Role of static hexadecapole deformations in subbarrier fusion reactions between heavy ions

J. Fernández-Niello
Sektion Physik der Universität München, D-8046 Garching, West Germany and TANDAR, Comisión Nacional de Energía Atómica, 1429 Buenos Aires, Argentina

C. H. Dasso
The Niels Bohr Institute, University of Copenhagen, DK-2100, Copenhagen Ø, Denmark and Sektion Physik der Universität München, D-8046 Garching, West Germany
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A systematic study of the contribution of $\lambda=4$ deformations to the enhancement of subbarrier fusion cross sections is carried out. The analysis is based on calculations that cover the full range of values of hexadecapole deformations found in actual nuclear systems. The interplay of this shape degree of freedom with the presence of prolate quadrupole deformations is also comtemplated.

Two recent publications[1,2] have brought attention to the possible role of static hexadecapole deformations in the enhancement of subbarrier fusion cross sections. There is, however, a clear conflict in the nature of the shape distortions that are emphasized. While in Ref. 1, "strong effects" are claimed from *negative* $\lambda=4$ deformations in the reaction $^{16}O+^{184}W$, even more pronounced enhancements are attributed in Ref. 2 to the *positive* hexadecapole deformation of thorium in the reaction $^{16}O+^{232}Th$.

Some qualitative arguments are advanced to explain the origin of these effects. These, however, were not quite complete. In this report we reexamine the arguments that can be put forward to understand the role of both positive and negative hexadecapole shapes. We also study the spectrum of possibilities that result from combining the hexadecapole distortions with characteristic prolate quadrupole deformations. To conduct this extensive survey we take advantage of a recently developed, updated version of the finite-range simplified coupled channel code CCFUS (Ref. 4) (term CCDEF, cf. Ref. 5), which allows for static deformations in both projectile and target. The accuracy of its predictions, tested against the calculations in Ref. 2, are found to be excellent.

For the purposes of this study we consider one of the reaction partners to be spherical. This specification is the most convenient to investigate our problem as it singles out the shape effects that are to be attributed to only one of the colliding ions. In what follows, the nuclear shapes are specified by a pair of parameters β_2,β_4 according to

$$R(\theta)=R_0[1+\beta_2 Y_{20}(\theta)+\beta_4 Y_{40}(\theta)] \ . \tag{1}$$

The role played by static deformations of the nuclear surfaces can be adequately investigated within the sudden approximation. This is because the moment of inertia of the deformed nucleus is typically so large that practically no rotation of the system takes place during the characteristic time in which the nuclei remain in contact. In this limit, the relevant cross sections are constructed by averaging over the results of calculations performed for all possible relative orientations between the projectile and target.[3]

There are fundamentally two ingredients which determine the outcome of this sampling procedure. On the one hand we have the effective modulation of the barrier height associated with a given spatial orientation. This is essentially determined by the shift of the nuclear surfaces measured with respect to a spherical (or prolate-deformed) reference. The barrier is normally lowered when the effect of the deformation is to bring the surfaces closer together while it is raised if the gap between the nuclei is broadened. (Other lower-order effects enter also in the picture; these are, for example, changes in the curvatures of the surfaces at the point of contact, deviations of this spot from the core-core radius, etc.) We have, on the other hand, the statistical weight of the different orientations that clearly favor hitting the equator of the deformed system rather than its poles.

One way to visualize the blending of these two ingredients is to plot[2] both the height of the effective barriers and their statistical weights as a function of the angle χ between the relative motion variable **r** and the symmetry axis of the rotor. We display in Figs. 1(a) and 1(b) the typical situation resulting from positive and a negative hexadecapole moments of the same magnitude. In this example we have also added a quadrupole deformation with $\beta_2=0.2$, which makes the illustration useful for characteristic situations encountered in real nuclear systems. The modulation of the effective barrier that appears *a priori* more interesting corresponds to $\beta_4<0$, since it lowers the height of the barrier for the orientations that are statistically favored. The positive deformations, instead, reduce the effective barriers mostly at the pole, a configuration which is formed with low probability.

On the other hand, the exponential character of the transmission coefficient as a function of the bombarding energy is such that the gain in fusion probability that results from orientation angles in the vicinity of the poles compensates by far for their rare occurrence. For this reason, rather than plotting $\tilde{V}_B$ and $\sin\chi$ as a function of χ we propose to use the quantity

$$\mathcal{F}(\chi)=\sin\chi\exp\left[\frac{E-\tilde{V}_B(\chi)}{\epsilon}\right]_{\beta_2,\beta_4} -\sin\chi\exp\left[\frac{E-\tilde{V}_B(\chi)}{\varepsilon}\right]_{\beta_2,\beta_4=0}, \tag{2}$$

which directly reflects where and by how much the fusion cross section is affected. Here E stands for the center-of-mass energy of the system and ϵ is related to the curvature of the potential barrier. The indicator $\mathcal{F}$ is plotted in Figs. 1(c) and (d) for the same cases as depicted in Figs. 1 (a) and (b).

It is apparent for this picture that the range of latitudes that significantly enhances the fusion in the case of positive hexadecapole deformations is about 20° wide. This identifies the "poles" of the deformed nucleus as indeed the most favorable orientation to increase the fusion rates. For negative deformations one can immediately appreciate from Fig. 1(d) that (i) the enhancement effects are not particularly large at $\chi\approx45°$, in spite of the larger weights; (ii) these are in any case almost canceled by the loss of fusion cross section, resulting from the flattening of the nucleus where it counts most (i.e., at $\chi\approx0°$). Note also that the surface of the deformed nucleus recedes back from the spherical reference at $\chi=90°$ for either sign of β_4, due to the prolate shape defined by $\beta_2=0.2$. Thus, this region is of no relevance for modulations of the enhancements when measured in absolute terms with respect to $\beta_4=0$, as indicated in Eq. (2).

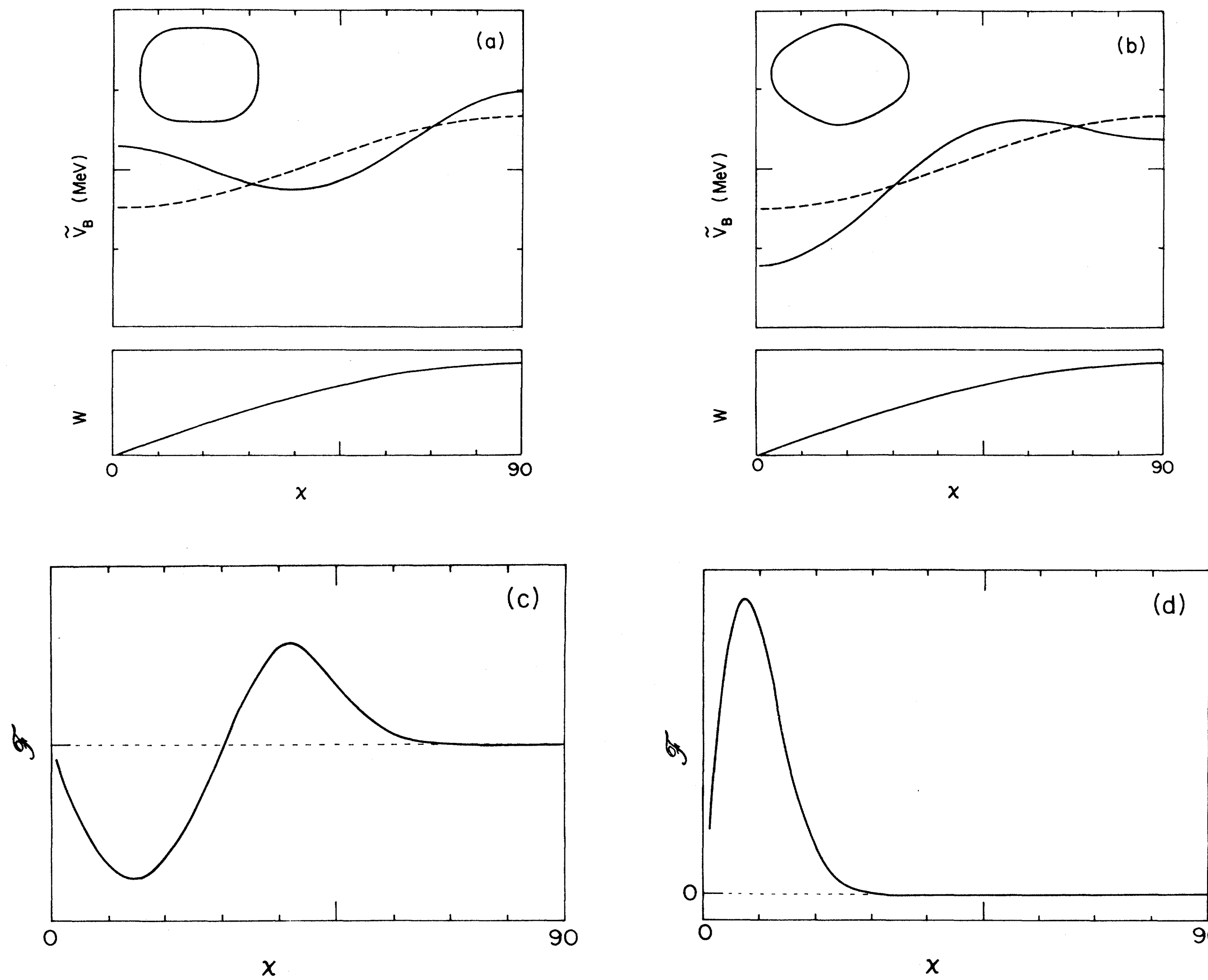

FIG. 1. Effective barriers $\tilde{V}_B$ statistical weights, and the indicator $\mathcal{F}$ as a function of the angle χ between the symmetry axis of a prolate-deformed target with $\beta_2=0.2$ and the vector **r** connecting the centers of the two ions. The plots in the left half of the figure [i.e., (a) and (c)] correspond to a negative hexadecapole deformation $\beta_4=-0.1$ while the right half [i.e., (b) and (d)] correspond to a positive hexadecapole deformation of the same magnitude, $\beta_4=+0.1$ for the system $^{16}O+^{232}Th$. The insets represent a cut of the nuclear profile for the given values of β_2, β_4. In these plots the dashed curve indicates the reference, obtained for $\beta_4=0$.

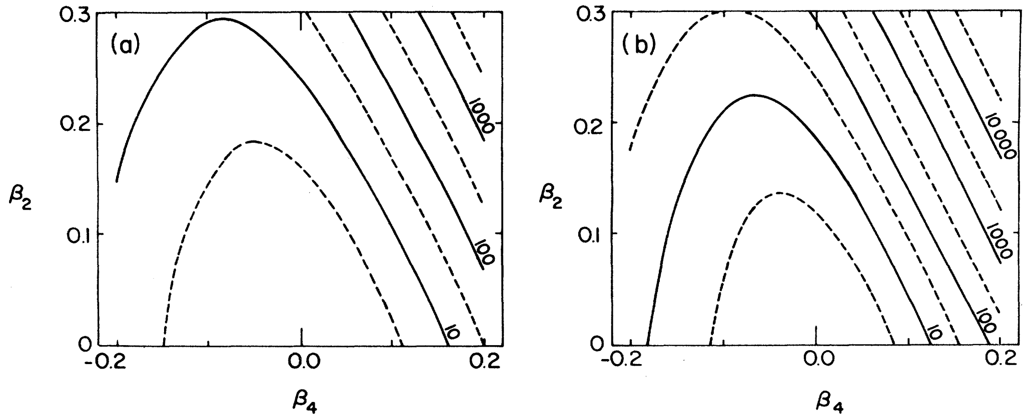

FIG. 2. Contour of the enhancement $\mathcal{E}$ as defined in the text for an energy E well below the coulomb barrier V_B, in the two-dimensional plane spanned by β_2,β_4. The ranges of these parameters have been chosen to cover the typical situations encountered in heavy ions. In (a) the numerical calculation was performed for a combination of projectile and target masses identical to the system investigated in Ref. 1. The contours in (b) correspond, instead, to masses as the ones used in Ref. 2. In either case, the solid lines mark the enhancements for exact powers of 10, as indicated. The actual values of β_2,β_4 for ^{184}W and ^{232}Th are 0.26, -0.19 and 0.21, 0.11, respectively.

From this analysis one concludes that positive hexadecapole deformations will always contribute to further enhancements of the fusion cross sections. In this statement it is understood that one speaks about increases that are measured with respect to the ones already generated by the prolate nucleus. The situation for negative $\lambda=4$ moments is, on the other hand, not obvious. Following the previous considerations, one expects situations in which the addition of the hexadecapole degree of freedom may even *reduce* the enhancement of the fusion cross section.

One way to survey the range of possibilities that may arise from combining different quadrupole and hexadecapole deformations is by plotting the contours of the enhancement $\mathcal{E}$, defined as the ratio of fusion cross sections σ_F

$$\mathcal{E}=\frac{\sigma_F(E \ll V_B)|_{\beta_2,\beta_4}}{\sigma_F(E \ll V_B)|_{\beta_2=0,\beta_4=0}} \tag{3}$$

in the plane defined by β_2,β_4. The definition (2) yields a stable quantity, provided one takes a value of E well below the lowest effective barrier $\tilde{V}_B$. To establish a point of contact with the references that motivated this work, calculations were performed for projectile and target masses as in (a) the reaction ^{16}O+^{184}W and (b) the reaction ^{16}O+^{232}Th. The results are collected in Fig. 2. One observes that the actual values of the enhancement depend, naturally, on the sizes of the nuclear systems involved. The qualitative features of Figs. 2(a) and (b) are nonetheless similar. In both situations the minimum of the enhancements for a given value of β_2 follows a ridge that does not coincide with the condition $\beta_4=0$. That can be better appreciated by plotting a cut of the enhancement surface $\mathcal{E}(\beta_2,\beta_4)$ for constant β_2, as drawn in Fig. 3 for the system ^{16}O+^{232}Th. This curve confirms our previous contention since it shows that small negative deformations do indeed decrease the fusion cross sections. For positive β_4 the fusion rates are, on the other hand, always improved.

Systematic measurements in nuclei with hexadecapole deformations are needed to check the role of this shape degree of freedom in subbarrier fusion reactions. Candidates for investigation could be systems in the rare-earths region, where the parameter β_4 ranges conveniently from negative to positive values.[6]

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Note added. A recent publication by J. R. Leight *et al.* [J. Phys. G **14**, L55 (1988)] addresses the subject of this Brief Report from an experimental point of view.

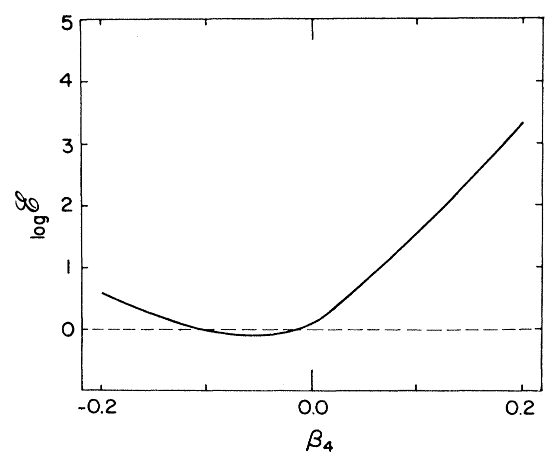

FIG. 3. Enhancement $\mathcal{E}$ as a function of β_4 for the reaction ^{16}O+^{232}Th. A value of $\beta_2=0.2$ was used, somewhat smaller to the prolate deformation assumed in Ref. 2. The enhancements are normalized to the value obtained for $\beta_4=0$ and thus reflect directly the role of the hexadecapole degree of freedom in this particular case.

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