\frac{1}{2}\mathbf{k})^2+\mu^2]}\ . \tag{4.4}$$

It is satisfying to find that (4.2) coincides exactly with the contact current and (4.3) with the pionic current. In the position space, they return to (1.7) and (1.8), respectively.

Thus the requirement of gauge invariance can produce the pionic current which is induced by the electromagnetic interaction of pions. It is sometimes argued that the pionic current is one of the processes which cannot be described by the nucleonic degrees of freedom only. The above exercise implies the contrary. To give an insight into this result, we show that the pionic current is also induced by a minimal replacement in the local potential (4.1). To this end we write the Fourier transform of (4.1) as

$$V^{\tau}_{\mathrm{OPE}}(r)=-\frac{f^2}{\mu^2}\int\frac{d^3q}{(2\pi)^3}e^{-i\mathbf{q}\cdot\mathbf{r}}\frac{1}{\mathbf{q}^2+\mu^2}=-\frac{f^2}{\mu^2}\frac{1}{-\nabla_1^2+\mu^2}\delta(\mathbf{r}_1-\mathbf{r}_2)\ . \tag{4.5}$$

In this form we can apply our rule: We make minimal replacements in $\nabla_1=-\vec{\nabla}_1-\overleftarrow{\nabla}_1$, expand it in terms of e and obtain the interaction term

$$\begin{aligned}
&-e\frac{f^2}{\mu^2}\tau_1\cdot\tau_2\frac{1}{-\nabla_1^2+\mu^2}i(\vec{e}_1-\overleftarrow{e}_1)[\nabla_1\cdot\mathbf{A}(\mathbf{r}_1)+\mathbf{A}(\mathbf{r}_1)\cdot\nabla_1]\frac{1}{-\nabla_1^2+\mu^2}\delta(\mathbf{r}_1-\mathbf{r}_2)\\
&=e\frac{f^2}{\mu^2}(\tau_1\times\tau_2)_z(\nabla_1-\nabla_2)\cdot\int d^3x\ \delta(\mathbf{x}-\mathbf{r}_1)\frac{1}{-\nabla_x^2+\mu^2}\mathbf{A}(\mathbf{x})\frac{1}{-\nabla_x^2+\mu^2}\delta(\mathbf{x}-\mathbf{r}_2)\\
&=e\frac{f^2}{\mu^2}(\tau_1\times\tau_2)_z(\nabla_1-\nabla_2)\cdot\int d^3x\ \mathbf{A}(\mathbf{x})\left[\frac{1}{-\nabla_x^2+\mu^2}\delta(\mathbf{x}-\mathbf{r}_1)\right]\left[\frac{1}{-\nabla_x^2+\mu^2}\delta(\mathbf{x}-\mathbf{r}_2)\right]\\
&=e\frac{f^2}{\mu^2}(\tau_1\times\tau_2)_z(\nabla_1-\nabla_2)\cdot\int d^3x\ \mathbf{A}(\mathbf{x})\frac{e^{-\mu|\mathbf{x}-\mathbf{r}_1|}}{4\pi|\mathbf{x}-\mathbf{r}_1|}\frac{e^{-\mu|\mathbf{x}-\mathbf{r}_2|}}{4\pi|\mathbf{x}-\mathbf{r}_2|}\\
&=e\frac{f^2}{\mu^2}(\tau_1\times\tau_2)_z\int d^3x\ \mathbf{A}(\mathbf{x})\cdot\left\{\frac{e^{-\mu|\mathbf{x}-\mathbf{r}_1|}}{4\pi|\mathbf{x}-\mathbf{r}_1|}\nabla_x\frac{e^{-\mu|\mathbf{x}-\mathbf{r}_2|}}{4\pi|\mathbf{x}-\mathbf{r}_2|}-\frac{e^{-\mu|\mathbf{x}-\mathbf{r}_2|}}{4\pi|\mathbf{x}-\mathbf{r}_2|}\nabla_x\frac{e^{-\mu|\mathbf{x}-\mathbf{r}_1|}}{4\pi|\mathbf{x}-\mathbf{r}_1|}\right\}\ .
\end{aligned} \tag{4.6}$$

In the momentum space we get the current

$$ie\frac{f^2}{\mu^2}(\tau_1\times\tau_2)_z\frac{2\mathbf{q}}{[(\mathbf{q}-\frac{1}{2}\mathbf{k})^2+\mu^2][(\mathbf{q}+\frac{1}{2}\mathbf{k})^2+\mu^2]}\ . \tag{4.7}$$

Thus it is not surprising that we can extract the pionic current from the nuclear potential. Brown and Franklin[24] are the first who showed that the pion-exchange currents can be derived from the minimal substitution in the one-pion exchange potential.

B. Charged scalar meson exchange

Next we consider the charged scalar meson exchange. Any other mesons can be treated analogously. The meson-nucleon coupling is described by the Hamiltonian

$$H=\alpha\cdot\mathbf{p}+\beta m+g\beta\varphi\ , \tag{4.8}$$

where α and β are the Dirac matrices, m is the nucleon mass, $\varphi=\tau\cdot\phi$ with ϕ being the scalar-meson field, and g is the meson-nucleon coupling constant. Applying the Foldy-Wouthuysen transformation[25] to order $1/m^2$ we obtain the meson-nucleon coupling Hamiltonian

$$H_g=g\varphi-\frac{g}{2m^2}\mathbf{p}^2\varphi-\frac{g}{4m^2}\sigma\times\nabla\varphi\cdot\mathbf{p}\ . \tag{4.9}$$

It is understood that $\mathbf{p}$ is the symmetrized momentum operator as defined in Sec. II. Since this Hamiltonian starts from the term of order $O(1/m^0)$ the retardation in meson propagation contributes to the one-meson-exchange potential. In fact the meson energy is considered to be $O(1/m)$, and we can make an expansion of the meson propagator,

$$D(q_\mu^2)=\frac{1}{q_\mu^2+\mu^2}=D(\mathbf{q}^2)-q_0^2D'(\mathbf{q}^2)+\cdots\ . \tag{4.10}$$

The retardation in the meson propagation is a conceptually important problem and will be treated in detail in the next subsection. In this subsection we retain the first term, i.e., the instantaneous part only. The scalar-meson-exchange potential to order $1/m^2$ is then given by the sum of the central potential

$$V_C^\tau=\left[1-\frac{\mathbf{p}_1^2}{2m^2}-\frac{\mathbf{p}_2^2}{2m^2}\right]v(r) \tag{4.11}$$

and the spin-orbit potential

$$V_{LS}^\tau=-\frac{1}{4m^2}\sigma_1\cdot\nabla v(r)\times\mathbf{p}_1+(1\leftrightarrow 2)\ , \tag{4.12}$$

where

$$v(r)=-g^2\int\frac{d^3q}{(2\pi)^3}e^{-i\mathbf{q}\cdot\mathbf{r}}D(\mathbf{q}^2)=-g^2\frac{e^{-\mu r}}{4\pi r} \tag{4.13}$$

is the static potential.

We are now in the position to apply our method developed in Sec. III. First consider the central part. Since it depends quadratically on the momenta, we can use the results of Sec. II C to obtain

$$\mathbf{J}_C^{\min}(\mathbf{x})=-\{e_1,\tau_1\cdot\tau_2\}\frac{1}{2m^2}[\mathbf{p}_1\delta(\mathbf{x}-\mathbf{r}_1)v(r)-\tfrac{1}{2}\sigma_1\times\nabla_x\delta(\mathbf{x}-\mathbf{r}_1)v(r)]+(1\leftrightarrow 2) \tag{4.14}$$

and

$$\mathbf{J}_C^{\tau,\mathrm{I}}(\mathbf{x})=e(\tau_1\times\tau_2)_z\frac{1}{8m^2}\nabla_x\delta(\mathbf{x}-\mathbf{r}_1)v(r)+(1\leftrightarrow 2)\ . \tag{4.15}$$

The spin-dependent term arises because we made an interpretation $\mathbf{p}_i^2=(\sigma_i\cdot\mathbf{p}_i)^2$. To write down the gauge-invariance condition in momentum space, we apply Eq. (2.54) with the result

$$\{V_C^\tau,e^{i\mathbf{k}\cdot\mathbf{r}_1}-e^{i\mathbf{k}\cdot\mathbf{r}_2}\}=2\left[1-\frac{\mathbf{p}_1^2}{2m^2}-\frac{\mathbf{p}_2^2}{2m^2}-\frac{\mathbf{k}^2}{8m^2}\right](e^{i\mathbf{k}\cdot\mathbf{r}_1}-e^{i\mathbf{k}\cdot\mathbf{r}_2})\ . \tag{4.16}$$

After this reordering, Eq. (2.82) gives us the condition

$$\mathbf{k}\cdot\mathbf{J}_C^\tau(\mathbf{q},\mathbf{k})=ie(\tau_1\times\tau_2)_z\left[1-\frac{\mathbf{p}_1^2}{2m^2}-\frac{\mathbf{p}_2^2}{2m^2}-\frac{\mathbf{k}^2}{8m^2}\right][v(|\mathbf{q}+\tfrac{1}{2}\mathbf{k}|)-v(|\mathbf{q}-\tfrac{1}{2}\mathbf{k}|)]\ . \tag{4.17}$$

Evaluating the divergence of $\mathbf{J}_C^{\tau,\mathrm{I}}$ leads us to

$$\mathbf{k}\cdot\mathbf{J}_C^{\tau,\mathrm{I}}(\mathbf{q},\mathbf{k})=-ie(\tau_1\times\tau_2)_z\frac{\mathbf{k}^2}{8m^2}[v(|\mathbf{q}+\tfrac{1}{2}\mathbf{k}|)-v(|\mathbf{q}-\tfrac{1}{2}\mathbf{k}|)]\ . \tag{4.18}$$

We therefore obtain

$$\mathbf{k}\cdot\mathbf{J}_C^{\tau,\mathrm{II}}(\mathbf{q},\mathbf{k})=ie(\tau_1\times\tau_2)_z\left[1-\frac{\mathbf{p}_1^2}{2m^2}-\frac{\mathbf{p}_2^2}{2m^2}\right][v(|\mathbf{q}+\tfrac{1}{2}\mathbf{k}|)-v(|\mathbf{q}-\tfrac{1}{2}\mathbf{k}|)]\ . \tag{4.19}$$

It is important to see that the $\mathbf{k}^2$ term in (4.17) is rigorously canceled by the divergence of $\mathbf{J}_C^{\tau,\mathrm{I}}$. Thus we find

$$\mathbf{J}_C^{\tau,\mathrm{II}}(\mathbf{q},\mathbf{k})=ieg^2(\tau_1\times\tau_2)_z\left[1-\frac{\mathbf{p}_1^2}{2m^2}-\frac{\mathbf{p}_2^2}{2m^2}\right]\frac{2\mathbf{q}}{[(\mathbf{q}-\tfrac{1}{2}\mathbf{k})^2+\mu^2][(\mathbf{q}+\tfrac{1}{2}\mathbf{k})^2+\mu^2]}\ , \tag{4.20}$$

where we have used $v(q)=-g^2/(\mathbf{q}^2+\mu^2)$ and (4.4).

As for the spin-orbit potential the minimal substitution in $\mathbf{p}_i$ produces

$$\mathbf{J}_{LS}^{\min}(\mathbf{x})=-\{e_1,\tau_1\cdot\tau_2\}\frac{1}{8m^2}\delta(\mathbf{x}-\mathbf{r}_1)\sigma_1\times\nabla v(r)+(1\leftrightarrow 2)\ , \tag{4.21}$$

and the minimal substitution in ∇ produces

$$\mathbf{J}_{LS}^{\tau,\mathrm{I}}(\mathbf{x})=e(\tau_1\times\tau_2)_z\frac{1}{4m^2}\delta(\mathbf{x}-\mathbf{r}_1)\sigma_1\times\mathbf{p}_1v(r)+(1\leftrightarrow 2)\ . \tag{4.22}$$

In the $\mathbf{q}$ space, the latter becomes

$$\mathbf{J}_{LS}^{\tau,\mathrm{I}}(\mathbf{q},\mathbf{k})=e(\tau_1\times\tau_2)_z\frac{1}{4m^2}[\sigma_1\times\mathbf{p}_1v(|\mathbf{q}+\tfrac{1}{2}\mathbf{k}|)-\sigma_2\times\mathbf{p}_2v(|\mathbf{q}-\tfrac{1}{2}\mathbf{k}|)]\ . \tag{4.23}$$

Given this form for $\mathbf{J}_{LS}^{\tau,\mathrm{I}}$, the current $\mathbf{J}_{LS}^{\tau,\mathrm{II}}$ is constrained by

$$\mathbf{k}\cdot\mathbf{J}_{LS}^{\tau,\mathrm{II}}(\mathbf{q},\mathbf{k})=-e(\tau_1\times\tau_2)_z[v(|\mathbf{q}+\tfrac{1}{2}\mathbf{k}|)-v(|\mathbf{q}-\tfrac{1}{2}\mathbf{k}|)]\frac{1}{4m^2}[\sigma_1\cdot(\mathbf{q}-\tfrac{1}{2}\mathbf{k})\times\mathbf{p}_1-\sigma_2\cdot(\mathbf{q}+\tfrac{1}{2}\mathbf{k})\times\mathbf{p}_2]\ . \tag{4.24}$$

We solve this for

$$\mathbf{J}_{LS}^{\tau,\mathrm{II}}(\mathbf{q},\mathbf{k})=-eg^2(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z\frac{2\mathbf{q}}{[(\mathbf{q}-\frac{1}{2}\mathbf{k})^2+\mu^2][(\mathbf{q}+\frac{1}{2}\mathbf{k})^2+\mu^2]}\frac{1}{4m^2}[\boldsymbol{\sigma}_1\cdot(\mathbf{q}-\tfrac{1}{2}\mathbf{k})\times\mathbf{p}_1-\boldsymbol{\sigma}_2\cdot(\mathbf{q}+\tfrac{1}{2}\mathbf{k})\times\mathbf{p}_2]\ , \tag{4.25}$$

in which we have again used the identity (4.4).

Our next task is to calculate exchange currents using the graphical method and to compare with the above results. In the external field the meson-nucleon system is described by

$$H=\boldsymbol{\alpha}\cdot(\mathbf{p}-e\,\mathbf{A})+\beta m+eA_0+g\beta\varphi\ . \tag{4.26}$$

We do not include the anomalous magnetic moment interaction for the sake of simplicity. The Foldy-Wouthuysen transformation yields the one-body interaction correct to $O(1/m^2)$,

$$H_e=eA_0-\frac{e}{m}\mathbf{p}\cdot\mathbf{A}-\frac{e}{2m}\boldsymbol{\sigma}\cdot\mathbf{H}-\frac{e}{8m^2}\nabla\cdot\mathbf{E}+\frac{e}{4m^2}\boldsymbol{\sigma}\cdot\mathbf{p}\times\mathbf{E}\ , \tag{4.27}$$

where $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic fields, respectively. H_e is manifestly gauge invariant, and therefore consistent with Eq. (2.14). Besides H_e the Foldy-Wouthuysen transformation of (4.26) produces at the same time the "seagull" or "contact" interactions

$$H_{eg}=\frac{g}{8m^2}(4\mathbf{p}\cdot\mathbf{A}\{e,\varphi\}+2\boldsymbol{\sigma}\cdot\mathbf{H}\{e,\varphi\}+i\nabla\cdot\mathbf{A}[e,\varphi]-\boldsymbol{\sigma}\cdot\mathbf{A}\times\nabla\{e,\varphi\}-2i\boldsymbol{\sigma}\cdot\mathbf{p}\times\mathbf{A}[e,\varphi])\ . \tag{4.28}$$

With the standard graphical method, we are ready to compute from (4.28) the contact current interaction

$$\begin{aligned}H_{\mathrm{em}}^{\mathrm{cont}}=&\{e_1,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\}\frac{1}{2m^2}\mathbf{p}_1\cdot\mathbf{A}(\mathbf{r}_1)v(r)+\{e_1,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\}\frac{1}{4m^2}\boldsymbol{\sigma}_1\cdot\mathbf{H}(\mathbf{r}_1)v(r)+e(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z\frac{1}{8m^2}\nabla_1\cdot\mathbf{A}(\mathbf{r}_1)v(r)\\&+\{e_1,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\}\frac{1}{8m^2}\boldsymbol{\sigma}_1\times\nabla v(r)\cdot\mathbf{A}(\mathbf{r}_1)-e(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z\frac{1}{4m^2}\boldsymbol{\sigma}_1\times\mathbf{p}_1\cdot\mathbf{A}(\mathbf{r}_1)v(r)+(1\leftrightarrow2)\ .\end{aligned} \tag{4.29}$$

One can check that all the terms in $H_{\mathrm{em}}^{\mathrm{const}}$ are those obtained from our approach: The first and second terms correspond to $\mathbf{J}_C^{\mathrm{min}}$, the third term corresponds to $\mathbf{J}_C^{\tau,\mathrm{I}}$, the fourth term to $\mathbf{J}_{LS}^{\mathrm{min}}$, and the last term to $\mathbf{J}_{LS}^{\tau,\mathrm{I}}$. The mesonic current coincides exactly with $\mathbf{J}_C^{\tau,\mathrm{II}}+\mathbf{J}_{LS}^{\tau,\mathrm{II}}$ found in our approach.

The gauge invariance we have achieved is not a trivial result. The meson retardation and the nucleon recoil are the problems to be discussed in the subsequent subsection. Another problem is the treatment of the nucleon negative-energy states. The nucleon-antinucleon "pair" excitation current is induced because reduction of the field-theoretic four-component equation is projected onto the two-component subspace. Apart from the conceptual question that the creation of antinucleon at low energies should be suppressed due to the compositeness of the nucleon, the pair current is inconsistent with the corresponding nonradiative potential as emphasized by Ohta,[26] Ohta and Kubota,[27] and more recently by Riska.[12] The contact interaction in the Foldy-Wouthuysen Hamiltonian and the pair interaction in the diagrammatical expansion scheme do not always coincide and the difference between them is not necessarily gauge invariant. Insofar as we use H_e, we have to use the contact interaction in the Foldy-Wouthuysen theory instead of the so-called pair. Stichel and Werner[28] and Riska[12] made use of the interaction caused by the minimal substitution in the meson-nucleon vertex, but the minimal and Foldy-Wouthuysen seagulls are different. In fact the minimal substitution in (4.9) gives us only three terms,

$$H_{eg}^{\mathrm{min}}=\frac{g}{8m^2}(4\mathbf{p}\cdot\mathbf{A}\{e,\varphi\}+i\nabla\cdot\mathbf{A}[e,\varphi]-\boldsymbol{\sigma}\cdot\mathbf{A}\times\nabla\{e,\varphi\})\ . \tag{4.30}$$

The gauge-invariant interaction

$$\frac{g}{4m^2}\boldsymbol{\sigma}\cdot\mathbf{H}\{e,\varphi\} \tag{4.31}$$

can be produced if we make the substitution in $(\boldsymbol{\sigma}\cdot\mathbf{p})^2$ rather than in $\mathbf{p}^2$. The last term in (4.28) is the result of a minimal substitution of ∇ in H_g. We again find that the minimal prescription for the explicit $\mathbf{p}$ dependence is not enough to generate the electromagnetic interaction as regards isospin-dependent potentials.

C. Meson retardation and nucleon recoil

The second term in the expansion (4.10) gives rise to the retarded potential

$$V_{\mathrm{ret}}^{\tau}=g^2\int\frac{d^3q}{(2\pi)^3}e^{-i\mathbf{q}\cdot\mathbf{r}}q_0^2D'(q^2)\ . \tag{4.32}$$

In the position space we write q_0 as Γ,

$$V_{\mathrm{ret}}^{\tau}=-\Gamma_1\Gamma_2w(r)\ , \tag{4.33}$$

where

$$w(r)=g^2\int\frac{d^3q}{(2\pi)^3}e^{-i\mathbf{q}\cdot\mathbf{r}}D'(q^2)=\tfrac{1}{2}\int_r^\infty r\,dr\,v(r)\ . \tag{4.34}$$

Since the meson energy can be written as the nucleon energy difference,

$$\Gamma\varphi=[T,\varphi]\ , \tag{4.35}$$

we obtain

$$\Gamma_i=-i\frac{1}{m}\mathbf{p}_i\cdot\nabla_i\ . \tag{4.36}$$

This leads to the retarded potential

$$V^{\tau}_{\rm ret} = -\frac{1}{m^2}\mathbf{p}_1\cdot\nabla\mathbf{p}_2\cdot\nabla w(r) \;. \tag{4.37}$$

If we insert

$$q_0^2 = \frac{1}{m^2}\mathbf{p}_1\cdot\mathbf{q}\mathbf{p}_2\cdot\mathbf{q} \tag{4.38}$$

into (4.32), we get (4.37).

We apply our method to Eq. (4.37) to construct the exchange current. The minimal substitution of $\mathbf{p}_i$ produces

$$\mathbf{J}^{\rm min}_{\rm ret}(\mathbf{x}) = -\{e_1, \tau_1\cdot\tau_2\}\frac{1}{2m^2}\delta(\mathbf{x}-\mathbf{r}_1)\nabla\mathbf{p}_2\cdot\nabla w(r) + (1\leftrightarrow 2) \tag{4.39}$$

and the minimal substitution of ∇ produces

$$\mathbf{J}^{\tau,\rm I}_{\rm ret}(\mathbf{x}) = -e(\tau_1\times\tau_2)_z\frac{1}{m^2}\delta(\mathbf{x}-\mathbf{r}_1)\mathbf{p}_1\mathbf{p}_2\cdot\nabla w(r) + (1\leftrightarrow 2) \;. \tag{4.40}$$

The latter is represented in the momentum space as

$$\mathbf{J}^{\tau,\rm I}_{\rm ret}(\mathbf{q},\mathbf{k}) = ie(\tau_1\times\tau_2)_z\frac{1}{m^2}[\mathbf{p}_1\mathbf{p}_2\cdot(\mathbf{q}+\tfrac{1}{2}\mathbf{k})w(|\mathbf{q}+\tfrac{1}{2}\mathbf{k}|) - \mathbf{p}_1\cdot(\mathbf{q}-\tfrac{1}{2}\mathbf{k})\mathbf{p}_2 w(|\mathbf{q}-\tfrac{1}{2}\mathbf{k}|)] \;. \tag{4.41}$$

On the other hand, the current

$$\mathbf{J}^{\tau}_{\rm ret}(\mathbf{q},\mathbf{k}) = \mathbf{J}^{\tau,\rm I}_{\rm ret}(\mathbf{q},\mathbf{k}) + \mathbf{J}^{\tau,\rm II}_{\rm ret}(\mathbf{q},\mathbf{k}) \tag{4.42}$$

must satisfy (2.82) or

$$\begin{aligned}\mathbf{k}\cdot\mathbf{J}^{\tau}_{\rm ret}(\mathbf{q},\mathbf{k}) &= ie(\tau_1\times\tau_2)_z[V^{\tau}_{\rm ret}(\mathbf{q}+\tfrac{1}{2}\mathbf{k}) - V^{\tau}_{\rm ret}(\mathbf{q}-\tfrac{1}{2}\mathbf{k})] \\ &= ie(\tau_1\times\tau_2)_z\frac{1}{m^2}[\mathbf{p}_1\cdot(\mathbf{q}+\tfrac{1}{2}\mathbf{k})\mathbf{p}_2\cdot(\mathbf{q}+\tfrac{1}{2}\mathbf{k})w(|\mathbf{q}+\tfrac{1}{2}\mathbf{k}|) - \mathbf{p}_1\cdot(\mathbf{q}-\tfrac{1}{2}\mathbf{k})\mathbf{p}_2\cdot(\mathbf{q}-\tfrac{1}{2}\mathbf{k})w(|\mathbf{q}-\tfrac{1}{2}\mathbf{k}|)] \;.\end{aligned} \tag{4.43}$$

Therefore we have the constraint

$$\mathbf{k}\cdot\mathbf{J}^{\tau,\rm II}_{\rm ret}(\mathbf{q},\mathbf{k}) = ie(\tau_1\times\tau_2)_z\frac{1}{m^2}\mathbf{p}_1\cdot(\mathbf{q}-\tfrac{1}{2}\mathbf{k})\mathbf{p}_2\cdot(\mathbf{q}+\tfrac{1}{2}\mathbf{k})[w(|\mathbf{q}+\tfrac{1}{2}\mathbf{k}|) - w(|\mathbf{q}-\tfrac{1}{2}\mathbf{k}|)] \tag{4.44}$$

which is solved for

$$\begin{aligned}\mathbf{J}^{\tau,\rm II}_{\rm ret}(\mathbf{q},\mathbf{k}) &= ieg^2(\tau_1\times\tau_2)_z\frac{1}{m^2}\mathbf{p}_1\cdot(\mathbf{q}-\tfrac{1}{2}\mathbf{k})\mathbf{p}_2\cdot(\mathbf{q}+\tfrac{1}{2}\mathbf{k}) \\ &\quad\times\frac{2\mathbf{q}}{[(\mathbf{q}-\tfrac{1}{2}\mathbf{k})^2+\mu^2][(\mathbf{q}+\tfrac{1}{2}\mathbf{k})^2+\mu^2]}\left[\frac{1}{(\mathbf{q}+\tfrac{1}{2}\mathbf{k})^2+\mu^2} + \frac{1}{(\mathbf{q}-\tfrac{1}{2}\mathbf{k})^2+\mu^2}\right] \;.\end{aligned} \tag{4.45}$$

In deriving this we have used $w(q) = -g^2/(\mathbf{q}^2+\mu^2)^2$ and

$$\begin{aligned}&\frac{\mathbf{q}}{\mathbf{k}\cdot\mathbf{q}}\left\{\frac{1}{[(\mathbf{q}+\tfrac{1}{2}\mathbf{k})^2+\mu^2]^2} - \frac{1}{[(\mathbf{q}-\tfrac{1}{2}\mathbf{k})^2+\mu^2]^2}\right\} \\ &\qquad = -\frac{2\mathbf{q}}{[(\mathbf{q}-\tfrac{1}{2}\mathbf{k})^2+\mu^2][(\mathbf{q}+\tfrac{1}{2}\mathbf{k})^2+\mu^2]}\left[\frac{1}{(\mathbf{q}+\tfrac{1}{2}\mathbf{k})^2+\mu^2} + \frac{1}{(\mathbf{q}-\tfrac{1}{2}\mathbf{k})^2+\mu^2}\right] \;.\end{aligned} \tag{4.46}$$

The current operators $\mathbf{J}^{\rm min}_{\rm ret}$ and $\mathbf{J}^{\tau,\rm I}_{\rm ret}$ obtained from minimal substitutions have the counter terms in the nucleon recoil current operator in the usual diagrammatical method. The recoil current, which is studied by Friar[29] in full detail, is the contribution from the time-ordered diagram in which a nucleon interacts with the electromagnetic field before a meson emitted by one of the two nucleons is absorbed by the other nucleon. The leading part is canceled by the wave function renormalization, but a finite quantity is left. The recoil current interaction becomes

$$H^{\rm rec}_{\rm em} = -\mathbf{\Gamma}^e_1\cdot\mathbf{\Gamma}_2 w(r) - \mathbf{\Gamma}_1\cdot\mathbf{\Gamma}^e_2 w(r) \;, \tag{4.47}$$

where

$$\mathbf{\Gamma} = \tau\mathbf{\Gamma} \tag{4.48}$$

and

$$\mathbf{\Gamma}^e\cdot\boldsymbol{\phi} = [H_e, \varphi] \;. \tag{4.49}$$

To order $1/m^2$

$$\begin{aligned}\mathbf{\Gamma}^e &= i\{e,\tau\}\frac{g}{2m}\mathbf{A}\cdot\nabla - [e,\tau]\frac{g}{m}\mathbf{p}\cdot\mathbf{A} \\ &\quad - [e,\tau]\frac{g}{2m}\sigma\cdot\mathbf{H} + [e,\tau]gA_0 \;.\end{aligned} \tag{4.50}$$

Thus we find

$$H_{\rm em}^{\rm rec}=\{e_1,\tau_1\cdot\tau_2\}\frac{1}{2m^2}\,\mathbf{A}(\mathbf{r}_1)\cdot\nabla\mathbf{p}_2\cdot\nabla w(r)$$
$$+e(\tau_1\times\tau_2)_z\frac{1}{m^2}\mathbf{p}_1\cdot\mathbf{A}(\mathbf{r}_1)\mathbf{p}_2\cdot\nabla w(r)$$
$$+e(\tau_1\times\tau_2)_z\frac{1}{2m^2}\sigma_1\cdot\mathbf{H}(\mathbf{r}_1)\mathbf{p}_2\cdot\nabla w(r)$$
$$-e(\tau_1\times\tau_2)_z\frac{1}{m}A_0(\mathbf{r}_1)\mathbf{p}_2\cdot\nabla w(r)+(1\leftrightarrow 2)\ . \qquad (4.51)$$

In Eq. (4.51) the first two terms exactly correspond to the currents $\mathbf{J}_{\rm ret}^{\rm min}$ and $\mathbf{J}_{\rm ret}^{\tau,\rm I}$ obtained from minimal substitutions in the retarded potential. The third term is absent in our approach because it is an interaction which is gauge invariant by itself and therefore is not accessible by our approach. The fourth term, which is proportional to the scalar potential, is the exchange charge density interaction caused by the nucleon recoil. The charge density operator is found to be

$$\rho_{\rm rec}(\mathbf{x})=-e(\tau_1\times\tau_2)_z\frac{1}{m}\delta(\mathbf{x}-\mathbf{r}_1)\mathbf{p}_2\cdot\nabla w(r)$$
$$+(1\leftrightarrow 2)\ . \qquad (4.52)$$

In the momentum space it is written as

$$\rho_{\rm rec}(\mathbf{q},\mathbf{k})=-ieg^2(\tau_1\times\tau_2)_z\left\{\frac{q_0'}{[(\mathbf{q}+\frac{1}{2}\mathbf{k})^2+\mu^2]^2}+\frac{q_0}{[(\mathbf{q}-\frac{1}{2}\mathbf{k})^2+\mu^2]^2}\right\}, \qquad (4.53)$$

where

$$q_0=\frac{1}{m}\mathbf{p}_1\cdot(\mathbf{q}-\tfrac{1}{2}\mathbf{k})\ , \qquad (4.54)$$

$$q_0'=\frac{1}{m}\mathbf{p}_2\cdot(\mathbf{q}+\tfrac{1}{2}\mathbf{k})\ , \qquad (4.55)$$

are the meson energies before and after the photo-absorption, respectively.

The nucleon recoil is not the only origin of the charge density operator. The mesonic interaction also brings about the density

$$\rho_{\rm mes}(\mathbf{q},\mathbf{k})=ieg^2(\tau_1\times\tau_2)_z\frac{q_0+q_0'}{[(\mathbf{q}-\frac{1}{2}\mathbf{k})^2+\mu^2][(\mathbf{q}+\frac{1}{2}\mathbf{k})^2+\mu^2]}\ . \qquad (4.56)$$

The sum of these density operators,

$$\delta\rho(\mathbf{q},\mathbf{k})=\rho_{\rm rec}(\mathbf{q},\mathbf{k})+\rho_{\rm mes}(\mathbf{q},\mathbf{k})\ , \qquad (4.57)$$

becomes

$$\delta\rho(\mathbf{q},\mathbf{k})=ieg^2(\tau_1\times\tau_2)_z\frac{2\mathbf{q}\cdot\mathbf{k}}{[(\mathbf{q}-\frac{1}{2}\mathbf{k})^2+\mu^2][(\mathbf{q}+\frac{1}{2}\mathbf{k})^2+\mu^2]}\left[\frac{q_0'}{(\mathbf{q}+\frac{1}{2}\mathbf{k})^2+\mu^2}-\frac{q_0}{(\mathbf{q}-\frac{1}{2}\mathbf{k})^2+\mu^2}\right]. \qquad (4.58)$$

The meson retardation also affects the current operator. In the mesonic current diagram, meson propagators are expanded in $1/m$,

$$\frac{1}{[(\mathbf{q}-\frac{1}{2}\mathbf{k})^2+\mu^2-q_0^2][(\mathbf{q}+\frac{1}{2}\mathbf{k})^2+\mu^2-q_0'^2]}$$
$$\cong\frac{1}{[(\mathbf{q}-\frac{1}{2}\mathbf{k})^2+\mu^2][(\mathbf{q}+\frac{1}{2}\mathbf{k})^2+\mu^2]}+\frac{1}{[(\mathbf{q}-\frac{1}{2}\mathbf{k})^2+\mu^2][(\mathbf{q}+\frac{1}{2}\mathbf{k})^2+\mu^2]}\left[\frac{q_0'^2}{(\mathbf{q}+\frac{1}{2}\mathbf{k})^2+\mu^2}+\frac{q_0^2}{(\mathbf{q}-\frac{1}{2}\mathbf{k})^2+\mu^2}\right]. \qquad (4.59)$$

The first term in this expansion gives us the static approximation of the mesonic current. In the preceding subsection we have seen that in the static approximation the mesonic current is exactly reproduced by our method. The second term brings about the correction due to meson retardation,

$$\mathbf{J}_{\rm ret}(\mathbf{q},\mathbf{k})=ieg^2(\tau_1\times\tau_2)_z\frac{2\mathbf{q}}{[(\mathbf{q}-\frac{1}{2}\mathbf{k})^2+\mu^2][(\mathbf{q}+\frac{1}{2}\mathbf{k})^2+\mu^2]}\left[\frac{q_0'^2}{(\mathbf{q}+\frac{1}{2}\mathbf{k})^2+\mu^2}+\frac{q_0^2}{(\mathbf{q}-\frac{1}{2}\mathbf{k})^2+\mu^2}\right]. \qquad (4.60)$$

We decompose this into two parts,

$$\mathbf{J}_{\rm ret}(\mathbf{q},\mathbf{k})=\mathbf{J}_{\rm ret}'(\mathbf{q},\mathbf{k})+\delta\mathbf{J}(\mathbf{q},\mathbf{k})\ , \qquad (4.61)$$

where

$$\mathbf{J}'_{\rm ret}(\mathbf{q},\mathbf{k})=ieg^2(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z q_0 q'_0 \times\frac{2\mathbf{q}}{[(\mathbf{q}-\frac{1}{2}\mathbf{k})^2+\mu^2][(\mathbf{q}+\frac{1}{2}\mathbf{k})^2+\mu^2]}\left[\frac{1}{(\mathbf{q}+\frac{1}{2}\mathbf{k})^2+\mu^2}+\frac{1}{(\mathbf{q}-\frac{1}{2}\mathbf{k})^2+\mu^2}\right] \quad (4.62)$$

and

$$\delta\mathbf{J}(\mathbf{q},\mathbf{k})=ieg^2(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z (q'_0-q_0) \times\frac{2\mathbf{q}}{[(\mathbf{q}-\frac{1}{2}\mathbf{k})^2+\mu^2][(\mathbf{q}+\frac{1}{2}\mathbf{k})^2+\mu^2]}\left[\frac{q'_0}{(\mathbf{q}+\frac{1}{2}\mathbf{k})^2+\mu^2}-\frac{q_0}{(\mathbf{q}-\frac{1}{2}\mathbf{k})^2+\mu^2}\right] . \quad (4.63)$$

One sees that $\delta\mathbf{J}(\mathbf{q},\mathbf{k})$ and $\delta\rho(\mathbf{q},\mathbf{k})$ obey the continuity equation

$$\mathbf{k}\cdot\delta\mathbf{J}(\mathbf{q},\mathbf{k})-(q'_0-q_0)\delta\rho(\mathbf{q},\mathbf{k})=0 . \quad (4.64)$$

The remaining current $\mathbf{J}'_{\rm ret}$, (4.62), exactly coincides with $\mathbf{J}^{\tau,\rm II}_{\rm rec}$, (4.45), just that found by our approach. Thus it is proved that our method can reproduce current operators up to divergence-free four-currents. It should be stressed that our results are valid beyond the nonrelativistic framework: The scalar-meson exchange gives rise to the two-body charge density, and yet the condition (1.2) is fulfilled.

To conclude this section we supplement two remarks. It is well known that there exists an ambiguity in the choice of the meson energy q_0. We note that any choice gives us one of unitary-equivalent potentials. We can transform (4.37) using a unitary transformation of the two-nucleon wave function,

$$\psi\rightarrow\psi'=e^{iS}\psi , \quad (4.65)$$

with

$$S=\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 S^{\tau} . \quad (4.66)$$

To order g^2 the retarded potential undergoes a modification,

$$V^{\tau}_{\rm ret}\rightarrow V^{\tau}_{\rm ret}+i[S^{\tau},T] . \quad (4.67)$$

If we choose a generator

$$S^{\tau}=\tfrac{1}{2}i[T,w(r)]=\frac{1}{2m}\mathbf{p}_1\cdot\nabla w(r)-\frac{1}{2m}\mathbf{p}_2\cdot\nabla w(r) , \quad (4.68)$$

we get a new retarded potential

$$V^{\tau}_{\rm ret}=\frac{1}{2m^2}[(\mathbf{p}_1\cdot\nabla)^2+(\mathbf{p}_2\cdot\nabla)^2]w(r) . \quad (4.69)$$

This is equivalent to the choice,

$$q_0^2=\frac{1}{2m^2}(\mathbf{p}_1\cdot\mathbf{q})^2+\frac{1}{2m^2}(\mathbf{p}_2\cdot\mathbf{q})^2 . \quad (4.70)$$

Applying our method to the potential (4.69) results in different current operators from those given above, but we can prove again that the current operators have their counter terms in the diagrammatical expansion scheme. Recall that, owing to the unitary transformation, the electromagnetic one-body interaction is also modified as

$$H'_e=H_e+i[S,H_e] . \quad (4.71)$$

The retardation in the meson exchange is strictly necessary in the proof of the Poincaré invariance of the nuclear interaction. There is in either of (4.37) or (4.69) a term of the form

$$-\frac{1}{4m^2}(\mathbf{P}\cdot\nabla)^2 w(r) .$$

Such a total-momentum-dependent term should not appear. In fact, it is canceled out by a relativistic correction: The nucleon position operators $\mathbf{r}_1$ and $\mathbf{r}_2$ are related to the relativistic internal position operator $\mathbf{r}$ to $O(1/m^2)$ as[30]

$$\mathbf{r}_1-\mathbf{r}_2=\mathbf{r}-\frac{1}{8m^2}\mathbf{r}\cdot\mathbf{P}\mathbf{P}-\frac{1}{8m^2}(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\times\mathbf{P} . \quad (4.72)$$

In terms of this the static potential becomes

$$v(|\mathbf{r}_1-\mathbf{r}_2|)=v(r)+\frac{1}{4m^2}(\mathbf{P}\cdot\nabla)^2 w(r) +\frac{1}{8m^2}(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\nabla v(r)\times\mathbf{P} . \quad (4.73)$$

The second term cancels the $\mathbf{P}$ dependence in the retarded potential and the third term cancels that in the instantaneous potential.

V. DISCUSSION

We have developed a method of constructing the exchange current operator from an arbitrary nucleon-nucleon potential. Taking the charged pseudoscalar and scalar mesons as illustrative examples, we showed that the contact and mesonic currents can be generated from the meson-exchange potentials without recourse to the explicit evaluation of diagrams. Close and Osborn[31] solved the continuity equation for isospin-independent potentials and it is found that current operators are in agreement with those deduced from the potentials by minimal substitutions. Our method proposed in this paper is more general and is applicable to charge-exchange potentials.

The diagrammatical method involves complications due to meson retardation, nucleon recoil, wave function renormalization, relativistic corrections, and so on. On the contrary, our method is simple and straightforward. Once the nonradiative potential is given, current operators are deduced without knowing such complications.

Furthermore, our method is not limited to one-boson-exchange models. It can be used as a guiding principle in the construction of current operators in microscopic nuclear interaction models.

It is a matter of course that the gauge invariance alone cannot determine all the currents. We have experienced such an instance in the scalar-meson-exchange retarded potential. For another example, the ρ meson has the anomolous magnetic moment interaction which has to be added to the usual minimal coupling. The anomoulous part is only responsible for the current which is gauge invariant by itself, and therefore it is not accessible by our approach. But our method can be utilized in the following way: We usually calculate exchange-current operators from some field theoretical model, and then take matrix elements of them between nuclear wave functions which are eigenfunctions of the nonradiative Hamiltonian $H_0=T+V$. Calculations of wave functions and current operators are performed on quite different basis. However, the nuclear potential V and the exchange-current interaction Hamiltonian are constrained by the gauge-invariance condition. If we employ exchange-current operators constructed in a field theoretical model while using a phenomenological potential, this condition is not satisfied and the gauge invariance is violated. If we calculate an exchange-current interaction by using some field theoretical model, we split it into two parts,

$$H_{\text{em}}^{(2)}=H_{\text{em}}^{V}+\delta H_{\text{em}} , \tag{5.1}$$

where

$$H_{\text{em}}^{V}=H_{\text{em}}^{\text{min}}+H_{\text{em}}^{\tau,\text{I}}+H_{\text{em}}^{\tau,\text{II}} \tag{5.2}$$

is written in terms of the theoretical potential V as prescribed in this paper. The four-divergence-free current interaction δH_{em} satisfies the requirement of gauge invariance by itself and is not related to the potential. After this separation the theoretical total Hamiltonian becomes

$$H=H_0+H_{\text{em}}^{(1)}+H_{\text{em}}^{V}+\delta H_{\text{em}} . \tag{5.3}$$

When we go to the phenomenological potential, we also make a replacement of V in H_{em}^{V}. Then the calculation of matrix elements does not violate gauge invariance even when we use any phenomenological nuclear forces. Furthermore, the model dependence is limited to δH_{em} which should be calculated from the underlying field theory.

[1]C. H. Blanchard, R. Avery, and R. G. Sachs, Phys. Rev. **78**, 292 (1950).
[2]N. Austern and R. G. Sachs, Phys. Rev. **81**, 710 (1951).
[3]H. Feshbach, Phys. Rev. **107**, 1626 (1957).
[4]M. Sugawara, Phys. Rev. **117**, 614 (1960).
[5]R. K. Bhaduri and P. van Leuven, Phys. Lett. **20**, 182 (1966).
[6]M. K. Liu and M. I. Sobel, Phys. Rev. C **7**, 2044 (1973).
[7]S. Mukherjee and R. Shyam, Phys. Rev. C **8**, 1149 (1973).
[8]J. C. Bergstrom, Phys. Rev. C **9**, 2435 (1974).
[9]R. R. Sheerbaum, Phys. Rev. C **11**, 255 (1975).
[10]D. W. L. Sprung, Nucl. Phys. **A242**, 141 (1975).
[11]Y. E. Pokrovskii, Pis'ma Zh. Eksp. Teor. Fiz. **21**, 608 (1975) [JETP Lett. **21**, 285 (1975)].
[12]D. O. Riska, Phys. Scr. **31**, 107 (1985); D. O. Riska and M. Poppius, *ibid*. **32**, 581 (1985).
[13]A. Buchmann, W. Leidemann, and H. Arenhövel, Nucl. Phys. **A443**, 726 (1985).
[14]R. G. Sachs, Phys. Rev. **74**, 433 (1948).
[15]R. H. Thompson and L. Heller, Phys. Rev. C **7**, 2355 (1973).
[16]D. O. Riska, Phys. Scr. **31**, 471 (1985).
[17]F. Gross and D. O. Riska, Phys. Rev. C **36**, 1928 (1987).
[18]K. Ohta, Nucl. Phys. A (in press).
[19]L. Heller, S. Kumano, J. C. Martinez, and E. J. Moniz, Phys. Rev. C **35**, 718 (1987).
[20]R. G. Sachs and N. Austern, Phys. Rev. **81**, 705 (1951).
[21]K. Ohta and M. Ichimura, Nucl. Phys. **A491**, 509 (1989).
[22]R. K. Osborn and L. L. Foldy, Phys. Rev. **79**, 795 (1950).
[23]L. Heller, in *The Two-Body Force in Nuclei,* edited by S. M. Austin and G. M. Crawley (Plenum, New York, 1972).
[24]V. R. Brown and J. Franklin, Phys. Rev. C **8**, 1706 (1973).
[25]L. L. Foldy and S. A. Wouthuysen, Phys. Rev. **78**, 29 (1950).
[26]K. Ohta, Phys. Rev. C **19**, 965 (1979).
[27]K. Ohta and T. Kubota, Nucl. Phys. **B150**, 383 (1979).
[28]P. Stichel and E. Werner, Nucl. Phys. **A145**, 257 (1970).
[29]J. L. Friar, Ann. Phys. (N.Y.) **104**, 380 (1977).
[30]F. E. Close and H. Osborn, Phys. Rev. D **2**, 2127 (1970).
[31]F. E. Close and H. Osborn, Phys. Lett. **34B**, 400 (1971).