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Cluster-model calculations of exotic decays from heavy nuclei

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A cluster model employing a local, effective cluster-core potential is used to investigate exotic decay from heavy nuclei as a quantum tunneling phenomenon within a semiclassical approximation. Excellent agreement with all reported experimental measurements of the decay widths for ^{14}C and ^{24}Ne emission is obtained. As an added bonus, the width for α particle emission for ^{212}Po is also calculated in good agreement with experiment.

Four years ago Rose and Jones[1] startled nuclear physicists by successfully observing a new type of naturally occurring radioactivity in the form of ^{14}C emission from a ^{223}Ra source (which was itself produced in the decay chain of ^{227}Ac). This result was soon reproduced and confirmed, with better statistics and unambiguous charge and mass identification, by Gales *et al.*[2] Subsequently, ^{14}C emission was also observed from several other isotopes of radium.[3-6] In addition, another new form of natural radioactivity involving ^{24}Ne emission from ^{231}Pa, ^{232}U, and ^{233}U (Refs. 7-9) has recently been identified, and has stimulated the search for more exotic decay modes in which there is ejection of even heavier nuclei like Mg and Si.

Several theoretical descriptions of these exotic decays, of varying degrees of complexity, have already appeared in the literature[10-12] and they generally produce lifetimes which are within factors of between 10 and 100 of the experimentally observed values. In this Rapid Communication we wish to point out that all the available lifetime measurements on exotic decays from heavy nuclei can be successfully reproduced, to within factors of between 0.1 and 0.5, by a simple extension of the Buck-Dover-Vary cluster model[13] to treat the heavy nucleus as an appropriate ^{14}C or ^{24}Ne cluster plus core system.

This model describes the interaction between a cluster and a core in terms of a simple, local potential $V(r)$ which may be obtained from a double folding integral involving the cluster and core densities $\rho_1(\mathbf{r}_1)$ and $\rho_2(\mathbf{r}_2)$, and an effective nucleon-nucleon potential $U(|\mathbf{r}_1-\mathbf{r}_2|)$. Hence

$$V(r)=\int\int\rho_1(\mathbf{r}_1)\rho_2(\mathbf{r}_2)U(|\mathbf{r}+\mathbf{r}_2-\mathbf{r}_1|)d^3r_1d^3r_2\,, \quad (1)$$

and this may itself by closely approximated by a simple three-parameter form such as

$$V(r)=\frac{-V_0[1+\cosh(R/a)]}{\cosh(r/a)+\cosh(R/a)}\,, \quad (2)$$

where V_0 is the depth of the potential and R and a are related to its radius and diffuseness. The single-particle Schrödinger equation is now solved with this potential, and the bound- and resonant-state solutions are identified with the states of relative motion of the cluster about the core.

The main requirements of the Pauli exclusion principle are satisfied by choosing the quantum numbers of relative motion n (the number of interior nodes in the radial wave function) and L (the orbital angular momentum) to obey a Wildermuth condition, $2n+L\geq N$, where N is a constant integer chosen large enough to correspond to the microscopic situation in which the cluster nucleons all occupy orbitals above those already occupied by the core nucleons. Any remaining effects of antisymmetrization may be absorbed into the effective potential.

One of the major successes of this model was its ability to predict α decay widths for a large number of states in ^{16}O and ^{20}Ne (Refs. 13-15) in excellent agreement with experiment, in contrast to earlier calculations employing Fermi function potentials, which typically overestimated these quantities by factors of 5 or more. In these cases the critical difference seems to be a far better description of the shape of the barrier by the folded potential, and we shall now investigate the emission of ^{14}C and ^{24}Ne from heavy nuclei with this model and show that similar successes may be achieved there.

For some very simple parametrizations of the nuclear densities and N-N interaction, the folding integral of Eq. (1) may be performed analytically in coordinate space. Some convenient two- and three-parameter representations of the charge densities of ^{14}C and ^{208}Pb are suggested in Ref. 16, and if these are adopted, together with a Dirac δ function for U, then the six-dimensional integral of Eq. (1) may be analytically reduced to a one-dimensional integral, which is ideal for numerical approximation using a Gauss-Hermite quadrature. More generally however, it is convenient to take the Fourier transform of Eq. (1) and then make use of the convolution theorem to transform the folding integral to a product of the individual transforms of the densities and interaction

$$\tilde{V}(q)=\tilde{\rho}_1(q)\tilde{\rho}_2(-q)\tilde{U}(q)\,. \quad (3)$$

This form is particularly convenient when we wish to convert nuclear charge densities ρ_{ch} obtained from electron scattering, to the desired matter or point densities ρ. Fol-

lowing Cook[17] we write

$$\tilde{\rho}(q)=(1+N/Z)\tilde{\rho}_{\rm ch}(q)/\tilde{\rho}_{N,\rm ch}(q) \quad (4)$$

for a nucleus containing N neutrons and Z protons where

$$\rho_{N,\rm ch}(r)=\alpha^3/8\pi\exp(-\alpha r) \quad (5)$$

and α is chosen to reproduce the nucleon mean square charge radius through

$$\langle r^2\rangle_N=12/\alpha^2=0.76-0.11(N/Z)\ \text{fm}^2. \quad (6)$$

Bearing this in mind, the universal symmetrized Fermi density function proposed by Burov, Eldyshev, Lukyanov, and Pol[18] to fit the electron scattering data for all nuclei,

$$\rho_{SF}(r)=\frac{\rho_0\sinh(R/b)}{\cosh(r/b)+\cosh(R/b)}, \quad (7)$$

is extremely useful since it admits an analytic evaluation of its Fourier transform as

$$\tilde{\rho}_{SF}(q)=-\frac{4\pi^2 bR\rho_0}{q\sinh(\pi bq)}\times\left[\cos(qR)-\frac{\pi b}{R}\sin(qR)\coth(\pi bq)\right]. \quad (8)$$

An N-N interaction with an analytic form for its Fourier transform, such as a Dirac δ function or a Gaussian, thus allows a completely analytic evaluation of $\tilde{V}(q)$, which can in turn be inverted by standard techniques.

In practice, we take the shape of $V(r)$ from this folding procedure, but adjust the precise value of its depth to reproduce exactly the cluster-core separation energy (as determined from the mass defect tabulations of Wapstra and Audi[19] for each individual case. When this is done, it is also found that the results are insensitive to the exact value of N ($=2n+L$) over a wide range.

Even when the cluster-core potential has been determined, the width of the resonant states for ^{14}C or ^{24}Ne+core systems cannot be calculated by the usual quantum-mechanical procedures of watching the phase shift rise through $\pi/2$ as the energy is varied, or by numerically solving the Schrödinger equation using complex arithmetic, since we are trying to obtain a width of 10^{-26} eV or less for a state whose energy is around 30 MeV (^{14}C clusters) or 60 MeV (^{24}Ne clusters). Instead, we make use of the two-potential approach to the decay of a quasistationary state recently developed by Gurvitz and Kalbermann.[20]

In the semiclassical limit, this leads to an expression for the decay width of a resonant state in terms of the three turning points r_0, r_1, and r_2 (in order of increasing distance from the origin) of the total cluster-core potential $V_T(r)$ (which contains nuclear, Coulomb, and centrifugal terms) and the energy E_0 of the equivalent bound state in the related potential $V_M(r)$ obtained by flattening off the barrier at its maximum value (occurring at $r=r_B$). In other words, $V_M(r)=V_T(r)$ for $r\le r_B$ and $V_M(r)=V_T(r_B)$ for $r\ge r_B$. This essentially leads to the Gamow formula with a well determined pre-exponential factor. If we assume that our heavy nucleus decays from its ground state (spin I_i) to produce a core in its ground state (spin I_f) and a spinless cluster, in a state of unique relative orbital angular momentum L, then this gives a width Γ where

$$\Gamma=\frac{(2I_i+1)}{(2I_f+1)(2L+1)}\frac{F\hbar^2}{4\mu}\exp\left[-2\int_{r_1}^{r_2}k(r)dr\right], \quad (9)$$

where μ is the reduced mass of the cluster-core system, $k(r)$ is the semiclassical wave number

$$k(r)=\left[\frac{2m}{\hbar^2}[E_0-V_T(r)]\right]^{1/2}, \quad (10)$$

and F is the semiclassical bound state normalization factor

$$F\int_{r_0}^{r_1}\frac{1}{k(r)}\cos^2\left[\int_{r_0}^{r}k(r')dr'-\frac{\pi}{4}\right]dr=1. \quad (11)$$

We make the Langer modification, in which $L(L+1)$ is replaced by $(L+\frac{1}{2})^2$, so that three turning points are always obtained and all our integrals are well defined.

We have checked this procedure against quantum-mechanical width determinations for states of the α-^{40}Ca and α-α systems, and found it to be satisfactory. Specifically, we found 0.045 and 29.0 eV for the widths of the 8^+ and 10^+ members of the yrast band in ^{44}Ti, compared with values of 0.051 and 27.1 eV obtained from a numerical solution of the Schrödinger equation using complex arithmetic.[21] Also, we obtained a value of 5.6 eV for the width of ^{8}Be in its 0^+ ground state, compared with the value of 5.8 eV found by Buck, Friedrich, and Wheatley.[22] We now proceed to use this method to calculate decay widths for ^{14}C and ^{24}Ne emission from heavy nuclei.

The Wilmermuth condition $2n+L\ge N$ requires us to place all the ^{14}C or ^{24}Ne cluster nucleons in orbitals outside the $Z=82$ and $N=126$ proton and neutron shell closures. Therefore, each cluster proton contributes at least 5 quanta to N, and each neutron contributes at least 6 quanta. Allowing minimum values of 10 and 28 quanta for internal motion of ^{14}C and ^{24}Ne in their respective ground states, we arrive at the conditions $2n+L\ge 68$ for ^{14}C and $2n+L\ge 106$ for ^{24}Ne clusters. As mentioned above, the results do not depend critically on the exact values chosen for $N=2n+L$.

We produce the simplest possible folded potential by using a Dirac δ function for the N-N interaction and Burov charge densities with $R=6.557$ fm and $b=0.515$ fm for the core and $R=2.214$ fm and $b=0.488$ fm for ^{14}C in Eqs. (7) and (8). The depth of the folded potential is fine tuned to reproduce the cluster-core separation energy with respect to the heavy parent nucleus. The energy E_0 and width Γ are then calculated as outlined above assuming a Coulomb potential appropriate to that of a uniformly charged spherical core of radius 5.5 fm interacting with a pointlike cluster.

For ^{14}C emission from ^{222}Ra, ^{224}Ra, and ^{226}Ra all ground-state spins are zero, so angular momentum and parity conservation lead us to the unique value of $L=0$ for these decays. For $^{223}\text{Ra}\rightarrow{}^{209}\text{Pb}+{}^{14}\text{C}$, the parent spin-parity is $\frac{1}{2}^+$ while the heavy core spin is $\frac{9}{2}^-$, so that we again have a unique assignment of $L=5$. Unfortunately, the spin of ^{221}Ra is unknown, but since ^{207}Pb has a spin of

TABLE I. Calculated and experimental values of the partial widths Γ (eV) for the emission of the indicated cluster from some heavy nuclei. A consistent geometry of $R=6.7$ fm and $a=0.75$ fm is used in all applications of the cosh potential of Eq. (2). Experimental data are from Refs. 1–9.

System	Γ (eV)		
	Folded potential	cosh potential	Experiment
$\alpha+{}^{208}\mathrm{Pb}={}^{212}\mathrm{Po}$	6.1×10^{-9}	1.0×10^{-8}	2.2×10^{-9}
${}^{14}\mathrm{C}+{}^{207}\mathrm{Pb}={}^{221}\mathrm{Ra}$	$\leq 4\times10^{-27}$	$\leq 6\times10^{-28}$	$<10^{-28}$
${}^{14}\mathrm{C}+{}^{208}\mathrm{Pb}={}^{222}\mathrm{Ra}$	5.8×10^{-26}	9.8×10^{-27}	6.4×10^{-27}
${}^{14}\mathrm{C}+{}^{209}\mathrm{Pb}={}^{223}\mathrm{Ra}$	2.6×10^{-30}	4.8×10^{-31}	4.0×10^{-31}
${}^{14}\mathrm{C}+{}^{210}\mathrm{Pb}={}^{224}\mathrm{Ra}$	6.5×10^{-31}	1.1×10^{-31}	9.0×10^{-32}
${}^{14}\mathrm{C}+{}^{212}\mathrm{Pb}={}^{226}\mathrm{Ra}$	4.1×10^{-36}	6.5×10^{-37}	4.2×10^{-37}
${}^{24}\mathrm{Ne}+{}^{207}\mathrm{Tl}={}^{231}\mathrm{Pa}$		4.2×10^{-38}	3.8×10^{-39}
${}^{24}\mathrm{Ne}+{}^{208}\mathrm{Pb}={}^{232}\mathrm{U}$		1.3×10^{-36}	3.0×10^{-37}
${}^{24}\mathrm{Ne}+{}^{209}\mathrm{Pb}={}^{233}\mathrm{U}$		$\leq 2\times10^{-40}$	9.9×10^{-41}

$\frac{1}{2}$, we can calculate an upper limit on the ${}^{14}\mathrm{C}$ emission width by treating it as an $L=0$ resonance and setting the statistical factor equal to 1. Our results are presented in Table I, and are clearly in very good agreement with the data.

Unfortunately, no reliable charge density is available for ${}^{24}\mathrm{Ne}$, so we have not been able to perform similar calculations for such emissions. However, we did apply our method to the α decay of ${}^{212}\mathrm{Po}$, using an α particle matter density of $\exp(-r^2/a^2)$ with $a=1.41$ fm (Ref. 23) and a Wildermuth condition of $2n+L\geq 22$ and obtained gratifyingly good agreement with experiment (Table I).

In view of its previous successes in light nuclei, we then looked for a cosh potential parametrization [Eq. (2)] of our folded potential to simplify our calculations even further. We found that values of $R=6.7$ fm and $a=0.75$ fm in Eq. (2) (with V_0 chosen separately for each decay so that the energy E_0 equalled the appropriate cluster-core separation energy) reproduced the shape of the folded potentials very closely. The corresponding ${}^{14}\mathrm{C}$ emission widths and the ${}^{212}\mathrm{Po}\rightarrow{}^{208}\mathrm{Pb}+\alpha$ decay width are shown in Table I and seen to be in excellent agreement with the experimental measurements. In fact, the values of R and a for the ${}^{208}\mathrm{Pb}+\alpha$ system should be a little smaller to optimize the fit to the folded potential, but we have not made any such adjustment so as to keep our number of free parameters to a minimum.

Since the same cosh potential geometry is adequate for Pb-like cores interacting with both α and ${}^{14}\mathrm{C}$ clusters, we decided to test it out for the ${}^{24}\mathrm{Ne}$ decays also. Our results (see Table I) are again in excellent agreement with the measured values. In these decays, ${}^{231}\mathrm{Pa}\rightarrow{}^{207}\mathrm{Tl}+{}^{24}\mathrm{Ne}$ and ${}^{232}\mathrm{U}\rightarrow{}^{208}\mathrm{Pb}+{}^{24}\mathrm{Ne}$ have unique assignments of $L=1$ and $L=0$, respectively. However, the decay ${}^{233}\mathrm{U}\rightarrow{}^{209}\mathrm{Pb}+{}^{24}\mathrm{Ne}$ is from a state of spin-parity $\frac{5}{2}^+$ to one of spin-parity $\frac{9}{2}^+$ and could have $L=2$, 4, or 6. We have adopted the value $L=2$, and hence only claim the status of upper limit for our result.

We were naturally curious to investigate the spectra produced by our folded and cosh potentials, to see if we could also describe the excited states of heavy (actinide) nuclei with our model. However, we found that the energies of the states in the rotational bands we generated were almost equal, thus not being as widely spaced as the measured values.[24] Also, our calculated reduced $E2$ transitions strengths $B(E2\uparrow;0^+\rightarrow 2^+)$ in the even-even nuclei were between a third and a half of the values deduced from Coulomb excitation experiments.[25]

The decay widths we have calculated range from 10^{-40} to 10^{-26} eV, and thus represent a substantial probing of the precise shape of the barrier. It would only be possible for us to correlate all these values so successfully with experiment if we did indeed have the shape of the barrier essentially correct. This is the crucial aspect of our calculation, resulting from our prescription for the cluster-core potential, which allows us to reproduce the decay widths so accurately. The inner and outer turning points are essentially determined by the centrifugal and Coulomb potentials, and it is the position of the critical middle turning point r_1 which truly determines the barrier penetrability and hence the decay width.

We conclude that a semiclassical barrier penetration calculation using a simple cluster model, where the cluster-core potential is produced by a double folding integral and may be accurately described by a simple three-parameter shape, is adequate to reproduce all the presently available experimental data on ${}^{14}\mathrm{C}$ and ${}^{24}\mathrm{Ne}$ emission from heavy nuclei. In addition, this same procedure is able to give the α decay width for ${}^{212}\mathrm{Po}\rightarrow{}^{208}\mathrm{Pb}+\alpha$ disintegration with similar accuracy.

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