Quark mass differences and isospin violation in the pion-nucleon coupling constants

Véronique Bernard
Centre de Recherches Nucléaires and Université Louis Pasteur de Strasbourg, Groupe de Physique Nucléaire Théorique, F-67037 Strasbourg CEDEX, France

Ulf-G. Meissner
Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
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We investigate the pseudoscalar pion-nucleon constants in the framework of various chiral models of the nucleon. We demonstrate that strong isospin violation ($m_u \neq m_d$) does not lead to a charge dependence of the pion-nucleon vertex as recently reported on the basis of *NN* scattering data. Using the equivalence theorem, we show that the pion mass difference leads to a 3% effect in the pseudovector πN coupling constants.

Recently it was pointed out[1] that there might be a charge dependence of the pion-nucleon coupling constants of the order of 10%. In particular, it is claimed that $g_{\pi^+ pn} > g_{\pi^0 pp}$. Such a large isospin breaking effect can come from two sources, namely strong isospin violation ($m_u \neq m_d$) or electromagnetic effects. Here, we wish to investigate to what extent strong isospin violation can account for differences of the coupling of charged or neutral pions to protons and neutrons. For that, we will consider various chiral models of the nucleon which account for the symmetries of low-energy strong interaction physics. As we will argue, it seems unlikely that strong isospin breaking in the light quark sector can generate an effect as claimed in Ref. 1.

To be specific, let us consider a generalized version of the Nambu and Jona-Lasinio model[2] which has been in great detail discussed in Ref. 3. The starting point is the following effective quark Lagrangian, which incorporates the spontaneous breakdown of chiral symmetry as well as the breaking of the axial $U(1)$ symmetry:[4]

$$\begin{aligned}
\mathcal{L}_{NJL} &= \mathcal{L}_{\rm kin} + \mathcal{L}_{\rm mass} + \mathcal{L}_{\rm int} \ , \\
\mathcal{L}_{\rm kin} &= \bar\psi i \partial\!\!\!/ \psi \ , \\
\mathcal{L}_{\rm mass} &= \hat m \bar\psi\psi, \quad \hat m = {\rm diag}(m_u, m_d, m_s) \ , \\
\mathcal{L}_{\rm int} &= G \sum_{a=0}^{8} [(\bar\psi\lambda^a\psi)^2 + (\bar\psi\lambda^a\gamma_5\psi)^2] \\
&\quad - K \det[\bar\psi(1+\gamma_5)\psi + \bar\psi(1-\gamma_5)\psi] \ .
\end{aligned} \tag{1}$$

The spinor ψ incorporates quarks of flavor u,d,s. G and K are coupling constants of dimension (mass)$^{-2}$ and (mass)$^{-5}$, respectively. The six fermion term proportional to K is entirely responsible for the $U(1)_A$ breaking[5] and flavor mixing (within the Hartree approximation we chose to work with). The current quark masses (at a renormalization point of ~ 1 GeV) are $m_u = 5.2$ MeV, $m_d = 8.9$ MeV, and $m_s = 175$ MeV.[6] The theory based on the Lagrangian (1) is not renormalizable, and loop integrals have to be rendered finite via a cutoff of the order of 1 GeV, which is also the typical scale of spontaneous chiral symmetry breaking, $\Lambda_{\chi SB} \approx 4\pi f_\pi \simeq 1$ GeV. The parameters $G\Lambda^2$, $K\Lambda^5$, and Λ can be fixed from fitting the properties of the pseudoscalar mesons (π, K, η, and η') as well as the quark condensates $\langle \bar q_i q_i \rangle_0$. For our purposes, it is most convenient to bring the interaction part of the Lagrangian (1) into the form of an effective two-body interaction. It reads

$$\begin{aligned}
\mathcal{L}_{\rm int}^{\rm eff} = &-G\left[1 - \frac{56}{9}\frac{K}{G}(S_s + S_u + S_d)\right](\bar\psi\lambda_0\gamma_5\psi)^2 - G\left[1 + \frac{28}{3}\frac{K}{G}S_s\right](\bar\psi\lambda_3\gamma_5\psi)^2 \\
&- G\left[1 - \frac{28}{9}\frac{K}{G}(S_s - 2S_u - 2S_d)\right](\bar\psi\lambda_8\gamma_5\psi)^2 \\
&+ \frac{28\sqrt{2}}{9}K(S_s - \tfrac{1}{2}S_u - \tfrac{1}{2}S_d)[(\bar\psi\lambda_0\gamma_5\psi)(\bar\psi\lambda_8\gamma_5\psi) + (\bar\psi\lambda_8\gamma_5\psi)(\bar\psi\lambda_0\gamma_5\psi)] \\
&+ \tfrac{28}{9}\sqrt{3/2}K(S_d - S_u)[\bar\psi\lambda_3\gamma_5\psi)(\bar\psi\lambda_0\gamma_5\psi - \sqrt{2}\bar\psi\lambda_8\gamma_5\psi) + (\bar\psi\lambda_0\gamma_5\psi - \sqrt{2}\bar\psi\lambda_8\gamma_5\psi)(\bar\psi\lambda_3\gamma_5\psi)] \ .
\end{aligned} \tag{2}$$

S_i $(i=u,d,s)$ is the regularized quark propagator for quarks of flavor i. Beyond some critical coupling G and/or K, the Lagrangian (1) generates nonperturbatively constituent quarks. The pseudoscalar mesons are then bound states of quarks and antiquarks, with their masses determined through the respective poles in the scattering matrix. The residues of these poles give the quark-meson couplings. Due to the different quark masses, the physical neutral pseudoscalar mesons are not eigenstates to the flavor states $\sim\lambda_0$, λ_3, and λ_8. In fact, the neutral pion mixes with the η and the η',

$$|\pi^0\rangle_p=\theta_\pi|\pi^0\rangle+\theta_\eta|\eta\rangle+\theta_{\eta'}|\eta'\rangle \;, \tag{3}$$

with $\theta_\pi\approx 1\gg\theta_\eta,\theta_{\eta'}$. Through this mixing, the neutral pion can couple to u, d, and s quarks, whereas the charged pions can only couple to the light quarks (u,d). It is important to note that for this mixing to occur, the light quark masses have to be different $(m_u\neq m_d)$, as one can easily see from the last line of the effective interaction (2). Similarly, the isospin breaking in the quark masses will induce isospin violation in the pion-quark coupling.

To lowest order, the nucleon is made of three constituent quarks with the pertinent flavor structure. To study possible higher-order effects, which might be large because of the coupling strengths G and K being large, we will also study the effects of pseudoscalar mesons surrounding the quark core in the spirit of the chiral bag model.[7] For that, we restrict ourselves to the two-flavor sector and neglect the kaon cloud. The nucleon wave function then reads

$$|N\rangle=\sqrt{Z_2}|(Q_iQ_jQ_k)_N\rangle+\sqrt{1-Z_2}|(Q_iQ_jQ_k)_{N,\Delta}\pi\rangle \;, \tag{4}$$

for $i,j,k=u,d$ and $(\ldots)_B$ means that the three constituent quarks are coupled to a baryon state B of given spin and isospin. The wave-function renormalization constant Z_2 measures the probability of finding a bare nucleon within a physical one. It is related to the self-energy of the nucleon by $Z_2^{-1}=[1-\partial\Sigma(E)/\partial E]_{E=M_N}$. The renormalized pion-nucleon coupling can then be evaluated to one-loop order by resumming all relevant Feynman-diagrams with intermediate N,Δ states. The bare pion-nucleon coupling follows from the coupling of pions to the bare states $\sim|(Q_iQ_jQ_k)_N\rangle$.

In Tables I and II we present our results for two sets of the input parameters m_u, m_d, m_s, Λ, G, and K. For set 1, we take the physical quark masses $m_u=5$ MeV, $m_d=8.9$ MeV, and $m_s=175$ MeV together with $\Lambda=900$ MeV and $(\pi^2/3G\Lambda^2)=0.6$, $(3K\Lambda^3/8\pi^2G)=0.96$. With set 2, we account for maximal isospin breaking by allowing $m_u=0$, and $m_d=14$ MeV (with all other parameters kept on their previous values). The pion mass difference is rather small, $m_{\pi^\pm}-m_{\pi^0}\sim 0.3$ MeV, in agreement with the calculation of Isgur.[8] The strong kaon mass difference comes out to be ~ 6 MeV, again not far from the value found in Ref. 8. The electromagnetic contributions to the mass splittings are also reasonably well reproduced because the electromagnetic form factors of the π and the K as predicted by the model are in good agreement with the data. In Table II, we give the pertinent pion- and kaon-quark couplings together with the decay constants f_π, f_k, and f_η. We also find that $f_{\pi^0}/f_{\pi^\pm}\approx 1.02$ and $f_{K^0}/f_{K^\pm}\approx 1.01$ for set 1. In fact, the neutral pion is mostly a λ_3 state [cf. Eq. (3)], and its coupling to quarks is roughly equal to the charged-pion–quark coupling. The situation is the same for the charged and neutral kaon. Using our model wave functions Eq. (4) for the nucleon, we find the following values for the bare (b) and renormalized (r) πN coupling constants:

$$\begin{aligned} g^b_{\pi^0pp}&=13.89, \quad g^r_{\pi^0pp}=10.32 \;,\\ g^b_{\pi^0nn}&=13.51, \quad g^r_{\pi^0nn}=10.09 \;,\\ g^b_{\pi^+np}&=13.70, \quad g^r_{\pi^+np}=10.21 \;. \end{aligned} \tag{5}$$

Within the accuracy of our model, these results rule out any explanation of the effects discussed in Ref. 1 from strong isospin violation.

Of course, we should also consider electromagnetic contributions to the isospin breaking since in the case of the pion mass these are more important than the strong ones. To single out the effects of the pion mass difference $(m_{\pi^\pm}-m_{\pi^0}\sim 5$ MeV) consider a chiral soliton model of the nucleon, in which pions, ρ and ω mesons interact nonlinearly and nucleons arise as topological solitons. For the so-called "minimal model" of Ref. 9 with $g_{\rho\pi\pi}=5.85$, $f_\pi=93$ MeV, and the pion mass m_{π^+},m_{π^0} as input parameters, one can directly calculate the strong πN coupling constant $g_{\pi NN}$ as a Fourier transform of the static soliton source to which the pions couple. Keeping $g_{\rho\pi\pi}$ and f_π fixed,[11] we find

$$g_{\pi^0NN}=14.716, \quad g_{\pi^+NN}=14.696 \;, \tag{6}$$

TABLE I. Constituent quark masses and pseudoscalar meson masses. The input parameters are $\{m_u$ (MeV), m_d (MeV), m_s (MeV), Λ (MeV), $\pi^2/3G\Lambda^2$, $3K\Lambda^3/8\pi^2G\}=\{5.2,8.9,175,900,0.6,0.96\}$ for set 1 and $\{0.0,14,175,900,0.6,0.96\}$ for set 2.

Set	M_u (MeV)	M_d (MeV)	M_s (MeV)	m_{π^+} (MeV)	m_{π^0} (MeV)	$m_{K^\pm}$ (MeV)	m_{K^0} (MeV)	m_η (MeV)	$m_{\eta'}$ (MeV)
1	253.71	260.85	490.83	144.80	144.37	506.18	511.44	441.83	953.0
2	244.19	269.33	490.67	144.94	139.83	499.54	518.08	440.96	877.0

TABLE II. Pseudoscalar meson-quark coupling constants and meson decay constants. The $\pi^0 qq$ coupling is split into its three components in flavor space proportional to the three Gell-Mann matrices λ_0, λ_3, and λ_8. For input parameters, see Table I.

Set	$g_{\pi^+ qq}$	g_{3qq}	g_{0qq}	g_{8qq}	f_{π^+} (MeV)	$g_{K^+ qq}$	$g_{K^0 qq}$	f_{K^+} (MeV)	f_η (MeV)
1	2.739	2.738	0.018	0.053	91.32	2.762	2.771	100.68	107.81
2	2.736	2.726	0.055	0.177	91.15	2.749	2.780	99.74	112.64

i.e., again a negligible effect. In Ref. 10, it was argued that the Skyrme model (or any similar chiral soliton model) could explain the large symmetry breaking in $g_{\pi^0 pp}$ and $g_{\pi^+ np}$. We feel, however, that these conclusions are too far reaching. To be more specific, what the chiral soliton model gives is the coupling constant $g_{\pi NN}$ related to the pseudoscalar coupling $\bar{\psi}_N \gamma_5 \tau \psi_N \phi_\pi$. If one then uses the equivalence theorem[12]

$$\frac{g_{\pi NN}}{2M_N} = \frac{f_{\pi NN}}{m_\pi} , \tag{7}$$

one finds that $f_{\pi^\pm NN}/f_{\pi^0 NN} = 1.03$ purely on the grounds that $m_{\pi^\pm}/m_{\pi^0} \simeq 1.03$ and that $g_{\pi^0 NN} \approx g_{\pi^\pm NN}$ as demonstrated in Eq. (6).

To summarize, we have used various chiral models to investigate the effect of strong isospin breaking on the charge dependence of the pion-nucleon coupling constants. On the quark level, the mixing of the neutral pion with the η and the η' induces a very small difference in $g_{\pi^0 qq}$ and $g_{\pi^\pm qq}$. This effect does not get enhanced when one goes to the level of pions coupled to protons and neutrons. To see that, we have used a model close in spirit to the renormalized chiral bag as well as the chiral soliton model. To a good approximation, one finds $g_{\pi^0 pp} \approx g_{\pi^+ np}$ [cf. Eqs. (5) and (6)]. The only sizeable effect comes in when one uses the pseudovector pion-nucleon coupling $f_{\pi NN}$ which can be converted from $g_{\pi NN}$ via the equivalence theorem (7). This effect is, however, of purely kinematical origin and should not be accounted for to explain the large isospin violation of $g_{\pi NN}$ which was inferred in Ref. 1 from NN scattering analysis. More effort is therefore needed to clarify this question from the experimental as well as the theoretical side.

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[11]Keeping f_π fixed is of course an approximation in light of our results. To compare to Ref. 10, however, we need to do so.

[12]See, for example, S. S. Schweber, H. A. Bethe, and F. de Hoffman, *Mesons and Fields* (Row, Peterson, Evanston, 1955), Vol. I.