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Chiral-quark-gluon description of the nuclear force

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We discuss the restoration of the chiral symmetry in the case of the *N-N* interaction on the basis of the cloudy bag model. A quantum-chromodynamics-based many-body theory of confined quarks, confined gluons, and pions is developed. In terms of the Green's functions and vertex functions of these fields, the energy shift due to gluon and pion exchange between quarks is calculated systematically and the short-range and long-range parts of the effective nuclear potential are obtained. For the short-range part, a soft repulsive core is obtained using the cloudy bag model parameters and the main contribution comes from the gluon exchange, as expected. The height of the core, however, is sensitive to the (poorly determined) subtraction required to remove the kinetic energy of the relative *N-N* motion. For the long-range part, an explicit formula, which is the same as one-pion-exchange potential, is obtained, but the strength of this part is 30% lower than that of one-pion-exchange potential when f_π is chosen as 93.0 MeV.

I. INTRODUCTION

A precise description of the *N-N* interaction is an unsolved fundamental problem in nuclear and hadronic physics. Since the meson-exchange theory of the nuclear force proposed by Yukawa in 1935,[1] there has been considerable progress[2] in unraveling its complexity. However, because of its complexity many problems are still open, especially the short-range behavior of the *N-N* interaction. Even if contributions from the vector mesons have been included, one can only deal with this part of the problem semiphenomenologically. On the other hand, the quark-gluon theory of hadronic structure has been developed since the 60's. Therefore one hopes that the nuclear force can be derived at the quark level. Even though some work based on the nonrelativistic potential model,[3-17] the bag model,[18-28] various nontopological soliton models, and the Skyrme model[29-36] have reproduced some of the properties of the nuclear force and low-energy scattering data, many problems remain and improvements are needed.

Obviously a more solid theoretical foundation is desired. QCD is the most promising theory of the strong interaction. In principle the nuclear force can be derived from fundamental QCD. Because of the obvious complexities of QCD, there is no reliable method to calculate it. Although some studies of the interaction between nucleons have been made by lattice calculations,[37] the approaches so far have been too simple. Hence semiphenomenological models which are QCD motivated are still used. The chiral symmetry is important for understanding the low-energy phenomena. As the first step, the cloudy bag model (also the chiral bag model)[38-46] is a good starting point. It contains the basic features of QCD such as confinement, asymptotic freedom, and chiral invariance. In addition, many analytical results have been obtained through this model. Using the model, we will develop a formulation in which a unified quark description of the short- and long-range parts of the nuclear force can be obtained. For the short-range part of the effective nuclear potential, the kinetic energy $\langle KE_{rel} \rangle$ of the relative motion of two nucleons should be subtracted from the energy of six-quark system.[47-48] In the bag model this correction has been argued to be of roughly the same size as the center-of-mass motion correction. Taking this to be exactly the case and using the Donoghue-Johnson method[49] for calculating it, a soft repulsion appears. The main contribution comes from one-gluon exchange. The size of this repulsion, however, could be significantly decreased if one uses the Wong-Liu method for estimating this correction. For the long-range part, an explicit formula of *N-N* interaction which is the same as the one-pion-exchange potential (OPEP) can be derived and its strength could be determined by the bag model parameters. In principle, this framework is also suitable for dealing with other meson fields and could provide a method for the meson exchange interaction at quark level. In the bag model, the nonperturbative effects are considered as boundary conditions. Then the "residual interaction" is calculated perturbatively. Unlike other works which treat the gluon and pion fields, as the classical fields and the energies are also calculated classically, we use the many-body theory of confined quark, confined gluon and pion to study the effects of gluon and pion exchanges. These fields are expanded in terms of their eigenmodes and quantized. The quarks and gluons are confined inside the cavities so that their eigenvalues are discrete. Then we find the corresponding Green's functions and vertex functions, respectively. After that, the contributions from the various Feynman diagrams (including the self-energies) are easy to calculate systematically. In the chiral or cloudy bag model, a baryon or in our case a nucleon—a cavity with quarks and gluons inside—is surrounded by pions. When two nucleons get close and overlap, a six-quark cavity is formed but at long range these two cavities are separated. In this paper we do not discuss the continuous transition from two separated cavities to overlapping cavities. However because the quark field couples to the pion at the bag surface, the Green's functions of the pion field which describes emission and absorption on one cavity and between two cavities are found.

The outline of this paper is as follows: in Sec. II the many-body theory of confined quark, confined gluon and pion fields is introduced. We emphasize the determination of the Green's functions for the one- and two-cavity cases. The details associated with the eigenmodes and the interaction matrix elements of these fields needed for perturbation theory can be found in Appendices A and B. In Sec. III the expressions of the short- and long-range parts of the effective nuclear potential are derived. Section IV contains numerical results and further discussion.

II. MANY-BODY THEORY OF PIONS, CONFINED QUARKS AND GLUONS FOR THE ONE- AND TWO-CAVITY CASES

In the original bag model each baryon is a cavity with quarks and gluons. First, consider a six-quark system with three quarks in one baryon and three in another baryon. The distributions of quarks in the two-baryon system depend upon the relative separation between the two baryons. When two bags approach each other they can overlap and consequently form a new bag which contains six quarks. The shape and size of this bag depends on the configuration of the six-quark state. For the long-range part of the interaction there are two individual bags. If only quarks and gluons are considered, there will not be any interaction between these two bags and the quarks are confined in two separate regions. So in the absence of a new mechanism the original bag model cannot describe the long-range interaction between baryons. Such a new mechanism is provided by chiral symmetry, i.e., by requiring the underlying dynamics to be chirally symmetric. This introduces the pion at a dynamical level. Because of the coupling of the pion to the quarks, the energy of the whole system, i.e., quarks, gluons, and pions, will change as the distance of two baryons changes. Namely, an effective nuclear potential can be derived at all distances. To calculate the energy of the six-quark system, one needs the Green's functions and the vertex functions for the one- and two-cavity cases. Our starting point is the Lagrangian density of the cloudy bag model. For the chiral bag model the formulation and the results are similar. The Lagrangian density of the cloudy bag model can be written as follows.

$$\mathcal{L}_{\rm HCB}=\mathcal{L}_B^0+\mathcal{L}_G^0+\mathcal{L}_\pi^0+\mathcal{L}_{\rm QG}^I+\mathcal{L}_{Q\pi}^I \tag{2.1}$$

and

$$\mathcal{L}_B^0=[-\tfrac{1}{2}\bar\psi\gamma_\mu(\partial_\mu\psi)+\tfrac{1}{2}(\partial_\mu\bar\psi)\gamma_\mu\psi-B]\theta(v)\ , \tag{2.2}$$

$$\mathcal{L}_\pi^0=[-\tfrac{1}{2}D_\mu^{ij}\pi_j\cdot D_\mu^{ik}\pi_k]\ ,$$

$$D_\mu^{ij}\equiv[\hat{\underline{\pi}}_i\hat{\underline{\pi}}_j+j_0(|\underline{\pi}|/f_\pi)(\delta_{ij}-\hat{\underline{\pi}}_i\hat{\underline{\pi}}_j)]\partial_\mu\ , \tag{2.3}$$

$$\mathcal{L}_G^0=(-1/4F_{\mu\nu}^aF^{\mu\nu a})\theta(v)\ ,$$

$$F_{\mu\nu}^a\equiv\partial_\mu A_\nu^a(x)-\partial_\nu A_\mu^a(x)-gf_{abc}A_\mu^b(x)A_\nu^c(x)\ , \tag{2.4}$$

$$\mathcal{L}_{\rm QG}^I=(ig_s\bar\psi\gamma_\mu\lambda^a/2I_f\psi A_\mu^a)\theta(v)\ , \tag{2.5}$$

$$\mathcal{L}_{Q\pi}^I=-\tfrac{1}{2}\bar\psi\exp(i\gamma_5\underline{\tau}\cdot\underline{\pi}/f_\pi)\psi I_c\theta(\partial v)\ , \tag{2.6}$$

where ψ, π, and A_μ^a are the quark, pion, and gluon fields, respectively, and f_π is the π decay constant and λ^a's are the generators of SU(3,C). I_f, I_c are the unitary matrices in the flavor and color space, respectively.

$$\theta(A)=\begin{cases}1 & \text{inside (or on) } A\ ,\\ 0 & \text{outside } A\ .\end{cases} \tag{2.7}$$

A represents the region occupied by the baryons. It is one cavity or two cavities for the short- and long-range part of the interaction, respectively. Here we use the surface coupling of quark and pion, as the usual chiral and cloudy bag model. It has been shown that[46] under certain transformation the surface coupling is connected with the volume coupling.

The Green's functions of the different fields are defined as follows:

$$iG_F(x_1x_2)\equiv\langle 0|T[\psi(x_1)\bar\psi(x_2)]|0\rangle =\psi^{\cdot}(x_1)\bar\psi^{\cdot}(x_2)\ , \tag{2.8}$$

$$\begin{aligned}i\Delta^0(x_1x_2)&\equiv\langle\phi_0|T[\pi(x_1)\pi(x_2)]|\phi_0\rangle\\ &=\langle 0|T[\pi(x_1)\pi(x_2)]|0\rangle\\ &=\pi^{\cdot}(x_1)\pi^{\cdot}(x_2)\ ,\end{aligned} \tag{2.9}$$

$$\begin{aligned}iD_{\mu\nu}^{ab}(x_2x_2)&\equiv iD_{\mu\nu}(x_1x_2)\delta^{ab}\\ &\equiv\langle\phi_0|T[A_\mu(x_1)A_\nu(x_2)]|\phi_0\rangle\\ &=A_\mu^{\cdot}(x_1)A_\nu^{\cdot}(x_2)\delta^{ab}\ ,\end{aligned} \tag{2.10}$$

where $|0\rangle$ is the vacuum state and $|\phi_0\rangle$ is the six-quark state. Because the configuration of the six-quark state can be mixed in single quark quantum numbers, in general,

$$\langle\phi_0|T[\psi(x_1)\bar\psi(x_2)]|\phi_0\rangle\neq\psi^{\cdot}(x_1)\bar\psi^{\cdot}(x_2) \tag{2.11}$$

for such configurations. In order to determine these Green's functions, the different fields should be expanded in terms of their eigenmodes. The eigenmodes of the quark and gluon fields confined in a spherical cavity have been discussed before.[50,51] The main results are summarized in Appendix A. Because the quark field couples to the pion field at the bag surface, one also needs the propagator of the field between two cavities. It will be discussed at the end of this section. Now we concentrate on the one-cavity case. After some calculation, the Fourier transformations of the quark and gluon propagators are the following:

$$G_F^0(\underline{r}_1\underline{r}_2,\omega)=\int d\tau\, e^{i\omega\tau}G_F^0(x_1x_2)$$
$$=\sum_n\left[\frac{U_n(\underline{r}_1)\bar{U}_n(\underline{r}_2)}{\omega-E_n+i\eta}+\frac{V_n(\underline{r}_1)\bar{V}_n(\underline{r}_2)}{\omega+E_n-i\eta}\right] \tag{2.12}$$

with

$$\tau\equiv t_1-t_2\ ,$$
$$D_{44}(\underline{r}_1\underline{r}_2)=-D^c(\underline{r}_1\underline{r}_2)$$
$$=-\sum_\kappa\frac{\phi_\kappa(\underline{r}_1)\phi_\kappa^*(\underline{r}_2)}{(k_\kappa^{\rm sc})^2}\ , \tag{2.13}$$

$$D_{ij}(\underline{r}_1\underline{r}_2,\omega)=\sum_{\lambda=\rm mg,el}\sum_\kappa\frac{A_\kappa^\lambda(\underline{r}_1)A_\kappa^{\lambda *}(\underline{r}_2)}{\omega^2-(k_\kappa^\lambda)^2}\ , \tag{2.14}$$

$$D_{4j}(\underline{r}_1\underline{r}_2)=0\ . \tag{2.15}$$

$U_n(\mathbf{r}), V_n(\mathbf{r})$ are the positive- and negative-energy solutions, respectively. $\phi_\kappa(\mathbf{r})$ represents the Coulomb mode functions of the gluon field. The $A_\kappa^\lambda(\mathbf{r})$ are the corresponding magnetic mode (λ=mg) and the electric mode (λ=el) functions. These modes are combinations of the harmonic, spin-harmonic, and vector-harmonic functions with the spherical Bessel functions which satisfy certain boundary conditions. Some of the eigenvalues and the detail expressions of these eigenfunctions can be found in Appendix A. In the cloudy bag model, the pion field can be expanded in terms of plane waves, i.e.,

$$\pi_1(x)=(2\pi)^{-3/2}\int\frac{d\mathbf{k}}{(2\omega_\kappa)^{1/2}}(a_{\underline{k}i}e^{i\kappa x}+a_{\underline{k}i}^+e^{-i\kappa x})\ , \tag{2.16}$$

where $\omega_\kappa\equiv(k^2+m_\pi^2)^{1/2}$. The Fourier transformation of the Green's function is

$$\Delta^0(\mathbf{r}\mathbf{r}',\omega)=\frac{1}{(2\pi)^3}\int d\mathbf{k}\frac{e^{i\mathbf{k}\cdot\mathbf{r}}\cdot e^{-i\mathbf{k}\cdot\mathbf{r}'}}{\omega^2-(\omega_k-i\eta)^2}\ . \tag{2.17}$$

In a spherical coordinate frame this becomes

$$\Delta^0(\mathbf{r},\mathbf{r}',\omega)=\sum_{lm}\Delta_l^0(rr',\omega)Y_{lm}^*(\theta',\phi')Y_{lm}(\theta,\phi)\ , \tag{2.18}$$

where

$$\Delta_l^0(rr',\omega)=\frac{2}{\pi}\int_0^\infty dk\cdot k^2\cdot\frac{j_l(kr)j_l(kr')}{\omega^2-(\omega_k-i\eta)^2}$$
$$=(i\omega)j_l(\omega r_<)[j_l(\omega r_>)-in_l(\omega r_<)]\ . \tag{2.19}$$

$j_l(x)$ and $n_l(x)$ are the spherical Bessel functions of first and second kind, respectively. When $\mathbf{r},\mathbf{r}'$ are on the same sphere, the expression is as follows:

$$\Delta_l^0(RR,\omega)=\frac{1}{R}[\xi j_l(\xi)n_l(\xi)+i\xi j_l^2(\xi)]\ , \tag{2.20}$$

where $\xi\equiv\omega R$. In particular, when $\xi=0$ we have

$$\mathrm{Re}\Delta_l^0(RR,0)=\lim_{\xi\to 0}\frac{1}{R}\cdot\xi\cdot j_l(\xi)n_l(\xi)$$
$$=-\frac{1}{2l+1}\cdot\frac{1}{R}\ . \tag{2.21}$$

For the long-range part there is a two cavity system. Generally speaking, due to the coupling of quark and pion these two bags should be deformed. For simplicity of the calculation, we still use the spherical approximation. When two bags are not very close, it should be a good approximation.

Because quarks and gluons are confined inside the bags, the quarks in different bags do not exchange gluons. The Green's functions of quark and gluon fields are the same as that in one bag case. The Green's function of the pion field between two bags can be expressed as

$$\Delta^0(x_1x_2)=\int\frac{d\omega}{2\pi}e^{-i\omega(t_1-t_2)}\cdot\Delta^0(\mathbf{r}_{1N}\mathbf{r}_{2N},\omega)\ , \tag{2.22}$$

where

$$\Delta^0(\mathbf{r}_{1N}\mathbf{r}_{2N},\omega)=(2\pi)^{-3}\int d\mathbf{k}\frac{e^{i\mathbf{k}\cdot\mathbf{r}_{1N}}\cdot e^{-i\mathbf{k}\cdot\mathbf{r}_{2N}}}{\omega^2-(\omega_k-i\eta)^2}\ . \tag{2.23}$$

When the separation, d, between two bags (N,N') is larger than the diameter, $2R$, of the bag. We obtain (see Fig. 1)

$$\Delta^0(\mathbf{r}_{1N}\mathbf{r}_{2N},\omega)=[2\pi]^{-3}\int d\mathbf{k}\frac{e^{i\mathbf{k}\cdot\mathbf{d}}\cdot e^{i\mathbf{k}\cdot(\mathbf{r}_N-\mathbf{r}_{N'})}}{\omega^2-(\omega_k-i\eta)^2}$$
$$=\sum_{ll'm}\Delta_{ll'}^m(r_Nr_N',\omega)$$
$$\times Y_{lm}^*(\theta_N,\phi_N)Y_{l'm}(\theta_N',\phi_N')\ . \tag{2.24}$$

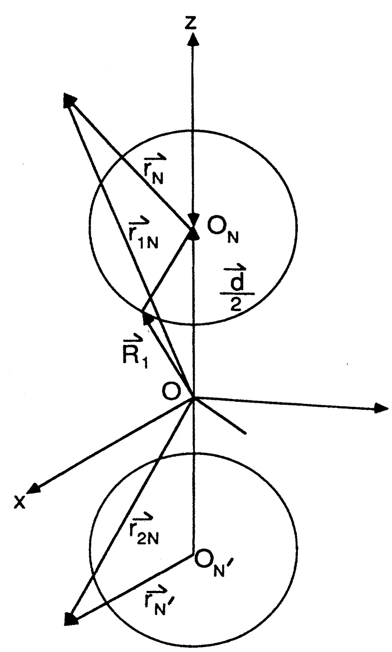

FIG. 1. The coordinate frame for two cavity case.

Here

$$\Delta_{ll'}^{m}(r_N r_{N'},\omega)=\frac{2}{\pi}(-1)^{l'-m}i^{l+l'}\cdot[(2l+1)(2l'+1)]^{1/2}\times\sum_L(2L+1)\begin{bmatrix} l & L & l' \\ -m & 0 & m\end{bmatrix}\begin{bmatrix} l & L & l \\ 0 & 0 & 0'\end{bmatrix}i^L\times\int dk\cdot k^2\frac{j_l(kr_N)j_L(kd)j_{l'}(kr_{N'})}{\omega^2-(\omega_k-i\eta)^2}\ . \tag{2.25}$$

III. EFFECTIVE NUCLEAR POTENTIAL

Many works have been done in order to understand the N-N interaction from the quark degree of freedom, as mentioned above. Some of these works employed the Born-Opperheimer approximation, while some did not in particular for scattering case. In this approximation, one calculates the energy of the two-quark system (i.e., two subsystems of three quarks at quark level) as a function of their static separation. The energy of the system calculated this way is the effective potential. As the first step, we compare the "nuclear effective potential" derived in the approximation with the empirical one and that from the meson-exchange theory.

The definition of the distance of two baryons is clear in the discussion of long-range part of their interaction. In this case, each nucleon is a cavity surrounded by its pion field. The distance of two nucleons is defined as the separation of the centers of these cavities. Because the pion field exerts pressure on these two cavities they may not be spherical and the shape could depend on the quantum numbers of the whole systems. However, in this paper we will treat them as spherical. For the short range the definition of "distance between two nucleons" is not clear. As two nucleons come close enough they will overlap and form a six-quark bag of deformed shape. These quarks are indistinguisable and therefore the meaning of "a nucleon" inside this deformed six-quark bag is ambiguous. The usual way is to construct a configuration of the system which contains two separate subgroups of three quarks and let each subgroup have the quantum numbers of a nucleon. The criteria of chosing the configuration could be either the quark distribution density of bicenter or the consideration of group classification. In both cases the P state of quark in the cavity should be included and the cavity could be deformed. But it is shown[18] that for the short range (<0.7 fm) the difference between the results of ellipsoidal cavity and that of the spherical cavity is small. So in our calculation the spherical cavity approximation is used. Of course in the medium range the deformation of the cavity is important, however we will not treat it here.

A. Six-quark states for two nucleon overlap case

Following DeTar's work the six-quark states are constructed for the calculation of the short-range nuclear force. In terms of s and p quark states, the "left" and "right" quark creation and annihilation operators are defined such that

$$b_{Lm\alpha}^{+}=b_{sm\alpha}^{+}-\sqrt{\mu}b_{pm\alpha}^{+}\ , \tag{3.1}$$

$$b_{Rm\alpha}^{+}=b_{sm\alpha}^{+}+\sqrt{\mu}b_{pm\alpha}^{+}\ , \tag{3.2}$$

where "L" and "R" indicate the "left" or "right" quark operator, s and p are the radial quantum numbers of the quark states, m is the magnetic quantum number, and α represents all other indices including the flavor and color variables. μ is a parameter which connects with the ratio of the s component and p component. μ changes from 0 to 1 while the "left" and "right" orbits vary from complete overlap to orthogonal. From the well-known spin-isospin states of a nucleon, the creation operators of "left" and "right" nucleons can be expressed in terms of the corresponding quark operators. For example, the spin-up state of a proton can be written as

$$P(\uparrow)=\frac{1}{\sqrt{18}}(2u\uparrow u\uparrow d\downarrow+2u\uparrow d\downarrow u\uparrow+2d\downarrow u\uparrow u\uparrow -u\uparrow d\uparrow u\downarrow-u\uparrow u\downarrow d\uparrow-u\downarrow d\uparrow u\uparrow -d\uparrow u\uparrow u\downarrow-u\downarrow u\uparrow d\uparrow-d\uparrow u\downarrow u\uparrow)\ . \tag{3.3}$$

Replacing each quark state by the "right" quark creation operator, there will be the "right" proton creation operator $P_R^{+}(\uparrow)$. Similarly, one can find all nucleon operators. With these operators in hand it is direct to write down two nucleon states for the different (T,M_T,S,M_S) channels. For instance, the six-quark state of ($T=0$, $S=1$, $M_S=0$) is as follows:

$$(N(\mu))^{1/2}|T=0,S=1,M_s=0\rangle =(p_R^{+}(\uparrow)n_L^{+}(\downarrow)+p_R^{+}(\downarrow)n_L^{+}(\uparrow) +n_R^{+}(\downarrow)p_L^{+}(\uparrow)+n_R^{+}(\uparrow)p_L^{+}(\downarrow))|0\rangle\ , \tag{3.4}$$

where $N(\mu)$ is the normalization coefficient and $|0\rangle$ is the vacuum state. It is easy to see each term in these states has the form of $b_{n_1}^{+}b_{n_2}^{+}b_{n_3}^{+}b_{n_4}^{+}b_{n_5}^{+}b_{n_6}^{+}|0\rangle$ where $b_{n_i}^{+}$ is the quark creation operator and n_i represents all quark quantum numbers. The coefficient of each term is a function of μ. The distance δ between two nucleons is defined as

$$\delta\equiv\frac{2\mu^{1/2}(1+\mu)}{(1+\mu^2)}\int\psi_s^{+}z\psi_p d^3r\ , \tag{3.5}$$

where ψ_s and ψ_p are the space parts of s and p state wave functions, respectively.

The E-δ curves calculated for those states are given in Sec. III. It should be pointed out that the framework of our work can also be used for other forms of six-quark states. Ma[24] has calculated the contribution from pion exchange to the short-range part of the nuclear force for the six-quark states constructed by group classification. Her results are quite similar to ours.

B. Energy shifts of six-quark system due to gluon and pion exchange

In terms of the Green's functions and vertex functions given above, one can calculate the energy shifts due to the coupling of the fields for a given physical state $|\Phi_0\rangle$. In the bag model the nonperturbative effects such as the confinement and vacuum pressure are represented by the boundary conditions. The remainder is regarded as "residual interaction" and can be calculated perturbatively. To the order of g_s^2 and f_π^{-2}, the interaction Hamiltonian density can be written as follows.

$$\mathcal{H}_I(x)=\mathcal{H}_I^{(1)}(x)+\mathcal{H}_I^{(2)}(x) , \tag{3.6}$$

$$\mathcal{H}_I^{(1)}(x)=\mathcal{H}_{I,\mathrm{GQ}}(x) = -ig_s\bar{\psi}(x)\gamma_\mu\frac{\lambda^a}{2}I_f\psi(x)A_\mu^a(x) , \tag{3.7}$$

$$\mathcal{H}_I^{(2)}(x)=\mathcal{H}_{I,\pi Q}(x) = if_\pi^{-1}\bar{\psi}(x)\gamma_5\underline{\tau}\cdot\underline{\pi}I_c\psi(x)\delta(r-R) . \tag{3.8}$$

According to Hubbard's modification[52] of the Gell-Mann-Low theorem[53] the energy shift due to $\mathcal{H}_I$ is

$$\Delta E=\langle\phi_0|\sum_{n=1}^{\infty}(-i)^n\cdot\frac{1}{n!}\int_{-\infty}^{+\infty}i\delta(t_1)d^4x_1,\ldots,\int_{-\infty}^{+\infty}d^4x_nT(\mathcal{H}_I(x_1)\mathcal{H}_I(x_2)\ldots\mathcal{H}_I(x_n))|\phi_0\rangle_{\mathrm{connected}} . \tag{3.9}$$

In our case $|\phi_0\rangle$ is the six-quark state considered. Because there are only quarks in $|\phi_0\rangle$ the first-order perturbation is zero. The lowest modification is the second-order perturbation which is given by

$$\Delta E^{(2)}=\Delta E_{\mathrm{QC}}^{(2)}+\Delta E_{Q\pi}^{(2)} , \tag{3.10}$$

$$\Delta E_{\mathrm{QG}}^{(2)}=-\tfrac{1}{2}\langle\phi_0|\int_{-\infty}^{+\infty}d^4x_1i\delta(t_1)\int_{-\infty}^{+\infty}d^4x_2T[\mathcal{H}_{I,\mathrm{QG}}(x_1)\mathcal{H}_{I,\mathrm{QG}}(x_2)]|\phi_0\rangle , \tag{3.11}$$

$$\Delta E_{Q\pi}^{(2)}=-\tfrac{1}{2}\langle\phi_0|\int_{-\infty}^{+\infty}d^4x_1i\delta(t_1)\int_{-\infty}^{+\infty}d^4x_2T[\mathcal{H}_{I,Q\pi}(x_1)\mathcal{H}_{I,Q\pi}(x_2)]|\phi_0\rangle . \tag{3.12}$$

By substituting the $\mathcal{H}_I$ into Eq. (3.9), one can find the contributions from the different diagrams according to Wick's theorem. The effect of the vacuum polarization has been represented by the bag parameter B and will not be calculated. The diagrams involved are in Fig. 2.

The expressions corresponding to these diagrams are as follows:

$$\Delta E_{SF,Q\pi}=-\frac{3}{32}\cdot\frac{1}{\pi^2}\cdot\frac{1}{f_\pi^2}\cdot\frac{1}{R^3}\cdot\sum_n\langle\phi_0|b_n^+b_n|\phi_0\rangle\cdot\sum_{\nu'\kappa'}\frac{x_{\nu'\kappa'}}{x_{\nu'\kappa'}+\kappa'}\cdot\frac{x_{\nu\kappa}}{x_{\nu\kappa}+\kappa}\cdot2|\kappa'|$$
$$\times\sum_J\begin{bmatrix} j' & J & j\\ -\frac{1}{2} & 0 & \frac{1}{2}\end{bmatrix}\cdot(2J+1)$$
$$\times\int d\xi\cdot\xi\left[\frac{j_J^2(\xi)}{x_{\nu'\kappa'}+\xi-x_{\nu\kappa}}\frac{1}{2}[1-(-1)^{J+l+l'}] -\frac{j_J^2(\xi)}{x_{\nu'\kappa'}+\xi+x_{\nu\kappa}}\cdot\frac{1}{2}[1-(-)^{J+l+\bar{l}'}]\right] , \tag{3.13}$$

$$\Delta E_{\mathrm{IN},Q\pi}=+\frac{1}{32\pi^2}\cdot\frac{1}{f_\pi^2R^3}\cdot\sum_{n_1,n_2,n_3,n_4}\langle\phi_0|b_{n_1}^+b_{n_3}^+b_{n_4}b_{n_2}|\phi_0\rangle\left[\sum_k\langle f_1|\tau_k|f_2\rangle\langle f_3|\tau_k|f_4\rangle\right]$$
$$\times\delta_{c_1c_2}\delta_{c_3c_4}\left[\frac{x_{\nu_1\kappa_1}}{x_{\nu_1\kappa_1}+\kappa_1}\cdot\frac{x_{\nu_2\kappa_2}}{x_{\nu_2\kappa_2}+\kappa_2}\cdot\frac{x_{\nu_3\kappa_3}}{x_{\nu_3\kappa_3}+\kappa_3}\cdot\frac{x_{\nu_4\kappa_4}}{x_{\nu_4\kappa_4}+\kappa_4}\right]^{1/2}\cdot[(2j_1+1)$$
$$\times(2j_2+1)(2j_3+1)(2j_4+1)]^{1/2}$$
$$\times\sum_{\mathrm{JM}}\cdot\tfrac{1}{4}[1-(-)^{J+l_1+l_2}][1-(-)^{J+l_3+l_4}]\cdot(-)^{\mu_4-\mu_1+1}\cdot\begin{bmatrix} j_1 & J & j_2\\ -\mu_1 & M & \mu_2\end{bmatrix}$$
$$\times\begin{bmatrix} j_3 & J & j_4\\ -\mu_3 & -M & \mu_4\end{bmatrix}$$
$$\times\begin{cases}-\dfrac{1}{2J+1} & \text{when } \xi=0\ (\text{i.e.},\omega_{\nu_4\kappa_4}=\omega_{\nu_3k_3}) ,\\ -\xi\cdot j_J(\xi)n_J(\xi) & \xi=0 ,\end{cases} \tag{3.14}$$

where $n_i \equiv (\nu_i \kappa_i \mu_i f_i c_i)$.

$$\Delta E_{\rm IN,QG} = \frac{g_s^2}{8}\int_{-\infty}^{+\infty}\delta(t_1)d^4x_1\int_{-\infty}^{+\infty}d^4x_2 \cdot \sum_{\lambda={\rm cl,el,mg}} D_{\mu\nu}^{\lambda,ab}(x_1,x_2)$$
$$\times \langle \phi_0|N[\bar\psi(x_1)\gamma_\mu\lambda^a I_f\psi(x_1)\bar\psi(x_2)\gamma_\nu\lambda^b I_f\psi(x_2)]|\phi_0\rangle$$
$$= +\frac{g_s^2}{8}\cdot\frac{1}{R}\cdot\sum_{n_1,n_2,n_3,n_4}\langle\phi_0|b_{n_1}^+b_{n_3}^+b_{n_4}b_{n_2}|\phi_0\rangle\delta_{f_1f_2}\delta_{f_3f_4}$$
$$\times\left[\sum_a\langle c_1|\lambda^a|c_2\rangle\langle c_3|\lambda^a|c_4\rangle\right]\cdot\sum_{\lambda={\rm cl,mg,el}}\sum_{\rm NJM}W_{\rm NJM}^{\lambda}, \tag{3.15}$$

where

$$W_{\rm NJM}^{\lambda}(\kappa_1\mu_1,\kappa_2\mu_2,\kappa_3\mu_3,\kappa_4\mu_4)=D_{\rm NJM}^{\lambda}(\kappa_1\mu_1,\kappa_2\mu_2)D_{\rm NJM}^{\lambda}(\kappa_3\mu_3,\kappa_4\mu_4)R^3\cdot\begin{cases}\dfrac{1}{(\omega_{\rm NJ}^{\rm sc})^2} & \text{for } \lambda={\rm cl}\\ \dfrac{1}{(\xi_{\kappa_3\nu_3}-\xi_{\kappa_4\nu_4})^2-(\omega_{\rm NJ}^{\lambda})^2} & \text{for } \lambda={\rm mg,cl}\ .\end{cases} \tag{3.16}$$

$D_{\rm NJM}^{\lambda}s$ represent the vertex functions of the coupling between quarks and gluons. The detail forms are given in Appendix B. The self-energy due to gluon exchange is

$$\Delta E_{SF,\rm QG} = -\frac{g_s^2}{8}\cdot\frac{1}{R}\cdot\sum_n\langle\phi_0|b_n^+b_n|\phi_0\rangle\cdot\langle c|(\lambda^a)^2|c\rangle\sum_{\nu'\kappa'}\cdot\sum_{\lambda={\rm cl,el,mg}}\Lambda^{\lambda}(\nu\kappa,\nu'\kappa')\ , \tag{3.17}$$

where the Λ matrices are defined in terms of the $D_{\rm NJM}^{\lambda}s$

$$\Lambda^{\rm cl}(\nu\kappa,\nu'\kappa')=\sum_{\rm NJ}\cdot\sum_{M\mu'}(\omega_{\rm NJ}^{\rm sc})^{-2}(|D_{\rm NJM}^{\rm cl}(\nu\kappa\mu,\nu'\kappa'\mu')|^2-|D_{\rm NJM}^{\rm cl}(\nu\kappa\mu,\bar\nu'\bar\kappa'\bar\mu')|^2)\ ,$$
$$\Lambda^{\lambda}(\nu\kappa,\nu'\kappa')=\sum_{\rm NJ}\cdot\sum_{M\mu'}(\omega_{\rm NJ}^{\lambda})^{-1}[(\epsilon_{\nu\kappa}-\epsilon_{\nu'\kappa'}-\omega_{\rm NJ}^{\lambda}]^{-1}|D_{\rm NJM}^{\lambda}(\nu\kappa\mu,\nu'\kappa'\mu')|^2$$
$$+(\epsilon_{\nu\kappa}+\epsilon_{\nu'\kappa'}+\omega_{\rm NJ}^{\lambda})^{-1}|D_{\rm NJM}^{\lambda}(\nu\kappa\mu,\bar\nu'\bar\kappa'\bar\mu')|^2)\quad\text{for }\lambda={\rm el,mg}\ , \tag{3.18}$$

where $\bar\nu',\bar\kappa',\bar\mu'$ denote negative-energy states.

It should be pointed out that the expressions of the self-energy are general and they can be simplified in certain cases. For instance, for the closed shells they are in the more compact form.

Because the spectra of quark and gluon are discrete, these expressions are the summations. Each term in the summations consists of two parts. The first part indicates the structure of the six-quark states, i.e., the $\langle\phi_0|b_{n_1}^+b_{n_3}^+b_{n_4}b_{n_2}|\phi_0\rangle$ and $\langle\phi_0|b_n^+b_n|\phi_0\rangle$ part. Because there is a parameter μ in $|\phi_0\rangle$, these two factors as well as

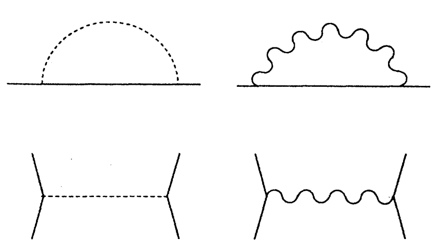

FIG. 2. The self-energies and interaction energies due to gluon and pion exchange.

the energy are functions of μ. Unlike the usual case in particle physics, $|\phi_0\rangle$ is not an eigenstate of the occupation representation but a linear combination of those eigenstates. Another part of these terms is the interaction matrix elements for the given quark states. They consist of Green's functions and vertex functions for the fields. The sum over ν', κ', and J in the self-energy due to pion exchange involve all possible intermediate states of a quark and a pion so it contains infinite terms. The sum over $\Delta E_{{\rm IN},Q\pi}$ also includes all possible angular momentum states of a pion. But the angular momentum of a quark in $|\phi_0\rangle$ is bounded; the whole summation has finite terms. For the gluon case the situation is similar. The sum over λ represents the different modes of gluon and NJM are their radial and angular momentum quantum numbers.

The contributions from the quark kinetic energy, gluon exchange, and pion exchange are polynomials of μ. The order is from 0 to 6. We define $G(\mu)$ as the contribution from the quark kinetic energy and gluon exchange.

$$E_{\rm QG}(\mu)\equiv\frac{G(\mu)}{N(\mu)}=\frac{\sum_{l=0}^{6}G_l\mu^l}{\sum_{l=0}^{6}N_l\mu^l}\ , \tag{3.19}$$

where $N(\mu)$ is the normalization coefficient. Similarly, the contribution from pion exchange is defined as

$$E_{\mathrm{IN},Q\pi}(\mu)\equiv\frac{P(\mu)}{N(\mu)}=\frac{\sum_{l=0}^{6} P_l\mu^l}{\sum_{l=0}^{6} N_l\mu^l} . \tag{3.20}$$

The $\{G_l\}$, $\{P_l\}$, and $\{N_l\}$ for the different (T,M_T,S,M_S) channel are given in Table I.

So far we have found the expressions for the energy shift due to pion and gluon exchange in terms of the perturbation calculation. They are the most important and difficult part of our task. To derive the effective nuclear potential, one needs the entire energy of the six-quark system. It can be written as follows:

$$E(\mu,R)=\frac{1}{R}[n_s(\mu)w_s+n_p(\mu)w_p]+\frac{\alpha_s^2}{R}[n_s(\mu)E_{s,\mathrm{QG}}+n_p(\mu)E_{p,\mathrm{QG}}+E_{\mathrm{IN,QG}}(\mu)]$$
$$+\frac{1}{f_\pi^2}\cdot\frac{1}{R^3}[n_s(\mu)E_{s,Q\pi}+n_p(\mu)E_{p,Q\pi}+E_{\mathrm{IN},Q\pi}(\mu)]+\tfrac{4}{3}\pi BR^3-\frac{Z_0}{R}-\frac{E_{\mathrm{c.m.c.}}}{R} , \tag{3.21}$$

where α_s, B, and Z are the parameters in the bag model. The first terms within a bracket are the kinetic energies of s and p quarks. The second set of terms is the effect due to gluon exchange which includes the self-energies of s,p quarks and their interaction energies. The third is due to the pion-exchange effects. The last three terms are the volume energy, zero-point energy and correction of center-of-mass motion, respectively. As mentioned in the Introduction, in order to get the effective nuclear potential one should subtract the contribution due to the relative motion of two nucleons. The effective potential should be expressed as the following:

$$V_{\mathrm{eff}}=M(6q)-2M(3q)-\langle \mathrm{KE_{rel}}\rangle . \tag{3.22}$$

So far there is no reliable estimate of $\langle \mathrm{KE_{rel}}\rangle$. It is believed to be roughly equal to the correction for the center-of-mass motion.[47] Since the center-of-mass motion correction in the bag model can be approximately expressed as const/R, some realistic calculations, such as the present one, which employ a value of Z_0 obtained from a fit to the hadron spectrum without an explicit center-of-mass motion correction, in fact already implicitly include this correction. The last term in (3.21), which has been called the center-of-mass motion correction above, should thus be viewed, not as this correction, but as an approximate representation of the correction associated with the subtraction of $\langle \mathrm{KE_{rel}}\rangle$. It can be seen that the energy of the six-quark system depends on the parameter μ which is related to the internuclear distance. Subtracting the masses of two free nucleons from it yields the effective nuclear potential.

C. The effective nuclear potential between nonoverlap bags

According to the energy shifts $\Delta E^{NN'}(d,R)$ (Refs. 52 and 53) the effective nuclear potential between nonoverlap bags can be expressed as

$$V_{NN'}(d,R)=E^N(R)+E^{N'}(R)+\Delta E^{NN'}(d,R)-m_N-m_{N'} , \tag{3.23}$$

where

TABLE I. The contributions from the quark kinetic energy, gluon exchange, and pion exchange for the different (TS) channels. $G(\mu)$: Contribution from quark kinetic energy and gluon exchange ($\alpha_S=1.55$). $P(\mu)$: Contribution from pion exchange. $N(\mu)$: Normalization coefficient.

	μ^0	μ^1	μ^2	μ^3	μ^4	μ^5	μ^6
			$T=0$ $MT=0$	$S=1$ $MS=1$			
$G(\mu)$	12.44	−8.28	165.93	−39.79	197.71	−8.28	19.54
$P(\mu)$	15.26	27.89	277.45	166.66	282.23	27.89	16.22
$N(\mu)$	1.0	0.0	15.0	0.0	15.0	0.0	1.0
			$T=0$ $MT=0$	$S=1$ $MS=0$			
$G(\mu)$	12.44	−9.33	164.32	−44.07	193.53	−9.33	19.05
$P(\mu)$	14.12	25.18	246.07	125.69	251.22	25.18	15.20
$N(\mu)$	1.0	0.0	15.0	0.0	15.0	0.0	1.0
			$T=1$ $MT=0$	$S=0$ $MS=0$			
$G(\mu)$	12.80	−9.14	166.78	−45.08	197.31	−9.14	19.68
$P(\mu)$	17.58	31.57	309.64	168.12	316.80	31.57	19.03
$N(\mu)$	1.0	0.0	15.0	0.0	15.0	0.0	1.0

$$\Delta E^{NN'}(d,R)=-\tfrac{1}{2}\,{}^{NN'}\!\left\langle\phi_0\left|\int_{-\infty}^{+\infty}d^4x_1 i\delta(t_1)\times\int_{-\infty}^{+\infty}d^4x_2\sum_j i\Delta^0(x_1x_2)\left[-f_\pi^2\delta\left[r_N-\left|\mathbf{R}_1-\frac{\mathbf{d}}{2}\right|\right]\times\delta\left[r_{N'}-\left|\mathbf{R}_2+\frac{\mathbf{d}}{2}\right|\right]\cdot\frac{1}{4}N[\bar\psi(x_1)\gamma_5\tau_j\psi(x_1)\times\bar\psi(x_2)\gamma_5\tau_j\psi(x_2)]\right]\right|\phi_0\right\rangle^{NN'} . \tag{3.24}$$

Assuming the quark's states all are in $1s_{1/2}$, one bag's energy is

$$E^N_{(R)}=\frac{3}{R}x_{1-1}+\frac{4}{3}\pi BR^3-\frac{Z_0}{R}-\frac{E_{\mathrm{c.m.c.}}}{R}+\frac{\alpha_s^2}{R}[3E_{QG}^{\mathrm{self}}+E_{QG}^{\mathrm{in}}]+\frac{1}{f_\pi^2}[3E_{Q\pi}^{\mathrm{self}}+E_{Q\pi}^{\mathrm{in}}]/R^3 . \tag{3.25}$$

If the radius R of the bag is equal to that of the isolated bag (of course, when a nucleon approaches another one, their radii may have to change) it implies $E^N(R)=m_N$, hence we have

$$V_{NN'}=\Delta E^{NN'}_{(d,R)}=\frac{1}{4f_\pi^2}\left[\frac{x_{1-1}}{4\pi R(x_{1-1}-1)}\right]^2\sum_{jll'm}\int d\Omega_N d\Omega_{N'}\cdot{}_{s-f}\langle NN'|\sum_{a\in N}\underline{\sigma}^a\cdot\hat{\underline{r}}_N\tau_j^a\times\sum_{b\in N'}\underline{\sigma}^b\cdot\hat{\underline{r}}_{N'}\tau_j^b|NN'\rangle_{s-f}\cdot\Delta^m_{ll'}(RR,0)\times Y^*_{lm}(\theta_N\phi_N)Y_{l'm}(\theta'_N,\phi'_N) . \tag{3.26}$$

Using

$${}_{s-f}\left\langle N\left|\sum_{a=1}^{3}\underline{\sigma}^a\cdot\hat{\underline{\gamma}}_{N'}\tau_j^a\right|N'\right\rangle_{s-f}=\tfrac{5}{3}\,{}_{s-f}\langle N|\underline{\sigma}\cdot\hat{\underline{\gamma}}\tau_j|N'\rangle_{s-f} \tag{3.27}$$

the effective nuclear potential of two separated nucleons being in various channels is given by

$$V^{T,S,M_S}_{(d,R)}=\frac{25}{72\pi^2R^2f_\pi^2}\left[\frac{x_{1-1}}{x_{1-1}-1}\right]^2\mathcal{F}_I , \tag{3.28}$$

where

$$\begin{aligned}\mathcal{F}_I&=\tfrac{1}{3}(I_0-2I_2),\quad T=1(M_T=1)S=1(M_s=1)\\&=\tfrac{1}{3}(I_0+4I_2),\quad T=1(M_T=1)S=1(M_s=0)\\&=\tfrac{1}{3}(I_0-2I_2),\quad T=1(M_T=0)S=1(M_s=1)\\&=\tfrac{1}{3}(I_0+4I_2),\quad T=1(M_T=0)S=1(M_s=0)|\\&=-I_0,\quad T=1(M_T=1)S=0\\&=-I_0,\quad T=1(M_T=0)S=0\\&=-(I_0-2I_2),\quad T=0\ \ S=1\ \ (M_s=1)\\&=-(I_0+4I_2),\quad T=0\ \ S=1\ \ (M_s=0)\\&=3I_0,\quad T=0\ S=0\end{aligned}$$

and

$$I_L=-\int\frac{j_1^2(kR)j_L(kd)}{(w_k-i\eta)^2}k^2dk . \tag{3.29}$$

IV. RESULTS AND DISCUSSIONS

A. Soft repulsive core and intermediate-range attraction

The bag model parameters used in our calculation are those taken from fitting to single baryon spectrum,[54] hence

$$\alpha_s=1.41,\quad B^{1/4}=0.151\ \mathrm{GeV},\quad Z_0=1.31 . \tag{4.1}$$

These parameters were achieved without the correction of center-of-mass motion so that part of Z_0 here should be regarded as this correction. Hence in this case the last term in Eq. (3.21) provides roughly the subtraction of $\langle \mathrm{K.E.}_{\mathrm{rel}}\rangle$. As usual in the chiral bag model, the correction of center-of-mass motion in our paper is calculated following Ref. 51 and it is roughly $0.75/R$ for nucleons.[55] It should be pointed out that in some papers, say Wong's,[56] the correction could be much larger than that and in this case the strength will be lower.

For a given μ, the energy of the six-quark system is minimized with respect to the bag radius. By substituting R_0 into Eq. (3.21) and substracting the masses of two free nucleons, we obtain the effective nuclear potential. There are some uncertainties in the calculations of the self-energies. The differences of those values given by the different calculations is still sizable. We shall discuss this situation in the next section. The results here do not include the effects of the self-energies. The tentative calculation shows that the inclusion of self-energies does not change the basic behavior of the effective potential.

The short-range part of the effective nuclear potential is shown in Fig. 3. In this figure the potentials of all possible channels (TM_TSM_S) for a two nucleon system are given. Figure 4 shows the contributions from pion exchange.

It can be seen that in our formulation the basic properties of the potential derived from the quark degrees of freedom are similar with that of the empirical potential used in nuclear physics. For the short range there is a "soft" repulsive core with a strength of about 300 MeV and the potential is attractive in the intermediate range.

It can be seen that the properties of the short-range potential are dominated by the gluon exchange between quarks as expected. The calculation also shows that the finite size of the nucleon has an important effect on the contribution of pion exchange. In the short range the effect of this exchange for all channels is repulsive. This is different from the pointlike model. But in the intermediate range the contribution turns out to be attractive just as the pointlike model.

B. Long-range part of the interaction

The line joining the centers of the two nucleons is chosen as the Z axis, the tensor and center potentials therefore can be expressed, respectively, as

$$V_C^{T,S=1}(d,R)=\tfrac{1}{3}[2V^{T,S=1,m_S=1}(d,R)+V^{T,S=1,m_s=0}(d,R)] , \quad (4.2)$$

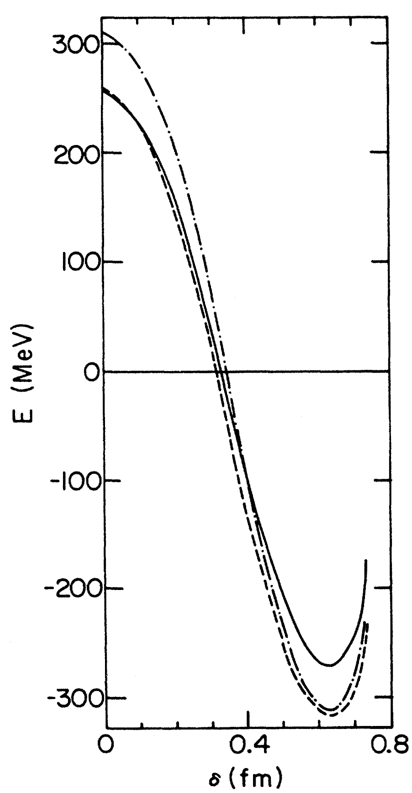

FIG. 3. The short-range effective nuclear potentials for the different (T,M_T,S,M_S) channels. The solid, dashed and dotted-dashed curves represent (0,0,1,1), (0,0,1,0), and (1,0,0,0) channels, respectively.

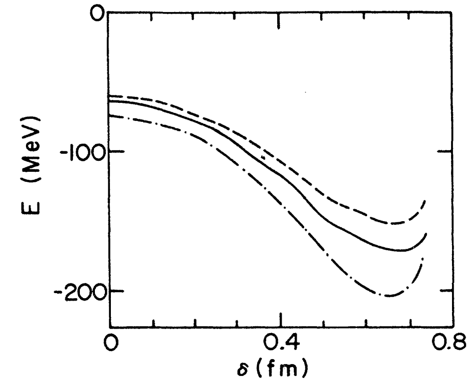

FIG. 4. The contributions from the pion exchange for the different (T,M_T,S,M_S) channels. The denotations of curves are the same as in Fig. 3.

$$V_T^{T,S=1}(d,R)=\tfrac{1}{3}[V^{T,S=1,m_s=1}(d,R)-V^{T,S=1,m_s=0}(d,R)] . \quad (4.3)$$

Using (3.27), each channel's potential is given by

$$\begin{aligned}
V_C^{T=0,\ S=1}(d,R)&=\frac{25}{72\pi^2R^2f_\pi^2}\left[\frac{x_{1-1}}{x_{1-1}-1}\right]^2(-I_0) ,\\
V_C^{T=1,\ S=1}(d,R)&=\frac{25}{72\pi^2R^2f_\pi^2}\left[\frac{x_{1-1}-1}{x_{1-1}-1}\right]^2\tfrac{1}{3}I_0 ,\\
V_T^{T=0,\ S=1}(d,R)&=\frac{25}{72\pi^2R^2f_\pi^2}\left[\frac{x_{1-1}}{x_{1-1}-1}\right]^2 I_2 ,\\
V_T^{T=1,\ S=1}(d,R)&=\frac{25}{72\pi^2R^2f_\pi^2}\left[\frac{x_{1-1}}{x_{1-1}-1}\right]^2(-\tfrac{1}{3}I_2) ,
\end{aligned} \quad (4.4)$$

and

$$\begin{aligned}
V_C^{T=0,\ S=0}(d,R)&=\frac{25}{72\pi^2R^2f_\pi^2}\left[\frac{x_{1-1}}{x_{1-1}-1}\right]^2 3I_0 ,\\
V_C^{T=1,\ S=0}(d,R)&=\frac{25}{72\pi^2R^2f_\pi^2}\left[\frac{x_{1-1}}{x_{1-1}-1}\right]^2 -I_0 .
\end{aligned} \quad (4.5)$$

It is easy to derive the I_0,I_2's analytical forms immediately

$$I_L=\frac{\pi}{2}\left[\left[\frac{1}{\mu R}ch\mu R-\frac{1}{(\mu R)^2}sh\mu R\right]^2\frac{m_\pi c^2}{\mu d}e^{-\mu d}\right] \times\begin{cases}1 & L=0 ,\\ -\left[1+\dfrac{3}{\mu d}+\dfrac{3}{(\mu d)^2}\right] & L=2 ,\end{cases} \quad (4.6)$$

where $\mu=(m_\pi c/\hbar)$.

Hence we obtain the effective nuclear potential for the long-range part from (4.2)–(4.5).

$$V_{(r)}=\frac{1}{3}\frac{25(\hbar c)^2}{72\pi^2R^2f_\pi^2}\left[\frac{x_{1-1}}{x_{1-1}-1}\right]^2\cdot\frac{\pi}{2}\frac{1}{(\mu R)^2}\left[ch\mu R-\frac{1}{\mu R}sh\mu R\right]^2 m_\pi c^2$$
$$\times\underline{\tau}_1\cdot\underline{\tau}_2\left[(\underline{\sigma}_1\cdot\underline{\sigma}_2)+\left[1+\frac{3}{\mu d}+\frac{3}{(\mu d)^2}\right]\hat{\underline{S}}_{12}\right]\cdot\frac{\exp(-\mu d)}{\mu d}$$
$$=\tfrac{1}{3}\cdot A\cdot\underline{\tau}_1\cdot\underline{\tau}_2\left[(\underline{\sigma}_1\cdot\underline{\sigma}_2)+\left[1+\frac{3}{\mu d}+\frac{3}{(\mu d)^2}\right]\hat{S}_{12}\right]m_\pi c^2\cdot\frac{\exp(-\mu d)}{\mu d}\ , \tag{4.7}$$

where

$$A=0.0564 \quad \text{for } f_\pi=93 \text{ MeV and } R=0.913 \text{ fm}$$

(Ref. 57).

In the OPEP model, $A^{\text{OPEP}}=0.081$. So the strength is about 30% lower than that of OPEP. This result is connected to the fact that the g_A is small in the cloudy bag model. In this model, $g_A=1.09$. With this value of g_A, f_π should be 77.13 MeV, to get the right value of $f_{N\pi\pi}$. In this case, $A=0.082$ which is close to that of OPEP.

C. Effects of the self-energies

There have been some discussions on the self-energy of quark for both gluon and pion exchange in the spherical cavity case.[58-62] It was shown that the quark self-energy operator associated gluon exchange can be written as follows.

$$\Sigma_f(q)=A\cdot m_q\cdot\lim_{\Lambda\to\infty}\ln\Lambda$$
$$+\text{an integral including a linear term of } k\ ,$$

where A is a constant, m_q is the mass of quark, and k is a four-dimension vector. Because of symmetry the later term vanishes after integration. So this self-energy is finite for the massless quark and it is logarithmic divergent for the massive quark. On the other hand, the self-energy due to pion exchange is linear divergent even for a massless quark. This may be connected with the surface coupling. In this case there is $\delta(r-R)$ in the interaction Hamiltonian density. To avoid this divergence, a nonlocal interaction is introduced due to the finite size of the pion. Although the finite self-energy is found the numerical results depend on the form of the nonlocal interaction.

The inclusion of self-energies leads to at least two consequences: (1) The self-energies of quark in the s state and p state are different. The ratio of s state component to p state component in the system is dependent on the separation of two nucleons. The greater the distance the more p state quark component there is in the configuration. So if the self-energies of a p quark is bigger than that of a s quark there will be more attraction. On the other hand, if the self-energies of a s quark are bigger the repulsion will increase. (2) When the self-energies are included, the parameters of bag model will change. For example, employing a four-dimensional Gaussian interaction associated with the pion-quark coupling the parameters are the following:[63,64]

$$\alpha_s=1.66,\quad Z_0=1.55,\quad B^{1/4}=116\text{ MeV}\ ,$$

and the ratio of the contribution from gluon exchange to that from pion also changes.

A tentative calculation has been made to include the effects of self-energies. No significant difference was found.

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APPENDIX A: EIGENMODES OF QUARK AND GLUON FIELDS IN A SPHERICAL CAVITY

Some parts in this appendix are not new. For convenience the most useful results are collected here. The equations and boundary conditions satisfied by the free quark and gluon fields in a bag are the following:

$$i\partial\!\!\!/\psi=0,\quad\text{inside the bag}\ ,$$
$$\left.\begin{array}{l} i n\!\!\!/\psi=\psi\ ,\\ n\cdot\partial\bar{\psi}\cdot\psi=2B\ ,\end{array}\right\}\text{at the bag surface}\ , \tag{A1}$$

$$\Box^2 A_\mu^\lambda=0,\quad\text{inside the bag}\ ,$$
$$n_\mu\cdot F^{\mu\nu a}=0,\quad\text{at the bag surface}\ , \tag{A2}$$

where ψ is quark field and A_μ^λ is gluon field. $F^{\mu\nu a}$ is defined as

$$F_{\mu\nu}^a(x)=\partial_\mu A_\nu^a(x)-\partial_\nu A_\mu^a(x)-gf_{abc}A_\mu^b(x)A_\nu^c(x) \tag{A3}$$

and

$$\left[\frac{\lambda^a}{2},\frac{\lambda^b}{2}\right]=if_{abc}\frac{\lambda^c}{2}\ . \tag{A4}$$

Solving the Dirac equation in a spherical cavity, the solution of positive energy for massless quark is as follows.

$$U_{\nu\kappa\mu}(\mathbf{r})=\begin{bmatrix} g_{\nu\kappa}(r)\chi_\kappa^\mu(\hat{r})\\ if_{\nu\kappa}(r)\chi_{-\kappa}^\mu(\hat{r})\end{bmatrix}\ , \tag{A5}$$

TABLE II. The quark spectrum.

ν/κ	$-1(S_{12})$	$1(P_{1/2})$	$-2(P_{3/2})$	$2(d_{3/2})$	$-3(d_{5/2})$	$3(f_{5/2})$
1	2.0428	3.8115	3.2039	5.1231	4.3273	6.3771
2	5.3960	7.0020	6.7578	8.4076	8.0596	9.7536
3	8.5776	10.1633	10.0042	11.6120	11.3764	13.0089
4	11.7365	13.3165	13.1969	14.7894	14.6096	16.2180
5	14.8878	16.4639	16.3691	17.9542	17.8083	19.4046
6	18.0356	19.6101	19.5311	21.1121	20.9892	22.5784
7	21.1816	22.7550	22.6873	24.2657	24.1595	25.7443
8	24.3264	25.8992	25.8399	27.4166	27.3230	28.9048
9	27.4704	29.0428	28.9901	30.5656	30.4819	32.0616
10	30.6139	32.1860	32.1386	33.7132	33.6376	35.2157

where

$$g_{\nu\kappa}(r)=N_{\nu\kappa}R^{-3/2}j_l(x_{\nu\kappa}r/R)\ ,$$
$$f_{\nu\kappa}(r)=N_{\nu\kappa}R^{-3/2}\mathrm{sgn}(\kappa)j_{\bar{l}}(x_{\nu\kappa}r/R)\ , \qquad \text{(A6)}$$

$\chi^{\mu}_{\kappa}(r)$ is the spin-harmonic function. The relations between j, l, $\bar{l}$, and κ are

$$j=|\kappa|-\tfrac{1}{2}\ ,$$
$$l=j+\tfrac{1}{2}\mathrm{sgn}(\kappa)\ , \qquad \text{(A7)}$$
$$\bar{l}=j-\tfrac{1}{2}\mathrm{sgn}(\kappa)\ .$$

$$N_{\nu\kappa}^{-2}=2j_l^2(x_{\nu\kappa})\cdot(x_{\nu\kappa}+\kappa)/x_{\nu\kappa}\ . \qquad \text{(A8)}$$

TABLE III. The spectrum of the gluon propagator.

	0	1	2	3	4
		(I) Scalar or longitudinal mode			
1	0.0000	2.0816	3.3421	4.5141	5.6467
2	4.4934	5.9404	7.2899	8.5838	9.8404
3	7.7253	9.2058	10.6139	11.9729	13.2955
4	10.9041	12.4044	13.8461	15.2445	16.6093
5	14.0662	15.5792	17.0429	18.4681	19.8624
6	17.2208	18.7426	20.2219	21.6666	23.0828
7	20.3713	21.8997	23.3905	24.8501	26.2833
8	23.5195	25.0528	26.5526	28.0239	29.4706
9	26.6661	28.2033	29.7103	31.1910	32.6489
10	29.8116	31.3521	32.8648	34.3534	35.8205
		(II) Magnetic mode			
1	2.7437	3.8702	4.9734	6.0619	7.1402
2	6.1168	7.4431	8.7218	9.9675	11.1890
3	9.3166	10.7130	12.0636	13.3801	14.6701
4	12.4859	13.9205	15.3136	16.6742	18.0085
5	15.6439	17.1027	18.5242	19.9154	21.2815
6	18.7963	20.2720	21.8539	23.1278	24.5178
7	21.9455	23.4337	25.0128	26.3224	27.7313
8	25.0928	26.5906	28.1678	29.5053	30.9294
9	28.2389	29.7441	31.3201	32.6801	34.1167
10	31.3840	32.8954	34.4705	35.8489	37.2960
		(III) Electric mode			
1	4.4934	5.7635	6.9879	8.1826	9.3558
2	7.7253	9.0950	10.4171	11.7049	12.9665
3	10.9041	12.3229	13.6980	15.0397	16.3547
4	14.0662	15.5146	16.9236	18.3013	19.6532
5	17.2208	18.6890	20.1218	21.5254	22.9046
6	20.3713	21.8539	23.3042	24.7276	26.1278
7	23.5195	25.0128	26.4768	27.9155	29.3325
8	26.6661	28.1678	29.6426	31.0939	32.5246
9	29.8116	31.3201	32.8037	34.2654	35.7076
10	32.9564	34.4705	35.9614	37.4317	38.8836

The eigenvalues $\chi_{\nu\kappa}$ are given by

$$j_l(\chi_{\nu\kappa})+\text{sgn}(\kappa)j_{\bar{l}}(\chi_{\nu\kappa})=0\ . \qquad \text{(A9)}$$

The negative-energy solution $V_{\nu\kappa\mu}(\mathbf{r})$ has a form similar to $U_{\nu\kappa\mu}(\mathbf{r})$ except that the eigenvalue is $-\chi_{\nu\kappa}$.

For the gluon field, we have the following results.

In the Coulomb gauge, the eigenmodes connected with the instantaneous Coulomb interaction can be expressed as

$$\phi_{\text{NJM}}(\mathbf{r})=N_{\text{NJ}}R^{-3/2}j_J(\omega_{\text{NJ}}^{\text{cl}}r/R)Y_{\text{JM}}(\hat{r})\ . \qquad \text{(A10)}$$

The eigenvalue is given by the boundary condition

$$\frac{d}{dr}[j_J(\omega_{\text{NJ}}^{\text{cl}}r/R)]|_{r=R}=0\ . \qquad \text{(A11)}$$

The normalized constant is

$$(N_J^{\text{cl}})^{-2}=\begin{cases}\frac{1}{2}(\omega_{10}^{\text{cl}})^{-2}[1-j_0(2\omega_{10}^{\text{cl}})], & N=1, J=0\ ,\\ \frac{1}{2}j_J^2(\omega_{\text{NJ}}^{\text{cl}})[1-J(J+1)/(\omega_{\text{NJ}}^{\text{cl}})^2], & \text{otherwise}\ .\end{cases} \qquad \text{(A12)}$$

The electric and magnetic modes of the transverse wave are

$$\begin{aligned}\mathbf{A}_{\text{NJM}}^{\text{el}}(r)=&\left[\frac{J+1}{2J+1}\right]^{1/2}N_{\text{NJ}}^{\text{el}}R^{-3/2}\\ &\times j_{J-1}(\omega_{\text{NJ}}^{\text{el}}r/R)\mathbf{Y}_{\text{JJ}-1M}(\hat{r})\\ &-\left[\frac{J}{2J+1}\right]^{1/2}N_{\text{NJ}}^{\text{el}}R^{-3/2}\\ &\times j_{J+1}(\omega_{\text{NJ}}^{\text{el}}r/R)\mathbf{Y}_{\text{JJ}+1M}(\hat{r})\end{aligned} \qquad \text{(A13)}$$

and

$$\mathbf{A}_{\text{NJM}}^{\text{mg}}(\mathbf{r})=N_{\text{NJ}}^{\text{mg}}R^{-3/2}j_J(\omega_{\text{NJ}}^{\text{mg}}r/R)\mathbf{Y}_{\text{JJM}}(\hat{r})\ , \qquad \text{(A14)}$$

where $\mathbf{Y}_{\text{JJM}}$ and $\mathbf{Y}_{\text{JJ}\pm 1M}$ are the vector-harmonic function. The eigenvalues and normalized constants of those modes are given by

$$\frac{d}{dr}[rj_J(\omega_{\text{NJ}}^{\text{mg}}r/R)]|_{r=R}=0\ , \qquad \text{(A15)}$$

$$j_J(\omega_{\text{NJ}}^{\text{el}})=0\ , \qquad \text{(A16)}$$

and

$$(N_{\text{NJ}}^{\text{mg}})^{-2}=\tfrac{1}{2}j_J^2(\omega_{\text{NJ}}^{\text{mg}})[1-J(J+1)/(\omega_{\text{NJ}}^{\text{mg}})^2]\ , \qquad \text{(A17)}$$

$$(N_{\text{NJ}}^{\text{el}})^{-2}=\tfrac{1}{2}j_{J+1}^2(\omega_{Nj}^{\text{el}})\ . \qquad \text{(A18)}$$

The lowest-lying states of a quark and a gluon are given in Tables II and III. It should be mentioned that the lowest eigenvalues of Coulomb mode of gluon field is zero. It cannot be substituted into the calculation. It was shown that for color singlet states this contribution vanishes. So in the summation this eigenvalue can be simply omitted.

APPENDIX B: THE MATRIX ELEMENTS OF QUARK-GLUON COUPLING AND QUARK-PION COUPLING

Employing the Racah-Wigner technique, one can calculate the vertex function $D_{\text{NJM}}^{\lambda}(\nu\kappa\mu,\nu'\kappa'\mu')$. For simplicity, the radial function of gluon field can be rewritten as follows.

$$\begin{aligned}&\phi_{\text{NJ}}(r)=N_{\text{NJ}}^{\text{cl}}R^{-3/2}j_J(k_{\text{NJ}}^{\text{cl}}r)\ ,\\ &h_{\text{NJ}}^{\text{mg}}(r)=N_{\text{Nj}}^{\text{mg}}R^{-3/2}j_J(k_{\text{NJ}}^{\text{mg}}r)\ ,\\ &h_{\text{NJ}\pm 1}^{\text{el}}(r)=N_{\text{NJ}}^{\text{el}}R^{-3/2}j_{J\pm 1}(k_{\text{NJ}}^{\text{el}}r)\ .\end{aligned} \qquad \text{(B1)}$$

The simplest one is in the Coulomb case

$$\begin{aligned}D_{\text{NJM}}^{\text{cl}}(\nu\kappa\mu,\nu'\kappa'\mu')=&\int d^3r\phi_\kappa U_{\nu\kappa\mu}^{+}U_{\nu'\kappa'\mu'}\\ =&\int_0^R dr\cdot r^2\cdot\phi_{\text{NJ}}(r)[g_{\nu\kappa}(r)g_{\nu'\kappa'}(r)+f_{\nu\kappa}(r)f_{\nu'\kappa'}(r)](-)^{J+l'+\bar{l}+1/2-\mu}\\ &\times\tfrac{1}{2}[1+(-)^{J+l+l'}]\begin{bmatrix}J' & J & J\\ -\frac{1}{2} & 0 & \frac{1}{2}\end{bmatrix}\cdot\begin{bmatrix}J' & J & J\\ -\mu' & M & \mu\end{bmatrix}\left[\frac{(2j+1)(2j'+1)(2J+1)}{4\pi}\right]^{1/2}.\end{aligned} \qquad \text{(B2)}$$

For the electric and magnetic modes, using the following useful formula

$$\langle l\tfrac{1}{2}j|T_K(Y_k,\sigma)|l'-\tfrac{1}{2}j'\rangle=a_K\cdot(-)^{j'-K-1/2}(2j'+1)^{1/2}\cdot(2k+1)^{1/2}\cdot\frac{1}{(4\pi)^{1/2}}\cdot\begin{bmatrix}j & j' & K\\ \frac{1}{2} & -\frac{1}{2} & 0\end{bmatrix}\ , \qquad \text{(B3)}$$

where

$$\begin{aligned}&a_k=(x'-x)/[k(k+1)]^{1/2}\ ,\\ &a_{k-1}=-(k+x+x')/[k(2k+1)]^{1/2}\ ,\\ &a_{k+1}=(k+1-x-x')/[(k+1)(2k+1)]^{1/2}\ ,\\ &x=(l-j)(2j+1)\ ,\end{aligned}$$

and

$$x'=(l'-j')(2j'+1) ,$$

we have

$$\begin{aligned} D^{\rm el}_{\rm NJM}(\nu\kappa\mu,\nu'\kappa'\mu') &= \int d^3r\, \mathbf{A}^{\rm el}_{\rm NJM}\cdot U^+_{\nu\kappa\mu}\boldsymbol{\alpha} U_{\nu'\kappa'\mu'} \\ &= i\int dr\cdot r^2 \Bigg[[h^{\rm el}_{\rm NJ-1}(r)+h^{\rm el}_{\rm NJ+1}(r)][g_{\nu\kappa}(r)f_{\nu'\kappa'}(r)-g_{\nu'\kappa'}(r)f_{\nu\kappa}(r)] \\ &\quad +(\kappa'-\kappa)[g_{\nu\kappa}(r)f_{\nu'\kappa'}(r)+g_{\nu'\kappa'}(r)f_{\nu\kappa}(r)]\left[\frac{1}{J}h^{\rm el}_{\rm NJ-1}(r)-\frac{1}{J+1}h^{\rm el}_{\rm NJ+1}(r)\right]\Bigg] \\ &\quad \cdot(-)^{j+j'-J-\mu-1/2}\frac{[(2j+1)(2j'+1)J(J+1)]^{1/2}}{[4\pi(2J+1)]^{1/2}} \\ &\quad \cdot\begin{bmatrix} j & J & j' \\ -\mu & M & \mu' \end{bmatrix}\cdot\begin{bmatrix} j & j' & J \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{bmatrix}\cdot\tfrac{1}{2}[1+(-)^{l'+J+l}] . \end{aligned} \tag{B4}$$

And for the magnetic mode one gets

$$\begin{aligned} D^{\rm mg}_{\rm NJM}(\nu\kappa\mu,\nu'\kappa'\mu') &= \int d^3r\, \mathbf{A}^{\rm mg}_{\rm NJM}\cdot U^+_{\nu\kappa\mu}\boldsymbol{\alpha} U_{\nu'\kappa'\mu'} \\ &= i\int dr\cdot r^2\cdot h^{\rm mg}_{\rm NJ}(r)\cdot[g_{\nu\kappa}(r)f_{\nu'\kappa'}(r)+g_{\nu'\kappa'}(r)f_{\nu\kappa}(r)]\cdot(\kappa+\kappa') \\ &\quad \times(-)^{j+j'-J-\mu-1/2}\cdot\frac{[(2J+1)(2j+1)(2j'+1)]^{1/2}}{[4\pi\cdot J\cdot(J+1)]^{1/2}}\cdot\begin{bmatrix} j & J & j' \\ -\mu & M & \mu' \end{bmatrix}\begin{bmatrix} j & j' & J \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{bmatrix} \\ &\quad \times\tfrac{1}{2}[1-(-)^{l'+J+l}] . \end{aligned} \tag{B5}$$

Because quark field and pion field are coupled at the bag surface, it is useful to write down the expressions of the quark wave function and pion Green's function at the surface. They are

$$U_{\nu\kappa\mu}(\mathbf{r})=g_{\nu\kappa}(R)\begin{bmatrix} 1 \\ i\underline{\sigma}\cdot\hat{\underline{r}} \end{bmatrix}\chi^\mu_\kappa=g_{\nu\kappa}(R)\begin{bmatrix} \chi^\mu_\kappa \\ -i\chi^\mu_{-\kappa} \end{bmatrix} , \tag{B6}$$

$$V_{\nu\kappa\mu}(\mathbf{r})=g_{\nu\kappa}(R)\begin{bmatrix} 1 \\ i\underline{\sigma}\cdot\hat{\underline{r}} \end{bmatrix}\chi^\mu_{-\nu}=g_{\nu\kappa}(R)\begin{bmatrix} \chi^\mu_{-\kappa} \\ -i\chi^\mu_{\kappa} \end{bmatrix} , \tag{B7}$$

$$g_{\nu\kappa}(\mathbf{r})=R^{-3/2}\left[\frac{x_{\nu\kappa}}{2(x_{\nu\kappa}+\kappa)}\right]^{1/2}\cdot\frac{j_l(\chi_{\nu\kappa})}{|j_l(\chi_{\nu\kappa})|} , \tag{B8}$$

$$\Delta^0_l(RR,\omega) = \begin{cases} \dfrac{1}{R}\cdot[\xi j_l(\xi)n_l(\xi)+i\xi j_l^2(\xi)] & \xi=\omega R\neq 0 , \\ -\dfrac{1}{2l+1}\cdot\dfrac{1}{R} & \omega=0 . \end{cases} \tag{B9}$$

From these, it is easy to get the matrix elements of quark-pion coupling.

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