Effective Lagrangian approach to K^+-nucleon scattering and K^+ photoproduction

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(Received 5 December 1988)

We study K^+N low-energy scattering using the effective Lagrangian approach. The scattering mechanism model consists of hyperon and meson exchanges (this model is borrowed from π-N scattering calculations). We limit ourselves to the tree level, motivated by nuclear physics applications. Canonical values of the required coupling constants give rise to scattering parameters which disagree strongly with experiment. Kaon photoproduction ($\gamma+p\rightarrow K^+ +\Lambda$) studies using a similar model also yield coupling constants which differ from standard compilations values. Using these values in the K^+N scattering calculations, we find a better agreement with experiment (the level of agreement is still only within a factor of 2–3, however). We discuss the role of ω and ρ t-channel mesonic exchanges in the scattering process and elaborate on the implications to some recent work in the current literature. Possible reasons for the results and discrepancies found in the present work, as well as possible implications, are discussed.

I. INTRODUCTION

Effective Lagrangian approaches (or tree level Feynman diagrams) are a popular tool in intermediate-energy nuclear physics. In particular, the Weinburg Lagrangian[1] and other similar phenomenological Lagrangians[2] have been applied to a variety of medium-energy nuclear calculations. Noteworthy are the attempts to work out a complete theoretical nuclear model based on an effective Lagrangian for mesons and nucleons.[3–10]

The present work does not elaborate on any of the nuclear physics applications of effective Lagrangians. Instead, we obtain here some results based on an effective Lagrangian used in the literature for K^+-nuclear related calculations and confront these results with available experimental data. The motivation for this work comes from the large discrepancies between kaon-nucleon-hyperon (KNY) coupling constants obtained via the reaction $\gamma+p\rightarrow K^+ +\Lambda$ when analyzed using a model borrowed from pion photoproduction, against the canonical, tabulated values of the same coupling constants, as obtained by a variety of other methods (a detailed discussion is given in the main body of this work, following the Introduction). However, our work is carried out with possible simple nuclear physics applications in mind. Further motivation is provided by a recent work[11] dealing with K^+ scattering on nucleons and nuclei. Consequently, we study the role of the ρ and ω mesons in K^+N low-energy scattering.

The thrust of this paper is a comparison of low-energy K^+-nucleon scattering parameters obtained from tree level Feynman diagrams based on the commonly-used Lagrangian with experimental results; a relation to the $\gamma p\rightarrow K^+\Lambda$ reaction also emerges. We then discuss the role of the ρ and ω mesons, and argue that the current level of our understanding is not sufficient to allow for a *quantitative* description (some data require a 10% accuracy or better) of strong-interaction processes involving the K meson in terms of tree level Feynman diagrams (indeed, this is the reason we disagree with the conclusions of Ref. 11).

As discussed in Ref. 7, nuclear physics applications are greatly simplified when the tree level meson-nucleon scattering matrix is sufficient for a reliable description of this physical system. (Indeed, this is the starting point of Ref. 7 regarding the pion-nucleon dynamics.) Since the K^+-nucleus interaction is an interesting part of intermediate-energy physics, we feel that a similar study involving K mesons is likewise important. We *do not* attempt here to build a new Lagrangian for accurately describing the elementary-particle reactions dealt with, but rather to test an existing model and shed light on existing, largely unnoticed, disagreements in the literature. Our starting point is common to many nuclear physics calculations, so we feel that our results should be brought to the attention of nuclear physicists.

II. THE LOW-ENERGY MESON-BARYON SCATTERING PARAMETERS

In order to provide a self-contained presentation, we introduce our notation first. We use the conventions of Bjorken and Drell[12] throughout. Denoting the nucleon momentum by p and the kaon momentum by K, and using the subscripts i and f for the initial and final states, respectively, the Mandelstam kinematical variables are

$$\begin{aligned} s &\equiv (p_f+K_f)^2=(p_i+K_i)^2 \ , \\ t &\equiv (K_i-K_f)^2=(p_i-p_f)^2 \ , \\ u &\equiv (p_i-K_f)^2=(K_i-p_f)^2 \ . \end{aligned} \tag{1}$$

On the mass shell, the relation

$$s+t+u=2M_N^2+2m_K^2 \ ,$$

where m_K is the kaon mass and M_B is the baryon mass, holds. [In the present case $B=N$ (nucleon).]

The Lorentz-invariant T matrix, which is a Dirac matrix (dimension 4×4) in the spinor space, is decomposed as in Chew, Goldberg, Low, and Nambu:[13]

$$T=A(s,t)+\tfrac{1}{2}[(K_i+K_f)\cdot\gamma]B(s,t) \ . \tag{2}$$

This form is valid for on-shell scattering or partly off-shell where the nucleon is on-mass-shell but the meson is not. The functions A and B depend on the Mandelstam variables s and t of Eq. (1).

Our goal is to relate the tree level expressions for $A(s,t)$ and $B(s,t)$ to threshold scattering parameters (namely, scattering lengths and volumes and effective ranges). These are defined via the partial-wave amplitudes $f_{l\pm}$,[14] and for low c.m. momentum transfer $\mathbf{q}$ we write

$$\begin{aligned}\mathrm{Re} f_{0+}&=a_{0+}+r_{0+}\mathbf{q}^2+\cdots \ ,\\ \mathrm{Re} f_{1\pm}&=a_{1\pm}\mathbf{q}^2+\cdots \ .\end{aligned} \tag{3}$$

where a_{0+} is the scattering length ($l=0$, s wave), r_{0+} is the s-wave effective range, and $a_{1\pm}$ are the scattering volumes ($l=1$, p wave). These can be expressed in terms of the amplitudes A and B and their partial derivatives with respect to s and t at threshold: $s=(M_N+m_K)^2$, $t=0$. Such expressions are given, for example, in Matsui and Serot[7] for the πN system. Thus, in the present case, the s-wave scattering length is

$$a_{0+}^{(I)}=\eta[m_K B^{(I)}+A^{(I)}]_0 \ , \tag{4}$$

where

$$\eta=\frac{1}{4\pi\left[1+\dfrac{m_k}{M_N}\right]} \ , \tag{5}$$

and where the subscript 0 stands for evaluation at threshold and I is the s-channel isospin. The expressions for $r_{0+}^{(I)}$ and for $a_{1\pm}^{(I)}$ are

$$r_{0+}^{(I)}=\eta\left\{-2C_0^{(I)}+\frac{(M_N+m_K)^2}{M_N m_K}D_0^{(I)}-\frac{1}{2M_N m_K}\left[\left[1-\frac{m_k}{2M_N}\right]A_0^{(I)}-\left[M_N+\frac{m_K^2}{2M_N}\right]B_0^{(I)}\right]\right\} \ , \tag{6}$$

$$a_{1+}^{(I)}=\tfrac{2}{3}\eta C_0^{(I)} \tag{7}$$

and

$$a_{1-}^{(I)}=\tfrac{2}{3}\eta C_0^{(I)}-\frac{\eta}{4M_N^2}[A_0^{(I)}-(2M_N+m_K)B_0^{(I)}] \ . \tag{8}$$

In Eqs. (6)–(8),

$$C_0^{(I)}=\left[\frac{\partial}{\partial t}(A^{(I)}+m_K B^{(I)})\right]_0 \ ,$$

and

$$D_0^{(I)}=\left[\frac{\partial}{\partial s}(A^{(I)}+m_K B^{(I)})\right]_0 \ , \tag{9}$$

where the subscript 0 again means evaluation at threshold.

III. CALCULATION OF THE LOW-ENERGY K^+N SCATTERING PARAMETERS

We evaluate the tree level Feynman diagrams based on a K^+N lagrangian which is equivalent to a pseudoscalar πN lagrangian in the σ model.[7,15] These diagrams include the Born terms (with a baryon propagator) and t-channel meson-exchange diagrams, including σ, ρ, and ω meson exchanges, and are shown in Fig. 1.

We consider the Born terms first. With possible exchanges of Λ and Σ hyperons, only the u-channel (crossed) diagrams contribute. (Exotic Z^* resonances are outside the scope and theoretical framework of our present study, are not well established, and will not be considered here.) The (s-channel isospin) $I=1$ amplitude is obtained directly from $K^+p\to K^+p$ scattering. For example, the resulting A and B amplitudes are, at threshold,

$$\begin{aligned}A_0^{(I=1)}&=\alpha_{KN\Lambda}(M_\Lambda-M_N)+\alpha_{KN\Sigma}(M_\Sigma-M_N) \ ,\\ B_0^{(I=1)}&=\alpha_{KN\Lambda}+\alpha_{KN\Sigma} \ ,\end{aligned} \tag{10}$$

where

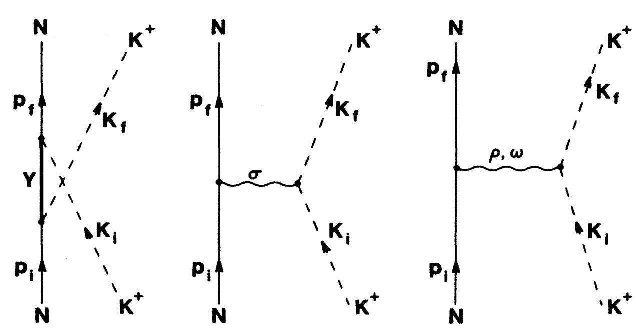

FIG. 1. The tree level diagrams for $K^+N\to K^+N$. The initial and final nucleon four momenta are p_i and p_f, respectively. The incoming and outgoing K^+ mesons have the momenta K_i and K_f. We show the u-channel Born term, where the exchange hyperon (Y) is Λ or Σ^0 for $K^+p\to K^+p$ and Σ^- for $K^+n\to K^+n$, as well as σ, ρ and ω-meson exchange diagrams in the t channel.

$$\alpha_{KN\Lambda}=\frac{g_{KN\Lambda}^2}{(M_N-m_K)^2-M_\Lambda^2}\ ,\qquad \alpha_{KN\Sigma}=\frac{g_{KN\Sigma}^2}{(M_N-m_K)^2-M_\Sigma^2}\ , \tag{11}$$

and g_{KNY} is the coupling constant for the kaon (K)-nucleon (N)-hyperon (Y) vertex. The $K^+n\to K^+n$ case, which receives equal contributions from the $I=0$ and $I=1$ amplitudes, is calculated in a similar way, and then used in order to obtain the $I=0$ amplitudes (namely, $A^{(I=0)}$ and $B^{(I=0)}$). For example, the resulting threshold expressions for the latter are

$$A_0^{(I=0)}=-\alpha_{KN\Lambda}(M_\Lambda-M_N)+3\alpha_{KN\Sigma}(M_\Sigma-M_N)\ ,\qquad B_0^{(I=0)}=-\alpha_{KN\Lambda}+3\alpha_{KN\Sigma}\ . \tag{12}$$

Using the threshold amplitudes $A_0^{(I)}$ and $B_0^{(I)}$, we readily obtain [using Eq. (4)] expressions for the ($l=0$) scattering lengths, $a_{0+}^{(I=0,1)}$. Furthermore, evaluating the partial derivatives of $A^{(I)}(s,t)$ and $B^{(I)}(s,t)$ with respect to s and t at threshold [see Eq. (9)], we obtain expressions for $r_{0+}^{(I)}$ and $a_{1\pm}^{(I)}$ [see Eqs. (6)–(8)]. All these expressions differ from the corresponding ones in the πN scattering case, since in the latter analysis both the s- and u-poles contribute, while in the present (K^+N) case only the u channel contributes to the Born scattering amplitude (the s channel is totally absent).

At this stage it is possible to actually calculate the predictions of the Born diagrams by themselves, and compare with available experimental results. Established values for the coupling constants $g_{KN\Lambda}$ and $g_{KN\Sigma}$ appear in the standard compilation by Dumbrajs *et al.*[16] and agree with theoretical efforts based on a quark model[17] or a potential approach.[18] All of these values are consistent with $g_{KN\Lambda}/\sqrt{4\pi}=4.1$ and $(g_{KN\Sigma}/g_{KN\Lambda})^2\ll 1$. Substituting these values in the expressions for the scattering parameters, we obtain the first line in Table I [pseudoscalar (PS) Born]. (Table I is a reprise of Table I in the preliminary paper on this work, Ref. 19.) The reason for the different sign of $a_{0+}^{(I)}$ for $I=0,1$ is simple. The only possible u-channel exchange for $K^+n\to K^+n$ is the Σ^- hyperon. Since, in this case, $g_{KN\Sigma}\ll g_{KN\Lambda}$ (note that this will *not* be the case in lines 4 and 5 of Table I), we get $a_{0+}^{(I=0)}+a_{0+}^{(I=1)}\simeq 0$. A comparison with the experimental data (taken from Ref. 16) reveals a very poor agreement.

[Note that the low-energy scattering parameters of Ref. 16 are defined with the (real) K matrix as a starting point, Eq. (2.15) in Ref. 16. Our definition, Eq. (3) (which follow Refs. 6, 7, and 14, for example), starts with the real parts of the partial-wave scattering amplitudes. These two definitions are *not* identical. The relations between the two sets of resulting parameters are as follows: The scattering length and volumes ($a_{1\pm}$) have the same values in both cases, while

$$r_{0+}=-\tfrac{1}{2}r_K a_{0+}^2-a_{0+}^3\ ,$$

where the r_K is the K matrix $l=0$ effective range parameter. (It is worthy of note that the units of r_{0+} are fm^3, while those of r_K are fm.)

In our previous work, Ref. 19, we have used wrong values for the experimental value of r_{0+}. The correct experimental numbers are given in Table I and Table III of the present work. This requires some changes in the ensuing discussion of theory versus experiment and of the range of the σ-meson mass in Ref. 19. The reader should therefore refer to the discussion of the present work, which supersedes that of Ref. 19.]

We now add a (t-channel) σ-exchange graph. This is analogous to the σ exchange in the pseudoscalar πN case,[7] where the σ provides a simple mechanism for intermediate-range attraction, "pair suppression" (and chirality). The σ meson contributes equally to $K^+p\to K^+p$ and to $K^+n\to K^+n$ through the $A^{(I)}$ amplitudes, adding

TABLE I. Numerical values of the K^+-N low-energy scattering parameters for the various cases considered in the text (Secs. III and VI). The experimental results are from Ref. 16.

	$\frac{g_{KN\Lambda}}{\sqrt{4\pi}}$	$\frac{g_{KN\Sigma}}{\sqrt{4\pi}}$	$a_{0+}^{(I)}$ (fm) $I=1$	$I=0$	$r_{0+}^{(I)}$ (fm^3) $I=1$	$I=0$	$a_{1-}^{(I)}$ (fm^3) $I=1$	$I=0$	$a_{1+}^{(I)}$ (fm^3) $I=1$	$I=0$
PS Born	4.1	Small	-1.39	$+1.39$	0.044	-0.044	-0.016	0.016	0.034	-0.034
σ model	4.1	Small	-2.78	0	0.134[a]		-0.016[a]	0.014[a]	0.018[a]	-0.051[a]
			-2.78	0	0.742[b]		-0.226[b]	-0.192[b]	-0.193[b]	-0.261[b]
PS Born	2.0	0.8	-0.38	0.19	0.012		-0.004		0.009	
PS Born & $M_\Lambda=M_\Sigma=M_N$	2.0	0.8	-0.431	0.20						
σ model	2.0	0.8	-0.56	0	0.024[a]		-0.009[a]		0.007[a]	
					0.018[b]		-0.037[b]		-0.021[b]	
Experiment			-0.29 ±0.02	(-0.04)– 0.08	0.007– 0.017		(-0.031)– (-0.017)	0.085	0.009– 0.023	-0.02

[a]Results for a σ-meson mass $m_\sigma=1500$ MeV.
[b]Results for a σ-meson mass $m_\sigma=400$ MeV.

$$-\frac{g_s g_{\sigma K} m_\sigma}{t-m_\sigma^2} \tag{13}$$

to both, because the σ is a scalar-isoscalar meson. In Eq. (13), m_σ is the mass of the σ meson related to the present K^+N process (here it will be unrestricted, and presumably unrelated to the mass of the σ meson in the πN process). The two coupling constants g_s and $g_{\sigma K}$ describe the σNN and σKK vertices in the second diagram of Fig. 1.

We can now use this additional [σ exchange, Eq. (13)] term to fit one of the scattering parameters. Since $a_{0+}^{(I=0)}$ is consistent with zero [or, equivalently, $a_{0+}^{K^+p\to K^+p} = 2a_{0+}^{K^+n\to K^+n}$; this is also the prediction of current algebra with partially conserved axial-vector current[20] (PCAC)], we set $a_{0+}^{(I=0)}=0$ and solve for $g_s g_{\sigma K}/m_\sigma$. We obtain the result,

$$\frac{g_s g_{\sigma K}}{m_\sigma} = \alpha_{KN\Lambda}(m_K+M_\Lambda-M_N) - 3\alpha_{KN\Sigma}(m_K+M_\Sigma-M_N). \tag{14}$$

This result approximately doubles the (negative) value of $a_{0+}^{(I=1)}$, as the σ contribution reduces $a_{0+}^{(I=0)}$ and $a_{0+}^{(I=1)}$ by almost equal amounts (see second line of Table I). Thus, any improvement in the former worsens the latter relative to experiment. To summarize this part, we have found that the Born (u-channel) terms, using standard values of the coupling constants, yield results in very poor agreement with experiment. Adding a σ-exchange (t-channel) graph, we can reproduce the experimental value for the isospin-0 scattering length, but only at the expense of worsening the result obtained for the corresponding isospin-1 quantity.

The values of $r_{0+}^{(I)}$ and $a_{1\pm}^{(I)}$ depend on the mass m_σ in addition to the combination $g_s g_{\sigma K}/m_\sigma$ of Eq. (14). This is a result of the partial derivatives of A and B that are necessary in order to obtain these parameters [see Eqs. (6)–(9)]. To find a numerical value for this mass, one should again require that any of these parameters is fitted to the experimental value. As we shall see, fitting each of these parameters requires different values of m_σ (which means that the underlying model fails).

The experimental value of $r_{0+}^{(I=1)}$ is in the range 0.07–0.017 fm^3. This can by accomplished by choosing a mass $m_\sigma \gg 1.5$ GeV. Turning now to $a_{1\pm}^{(I=0,1)}$, we find that $m_\sigma \geq 1500$ MeV (which is an appropriate mass for a two-kaon resonance) yields a reasonable to good agreement between the calculated values of $a_{1\pm}^{(I=0,1)}$ and the experimental data. These results are shown in the second and third lines of Table I. Summarizing our results so far, we have found that values of m_σ needed to account for the quantities r_{0+} and $a_{1\pm}$ vary from one quantity to another. *It is clear that the proposed effective Lagrangian with the canonical values for the coupling constants cannot be used for a reliable and accurate description of processes involving K^+ mesons and nucleons at low energies.*

Our present results do not contradict a previous analysis by Alcock and Cottingham.[21] That analysis deals with high-energy K^+-N dynamics, and is therefore not directly related to our present calculation. Furthermore, Cottingham *et al.*[21] have mostly studied higher partial waves. These authors have, in fact, remarked[21] that the u-channel Λ exchange (which is included in our present work) could be significant for low partial waves.

Our motivation for using the tree level Feynman diagrams has been introduced above. One should realize, however, that in a more complete study it is necessary to use a unitarized t matrix. This may be obtained by treating the tree level Feynman diagrams as a quasipotential (or an effective potential[14]) (the driving term) to be iterated in the Bethe-Salpeter equation or[14] in some approximate dynamical scheme. Such treatments may be found in Davis *et al.*[22(a)] using particle exchanges and the Blankenbecler-Sugar equation, and in Veit *et al.*[22(b)] using a quark-model driving term in the Lippmann-Schwinger equation. In both cases, low partial wave KN dynamics is well reproduced. This treatment is outside the scope of the present work (we hope to address these questions in a future work; note that for the nuclear application, which is our final goal, it is necessary to use an arbitrary reference frame, and not the convenient c.m. frame).

It is possible to add the vector mesons, ω and ρ, to the calculation. In view of their crucial importance in a recent work[11] that we wish to discuss, we treat the $\omega+\rho$ contribution separately in the next section.

IV. THE ROLE OF THE ω- AND ρ-MESON EXCHANGES

In this section we study the effect of the ω- and ρ-meson exchanges on the low-energy K^+N scattering parameters.

The ω meson, which does not contribute in the $\pi+N \to \pi+N$ case, would contribute to $K^++N\to K^++N$ scattering. This t-channel exchange is shown in the third diagram of Fig. 1. Being a vector-isoscalar meson, the ω contributes equally to the $B^{(I=1)}$ and $B^{(I=0)}$ amplitudes, adding

$$\frac{2g_v g_{\omega K}}{t-m_\omega^2} \tag{15}$$

to both. [Comparing Eq. (15) with the σ contribution, Eq. (13), we note that the σ meson contributes equally to the A amplitudes, since it is a scalar-isoscalar meson, while the ω meson adds equal amounts to the B amplitudes, since it is a vector-isoscalar meson.] In Eq. (15), m_ω is the mass of the ω meson ($m_\omega = 783$ MeV),[23] and the two coupling constants g_v and $g_{\omega K}$ describe the coupling strengths at the ωNN and the ωKK vertices in the third diagram of Fig. 1.

The ρ-meson exchange will be treated in a similar fashion. However, it should be pointed out that the ρ meson has very subtle issues associated with it. The problem of including this meson in a renormalizable, chirally invariant Lagrangian, especially for use in the nuclear many-body problem (which may well be a desired goal of many readers), still remains unresolved.[6,7]

Being an isovector meson, the ρ contributes to the B amplitudes of $K^+p\to K^+p$ and $K^+n\to K^+n$ with opposite signs [unlike the (isoscalar) σ and ω mesons, cf. Eqs.

(13) and (15)]. In terms of the s-channel isospin, the ρ contributes

$$\frac{1}{2}\frac{g_\rho^2}{t-m_\rho^2} \quad (I=1) \tag{16a}$$

to $B^{(I=1)}$, and

$$-\frac{3}{2}\frac{g_\rho^2}{t-m_\rho^2} \quad (I=0) \tag{16b}$$

to $B^{(I=0)}$. In Eqs. (16) m_ρ is the mass of the ρ meson ($m_\rho=770$ MeV);[23] g_ρ is Sakurai's universal ρ-meson coupling constant,[16] which is equal to the $\rho\pi\pi$ coupling constant, while the ρKK and the ρNN coupling constants are[16] $g_{\rho K}=g_{\rho NN}=\frac{1}{2}g_\rho$ [this also comes out of simple quark counting considerations; I am grateful to J. V. Noble (private communication, 1988) for pointing this out to me]. We note that our results do not depend at all on Sakurai's universality assumption[16] for the ρ coupling. We do not make any use of this assumption, and the product of $g_{\rho K}$ and $g_{\rho NN}$ may be used instead.

The ratio of -1:3 for the ρ-meson contributions to the $I=1$ and $I=0$ B amplitudes, respectively, is in agreement with the isospin structure of the ρ-meson mediated K^+-N scattering. The nucleon and the K^+ meson have isospin $\frac{1}{2}$, so the pertinent isospin structure in the amplitude is $\tau_N\cdot\tau_K$ (which takes on the values of -3 for $I=0$ and $+1$ for $I=1$), giving rise to the above ratio.

The tensor coupling of the ρ meson to the nucleon will not be included in this work, for reasons similar to those discussed in Refs. 6 and 7, namely, nonrenormalizability. (Our goal, like theirs, is the nuclear many-body system.)

From the above discussion it follows that the combined contribution of the $\omega+\rho$ meson exchanges to the invariant T matrix [Eq. (2)] is through the B amplitudes, i.e.,

$$\begin{aligned} B^{(I=1)}(\omega+\rho)&=\frac{2g_v g_{\omega K}}{t-m_\omega^2}+\frac{1}{2}\frac{g_\rho^2}{t-m_\rho^2}\ , \\ B^{(I=0)}(\omega+\rho)&=\frac{2g_v g_{\omega K}}{t-m_\omega^2}-\frac{3}{2}\frac{g_\rho^2}{t-m_\rho^2}\ . \end{aligned} \tag{17}$$

Equation (17) is important for the subsequent discussion. We note that the $\omega+\rho$ by themselves could reproduce the experimental $a_{0+}^{(I=0,1)}$, since the ρ meson contribution has opposite signs for $I=0,1$, while the ω meson contribution is identical in the two cases. *There seems to be a common belief that this description of the K^+N scattering process is, indeed, satisfactory.*

Using Eqs. (4) and (17) to obtain expressions for $a_{0+}^{(I=0,1)}$, we can set $a_{0+}^{(I=0)}=0$, as in Sec. III [see Eq. (14)]. This *requirement* yields (using $m_\omega\simeq\omega_\rho$)

$$g_v g_{\omega K}=\tfrac{3}{4}g_\rho^2\ . \tag{18}$$

Using this result [Eq. (18)] in $a_{0+}^{(I=1)}$, we find that the experimental value for the latter, namely,

$$a_{0+}^{(I=1)}=-0.3\ \text{fm}\ ,$$

can be obtained for

$$\frac{g_\rho^2}{4\pi}=1.42\ .$$

This value is lower than any of the canonical values for g_ρ, which will be discussed in detail in the next section. Moreover, the resulting

$$\frac{g_v g_{\omega K}}{4\pi}=\frac{3}{4}\frac{g_\rho^2}{4\pi}=1.065\ ,$$

is only about 30–60 %of the canonical values.

The main point, however, is that the physics of the scattering mechanism with just the $\omega+\rho$ meson exchange is clearly incomplete. A more exhaustive and satisfactory description should include all the diagrams of Fig. 1. Results along these lines are given in the following sections.

In the next section we present numerical results (along the lines of Sec. III) for the K^+N scattering parameters with all the diagrams of Fig. 1 included, and remark on the real importance of the vector-meson exchange in the process.

V. FULL-MODEL RESULTS

In this section we present results for the K^+N scattering parameters based on the full model discussed in this work, as represented by all diagrams of Fig. 1. These will be referred to as the full-model results.

The various K^+N dynamical contributions discussed in Sec. III and IV may be combined to yield the full-model A and B amplitudes. The scattering parameters are then calculated as before. For example, the $I=1$ and $I=0$ scattering lengths are

$$a_{0+}^{(I=1)}=\eta\left[\alpha_{KN\Lambda}(m_K+M_\Lambda-M_N)+\alpha_{KN\Sigma}(m_K+M_\Sigma-M_N)+\frac{g_s g_{\sigma K}}{m_\sigma}-\frac{2g_v g_{\omega K}m_K}{m_\omega^2}-\frac{1}{2}\frac{g_\rho^2 m_K}{m_\rho^2}\right]\ , \tag{19a}$$

and

$$a_{0+}^{(I=0)}=\eta\left[-\alpha_{KN\Lambda}(m_K+M_\Lambda-M_N)+3\alpha_{KN\Sigma}(m_K+M_\Sigma-M_N)+\frac{g_s g_{\sigma K}}{m_\sigma}-\frac{2g_v g_{\omega K}m_K}{m_\omega^2}+\frac{3}{2}\frac{g_\rho^2 m_K}{m_\rho^2}\right]\ . \tag{19b}$$

The quantity η is defined in Eq. (5) and $\alpha_{KN\Lambda}$ and $\alpha_{KN\Sigma}$ are defined in Eq. (11). We now *set* $a_{0+}^{(I=0)}=0$ as before [see Eq. (14)]: this *requirement* yields

$$\frac{g_s g_{\sigma K}}{m_\sigma} - \frac{2g_v g_{\omega K} m_K}{m_\omega^2} = \alpha_{KN\Lambda}(m_K + M_\Lambda - M_N) - 3\alpha_{KN\Sigma}(m_K + M_\Sigma - M_N) - \frac{3}{2}\frac{g_\rho^2 m_K}{m_\rho^2} . \tag{20}$$

Substituting this result into Eq. (19a) yields

$$a_{0+}^{(I=1)} = 2\eta \left[\alpha_{KN\Lambda}(m_K + M_\Lambda - M_N) - \alpha_{KN\Sigma}(m_K + M_\Sigma - M_N) - \frac{g_\rho^2 m_K}{m_\rho^2} \right] . \tag{21}$$

Several important features in Eqs. (19)–(21) should be pointed out. (i) There is no contribution from ω meson exchange to the scattering lengths a_{0+} if the σ meson is also included (or vice versa). Adding the ω meson on top of the σ exchange contribution makes it necessary to change the combination $g_s g_{\sigma K}/m_\sigma$ when setting $a_{0+}^{(I=0)}$ to zero—cf. Eqs. (14) and (20), but $a_{0+}^{(I=0,1)}$ remain unchanged [see Eq. (21)]. This is, of course, a trivial result of the common isoscalar nature of the σ and the ω mesons. However, the above is only true for the scattering lengths a_{0+}, and we should expect the ω meson to affect the higher scattering parameters, which involve partial derivatives of the A and B amplitudes [see Eqs. (6)–(9)]. This is indeed the case and is discussed in the following (see Table III). As Eq. (21) makes very clear, the ρ-meson exchange contribution will make $a_{0+}^{(I=1)}$ even more negative, and thus will worsen the discrepancy between our theoretical full-model results and the experimental ones. (This discrepancy is already very large, see Table I.) The precise numerical effect of the ω and ρ mesons on the scattering parameters is described in the following.

For a quantitative study of the effects of the ω and ρ mesonic exchanges on the K^+N low-energy scattering parameters, the values of the pertinent coupling constants are required. Substantial uncertainty is found in the literature[16] regarding the precise values of these coupling constants, however lower and upper limits are available and will be used here. In Table II we quote these limits, taken from various sources.[16,24–26] It is interesting to note that Bütgen *et al.*[27] use values which satisfy the relation of Eq. (18), but they do not state the sources of their coupling constants.

Table III gives our full-model results for the scattering parameters. As expected, the effect of the ρ meson is to make $|a_{0+}^{(I=1)}|$ larger, and therefore worsen the discrepancy between our theory and the experiment. The ρ-meson contribution is non-negligible, but the dominant effects are already given by the Λ and σ exchanges (i.e., the first two diagrams of Fig. 1). This is similar to the π-N scattering case.[6,7]

We now turn to the higher scattering parameters in Table III. The quantititative results depend, of course, on the precise values chosen for the pertinent ω and ρ coupling constants, but the qualitative picture remains the same. To get the right value for $r_{0+}^{(I=1)}$ in this model requires large values of the σ meson mass, $m_\sigma \gg 1.5$ GeV, while the experimental values of $a_{1\mp}$ require $m_\sigma \geq 1.5$ GeV, *and* lower values for the relevant ω and ρ coupling constants. These results should be compared with our previous σ-model results of Table I (second and third lines).

Our previous conclusions at the end of Sec. III remain valid, and are not changed by the addition of the ω and ρ mesons to the calculation. Moreover, we conclude that the ω and ρ mesonic exchanges are of limited importance in the K^+N low-energy scattering process. The analysis of Ref. 11, based on a K^+N scattering model which takes into account only the vector $(\omega+\rho)$ meson exchanges need, therefore, to be modified. This is especially true since: (i) Ref. 11 deals with a small nuclear effect, of the order of 10%, so that a careful analysis is called for; (ii) Lower-energy data related to the same effect will be available in the very near future.[37] (In fact, the authors of Ref. 11 refer, in their second paragraph, to very low-energy K^+-nucleon scattering, the s-wave amplitude, and the $\rho+\omega$ exchanges.)

VI. RELATION TO THE $\gamma p \to K^+\Lambda$ REACTION

The kind of phenomenological Lagrangian considered here has been adopted for the description of another pro-

TABLE II. Lower and upper limits for the required ω- and ρ-meson coupling constants to the nucleon and the K meson taken from the quoted literature. Our notation has been introduced in Sec. IV, where Sakurai's universality for the ρ meson is also discussed.

$\frac{g_v^2}{4\pi}$	$\frac{g_v g_{\omega K}}{4\pi}$	$\frac{g_{\rho NN}^2}{4\pi}$ [a]	$\frac{g_\rho^2}{4\pi}$ [a]	$\frac{g_{\rho K} g_\rho}{4\pi}$	$\frac{g_{\rho K}}{g_\rho}$	Ref.
5.7–20		0.55–0.9				24
	1.5–3.0		2.3±0.7	1.5–3.7	0.5–1.3	16
8.7–12.5		0.63–0.95				25
6		0.67				26
			2.93			b

[a]Conventions for $g_{\rho NN}^2$ and g_ρ^2 differ by a factor of 4 among various sources in the literature.
[b]This value is extracted from $\rho \to 2\pi$ decay: see discussion in Sec. 7.6 of Ref. 6.

TABLE III. Numerical results of the K^+-N low-energy scattering parameters in the full model of Sec. V. The experimental results are from Ref. 16. (Compare with Table I.) The results for this table have been calculated for $g_{KN\Lambda}/\sqrt{4\pi}=4.1$ and $g_{KN\Sigma} \ll g_{KN\Lambda}$.

$\frac{g_\rho^2}{4\pi}$	$\frac{g_v g_{\omega K}}{4\pi}$	$a_{0+}^{(I)}$ (fm)[a] $I=1$	$a_{0+}^{(I)}$ (fm)[a] $I=0$	$r_{0+}^{(I)}$ (fm^3) $I=1$	$r_{0+}^{(I)}$ (fm^3) $I=0$	$a_{1-}^{(I)}$ (fm^3) $I=1$	$a_{1-}^{(I)}$ (fm^3) $I=0$	$a_{1+}^{(I)}$ (fm^3) $I=1$	$a_{1+}^{(I)}$ (fm^3) $I=0$	Remarks
2.0	1.5	−3.21	0	0.151	0.046	−0.057	0.0165	0.0004	−0.050	b
		−3.21	0	0.784	0.679	−0.268	−0.194	−0.211	−0.261	c
	3.5	−3.21	0	0.139		−0.096	−0.023	−0.012	−0.063	b
		−3.21	0	0.584		−0.244	−0.171	−0.161	−0.211	c
2.9	1.5	−3.40	0	0.163		−0.061	0.031	−0.003	−0.045	b
		−3.40	0	0.861		−0.294	−0.202	−0.236	−0.278	c
	3.5	−3.40	0	0.150	0.038	−0.101	−0.009	−0.016	−0.058	b
		−3.40	0	0.661	0.548	−0.271	−0.179	−0.186	−0.228	c
Experiment		−0.29±0.02	(−0.04)– 0.08	0.07– 0.017		(−0.031)– (−0.017)	0.085	0.009– 0.023	−0.02	

[a]Results for $a_{0+}^{(I)}$ are independent of the ω meson in this model: see Eq. (21). The other parameters *are* sensitive to the value of $g_v g_{\omega K}/4\pi$.
[b]Results for a σ-meson mass $m_\sigma=1500$ MeV.
[c]Results for a σ-meson mass $m_\sigma=400$ MeV.

cess involving K^+ mesons and baryons, namely, K^+ photoproduction. The description is again in complete analogy with the corresponding pionic reaction (i.e., π photoproduction based on an effective π-N Lagrangian). Such attempts[28–30] provide sets of coupling constants obtained by fitting the calculated cross section and Λ polarization to available experimental data for the reaction at photon energies around 1–2 GeV. The coupling constants obtained in this way differ substantially from the standard values discussed so far. In particular, calculated cross sections and Λ polarizations (on the basis of Feynman tree diagrams and the phenomenological Lagrangian) and experimental measurements appear to be in fair agreement for the values

$$\frac{g_{KN\Lambda}}{\sqrt{4\pi}} \simeq 2.0 \text{ and } \frac{g_{KN\Sigma}}{\sqrt{4\pi}} \simeq 0.8 \ .$$

[Strictly speaking, the (γ,K^+) process yields a value for the product of $g_{KN\Sigma}$ and μ_T, the Λ-Σ^0 transition moment. The latter is accurately measured,[31] so a value for $g_{KN\Sigma}$ has been deduced.] We have put these values to test in calculating the scattering parameters for $K^+ + N \to K^+ + N$.

Using the same expressions as in the previously discussed cases of Sec. III (lines 1–3 of Table I) we again calculate the Born (first diagram of Fig. 1) and σ model (first and second diagrams of Fig. 1) results for the scattering parameters of Table I. The ω and ρ mesonic exchanges will not be considered in this section since their effect has been shown to be relatively unimportant in the previous section (in the case of $a_{0+}^{(I)}$ the vector-meson exhanges enlarge the discrepancy between the calculated and experimental results). It is worthy of note that this time the Σ-pole contribution is of some importance in the calculation, especially in cases such as Eq. (12) where it is multiplied by a factor of 3. [For example, the ratios of the Λ-pole to Σ-pole Born term contributions are now approximately 5:1, 7.5:1, and 8:1 for $A_0^{(I=1)}$, $B_0^{(I=1)}$, and $C_0^{(I=1)}$, respectively; the only $K^+ n \to K^+ n$ Born diagram is a Σ^--pole one, and as a result the corresponding Σ contribution to the $I=0$ Born amplitudes is three times larger than for $I=1$. This puts the Λ:Σ contributions ratio at approximately 2:1.]

With the coupling constants determined this way, we obtain the *PS* Born numbers in the fourth line of Table I (which should be compared with the first line). The current numerical values of $a_{0+}^{(I=0,1)}$ are in much better agreement with the experimental results, considering the crudeness of the data used for obtaining the couplings, the uncertainties of the model, and the theoretical limitations of the method implied by the Kroll-Ruderman theorem.[32] *It is important to realize that the (γ,K^+) reaction is not a reliable source of information for determining the required coupling constants.* The Kroll-Ruderman theorem[32] states that photomeson production would provide an unambiguous means of measuring the (renormalized) meson-nucleon coupling constant only if the meson mass is much smaller than the nucleon mass. More precisely, the matrix element for a charged meson photoproduction at threshold, correct to all orders in the meson coupling constant and in the limit of a vanishing meson mass, is equivalent to the weak coupling result (obtained from second-order perturbation theory). Historically, this has been used to determine[32] the π-N coupling constant, where the mass ratio is 0.15. However, the precision ion is low, mainly due to theoretical uncertainty. As indicated by Kroll and Ruderman,[32] even the mass ratio of 0.15 is not small enough. This is certainly the case with K^+ mesons as well, where $m_{K^+}[\frac{1}{2}(M_N+M_\Lambda)]^{-1} \simeq 0.48$, and the coupling constant $g_{KN\Lambda}$ is comparable with the πNN one. Thus, *it is not clear a priori that the lowest order diagrams alone are enough to determine coupling constants in the $\gamma + p \to K^+ + \Lambda$ case.* We note, however, that soft kaon theorems seem to be a reasonable starting point in many calculations;[33] furthermore, one is dealing here with an effective, phenomenological Lagrangian, which incorporates current algebra and PCAC and may not be an appropriate basis for a complete field theory, so the method may eventually turn out to be a *a posteriori* justifiable. This question requires further clarification and would constitute an interesting line of research with the advent of the Advanced Hadronic Facilities.

The effect of the photoproduction coupling constants on the rest of the scattering parameters is some improvement relative to the experimental results.

At this point we note that results for $M_\Lambda = M_\Sigma = M_N$ (fifth line of Table I) are quite similar to the results of the previous line (where $M_\Sigma > M_\Lambda > M_N$). We thus find an indication that we do not have to worry about SU(3) symmetry-breaking effects at this point. This is a delicate (and potentially disturbing) point, as discussed by Schechter, Ueda, and Venturi.[34]

The σ-model results (first *and* second diagrams in Fig. 1) are obtained once again by satisfying the *requirement* $a_{0+}^{(I=0)}=0$, as before. The results appear in the sixth line of Table I. We note that using the σ model changes $a_{0+}^{(I=0)}$ from -0.38 to -0.56 fm, and ($a_{0+}^{(I=1)}$ from 0.19 fm to 0, respectively), and that these values are in fair agreement with the experimental results (that is, to within a factor of 2). They depend on the combination $g_s g_{\sigma K}/m_\sigma$ (and not on the mass m_σ itself). The rest of the parameters, however, depend on m_σ as well. Here the situation is different from the corresponding previous one (i.e., that of Sec. III and Table I, line 2). To fit $r_{0+}^{(I=1)}$ requires $m_\sigma \leq 400$ MeV, which might be too low a value if the particle is thought of as a two-kaon resonance. To get the right value for $a_{1-}^{(I=1)}$ we need $m_\sigma \simeq 400$ MeV, while $a_{1+}^{(I=1)}$ gets closest to the experimental value for $m_\sigma \to \infty$. Since all these quantities are quite sensitive to variations of m_σ, a universal agreement with experiment is ruled out.

We have thus found in this section that with the coupling constants obtained from the photoproduction data, the values obtained for the scattering lengths are in a much better agreement with experiment; but even in this case no good qualitative agreement between the theoretical results and the experiment could be achieved.

VII. DISCUSSION AND CONCLUSIONS

Before discussing the meaning of our results we should present a comparison with a similar calculation in the πN case.[7] While a_{0+} and a_{1+} are fairly well accounted for (to within a factor of 2), the experimental results for r_{0+} are essentially unexplained theoretically for low-energy πN scattering (a 1–2 orders of magnitude difference is found) and the results for a_{1-} are off by a factor of 3. The main success in the πN sector is the sum rule

$$\tfrac{3}{2}(a_{0+}^{(\pi^+ p)} + a_{0+}^{(\pi^- p)}) = 2a^{(I=3/2)} + a^{(I=1/2)} \simeq 0 ,$$

or, in terms of t-channel isospin,

$$a_{0+}^{\text{isoscalar}} \simeq 0 .$$

We keep this information in mind while phrasing our conclusions.

At this level of calculation we are able to make the following conclusions:

(i) A tree approximation description of K^+N low-energy scattering, using the standard values of coupling constants, does not provide a realistic model for the process. This conclusion should be borne in mind when nuclear physics applications based on the tree level meson-nucleus interaction are attempted for the K^+ meson.

(ii) The second set of coupling constants discussed here (obtained from the $\gamma p \to K^+\Lambda$ data analyzed at the tree level) is more favorable, while the standard values of the coupling constants seem to be ruled out by the data when this model is used. This is a somewhat surprising result, since the values of Ref. 16 appear to be well estabilished and cross checked. Based on this conclusion, the low-energy πN and K^+N scattering are theoretically accounted for at a similar level of accuracy. However, a universal agreement with experiment has not been found in either case.

(iii) The theoretical description K^+N scattering in terms of ρ and ω exchange alone is not possible, and is clearly incomplete. A more exhaustive and satisfactory description should include all diagrams of Fig. 1. Moreover, we conclude that the ω and ρ mesonic exchanges are of limited importance in the K^+N low-energy scattering process.

As noted earler, we do not attempt to build a model for the elementary processes ($K^+N \to K^+N$ and $\gamma p \to K^+\Lambda$). Rather, we have put to test a model that has been used in the field for a long time and is in current use as well. Although we have drawn several conclusions from this work, there is still a great deal of further work left to be done; it may well affect our present results. A number of pertinent points for consideration and further research are discussed below (we have indicated the more conservative ideas first, and then the more speculative ones).

The significance of the σ model in our calculation is not clear, since it makes some parameters come closer to experiment, while other values get worse.

As explained in Sec. III, the tree level may not be a satisfactory approximation to the scattering matrix, especially at higher energies (mainly due to lack of unitarity). The effects of iterations in the Bethe-Salpeter equation, and their implementation in nuclear calculations, are evidently an interesting question to be addressed in future investigations.

Further changes in the coupling constants $g_{KN\Lambda}$ and $g_{KN\Sigma}$ extracted from the photoproduction reaction can still be expected in view of the limitations of this reaction in providing information about these quantities. Such changes are likely to affect the results of Sec. VI.

The effective Lagrangian based on the σ model may not be a good starting point for the description of processes involving mesons and baryons. In that case, closely related models may also be inappropriate; this might be the reason behind the failures to describe π-nuclear dynamics in a consistent relativistic nuclear field theory based on chiral phenomenology.[7]

The origin of the problem may also lie in the K-baryon system because of the large mass of the kaon, but, at the same time, may not affect the πN system, due to the small value of m_π. To put this issue in the right perspective we note that, for example, calculating for $\pi\pi$ scattering in this approach is known to be problematic, possibly because the pion mass is no longer small compared with other masses in the problem[20,35] (as in πN scattering, where the approach is more successful); π-nuclear dynamics may also be wrongly described by the model because characteristic nuclear excitation energies are of the order of $m_\pi/15$, so that the pion mass is relatively very large in this case.[20,35] The consideration of hard-meson corrections may be essential. The role of pair suppression in the two cases should also be looked into.

A more speculative possibility concerns the contributions from Z^*'s (five-quark resonances) in the same fashion that Y^*'s contribute to $\bar{K}N$ scattering. We should then have to consider the s-channel Born diagram(s) in addition to the u- and t-channel ones, with a possibility of large contributions from this source and large corresponding changes in the calculated final values of the scattering parameters. [In such a case, the apparent disagreement between the two processes considered, namely, $K^+N \to K^+N$ and $\gamma p \to K^+\Lambda$, may be due to theoretical problems in extracting coupling-constants from photoproduction processes (see discussion at the end of Sec. VI).]

The issues considered in this work become increasingly important with the advent of the Advanced Hadronic Facilities.[36] We believe that additional tests and caution are needed for a reliable interpretation of our results. [Specifically, it might be necessary to carefully construct a chirally-SU(3)-symmetric model for dealing with the problem, and use it to study additional reactions such as $\pi^+ + N \to K^+ + \Lambda$ as well as reactions involving the Σ hyperon. A calculation of the K^+N scattering phase shifts using the present model could also provide an additional test of our conclusions.] It is not clear at present that the theoretical approach adopted here is completely satisfactory. We *have shown,* however, that formidable problems are associated with the use of an effective Lagrangian and tree level Feynman diagrams in the context of processes involving K^+ mesons, nucleons, and hyperons.

ACKNOWLEDGMENTS

I am grateful to Carl B. Dover for a useful correspondence regarding Ref. 11 and to Julian V. Noble, Al S. Rosenthal, Brian D. Serot, Roy Thaler, Charlotte Elster, and Hans-J. Weber for useful discussions. This work was supported by the U.S. Department of Energy (DOE) at Case Western Reserve University and the Los Alamos National Laboratory (LANL) and by the U.S. National Science Foundation at Indiana University. The LANL is operated by the University of California for the U.S. DOE.

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