Two-amplitude model for the energy dependence of double charge exchange on ^{18}O

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(Received 20 January 1988)

A naive two-amplitude model, in which one amplitude is independent of energy and the other is $2|T_{33}|^2 e^{2i\phi_{33}}$, gives an excellent fit to the energy dependence of the double charge exchange cross section on ^{18}O.

The mechanism of pion-induced double charge exchange (DCX) is still not understood. Even for what should be the simplest case—in which the final state is the double isobaric analog (DIAS) of the target ground state—the observed behavior is not that expected on theoretical grounds. Though DIAS data exist for about 20 nuclei, the reaction ^{18}O(π^+,π^-) ^{18}Ne(g.s.) is by far the best studied, [2–5] so most of the following remarks apply to that case.

Figure 1 displays the measured[2,3,5] 5° cross section as a function of incident-pion kinetic energy. The main features are (1) the large dip near 180 MeV, and (2) the fact that the cross section near 140 MeV is as large as that at 292 MeV. Lowest-order calculations,[5] along the lines of Miller and Spencer,[6] predict a monotonic increase in the cross section from 80 to 300 MeV.

At energies of about 200 MeV and above, the angular-distribution shape[2,3] is roughly as expected, but at energies in the range 160–180 MeV, the experimental angular distribution[2,3] has a minimum at a much more forward angle than expected theoretically.[6–8] One explanation of the data invokes the interference of two amplitudes.[9–11] To fit the data near 160–180 MeV, the two amplitudes[10] must be nearly totally out of phase. In fact, two separate fits to the angular distribution at 164 MeV produce a relative phase between the two amplitudes of 169° and 175°. In these two fits,[10] the ratios of nonanalog to DIAT amplitudes are 1.80 and 2.05, respectively.

In the spirit of the same two-amplitude model (TAM), the relative phase[11] at 292 MeV is 95°. For ^{18}O at 292 MeV, the relative magnitudes of the normal DIAT amplitude and the "other" amplitude are found to be about equal. Specifically, the ratio of nonanalog to DIAT amplitude at 292 MeV for ^{18}O is found to be 0.92.[11] This behavior of phase with energy is very similar to that of twice the $\Delta_{33}\pi N$ resonance phase, ϕ_{33}. Furthermore, the 33 amplitude squared at 180 MeV is roughly twice the value at 300 MeV.

Figure 2 contains a plot of the πNP_{33} scattering amplitude from Ref. 12. The phase reaches 90° at about $T_\pi = 190$ MeV. Thus, twice the phase is about 180° at 180 MeV and 265° at 292 MeV—very close to the behavior of the relative phase extracted for the interfering amplitude, and, if the 33 *cross section* is taken to be the magnitude of the *amplitude* for DCX (i.e., a second-order Δ process), this amplitude at 180 MeV is then about twice the amplitude at 300 MeV.

We demonstrate one possible pair of interfering amplitudes in Fig. 3. We show separately the cross section for an energy-independent amplitude of unit magnitude, and for a second amplitude equal to $2|T_{33}|^2$, i.e., the cross section from the second amplitude alone is $4|T_{33}|^4$. If the first amplitude also has an energy-independent phase, then the relative phase between it and the second amplitude is just $2\phi_{33}$. The other curve in Fig. 3 is just

$$|1+2T_{33}^2|^2 = |1+2|T_{33}|^2 e^{2i\phi_{33}}|^2 .$$

It looks remarkably like the data for ^{18}O, as can be seen in Fig. 4.

Figure 4 displays the ^{18}O data, along with the cross section computed assuming relative amplitudes at 300 MeV of 0.9, 1.0, and 1.1. The reason for this dramatic agreement with such a naive expression is not understood, but it must have a relatively simple explanation.

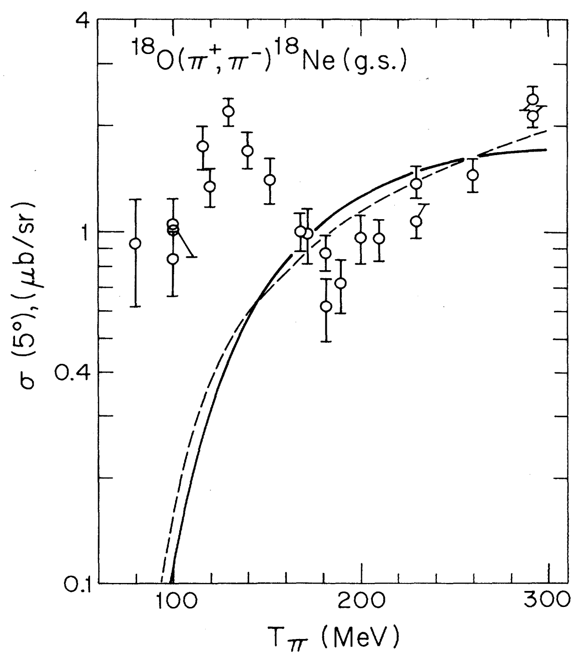

FIG. 1. Data points are cross sections for ^{18}O$(\pi^+,\pi^-)^{18}$Ne(g.s.) measured at a laboratory angle of 5°, from Refs. 2, 3, and 5. Curves are lowest-order predictions from Ref. 5, using the approach of Ref. 6.

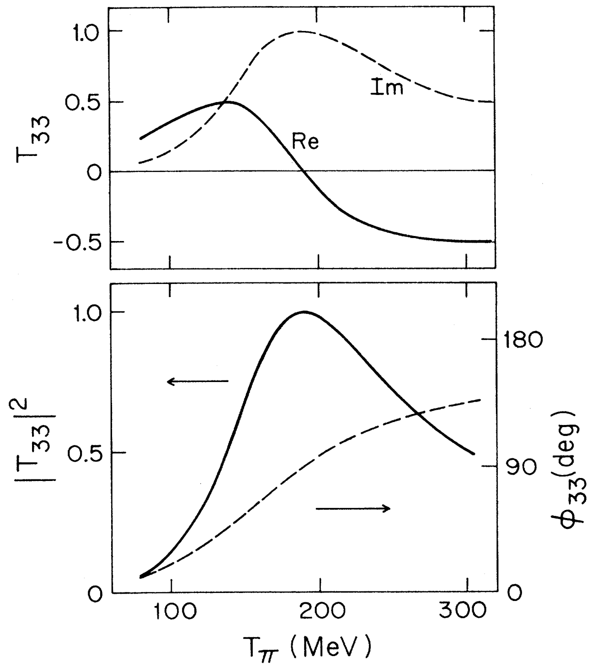

FIG. 2. (Top) Real (solid) and imaginary (dashed) parts of the $p_{33}\pi N$ scattering amplitude, from Ref. 11. (Bottom) Magnitude squared (solid, left-hand scale) and phase (dashed, right-hand scale) of the p_{33} scattering amplitude.

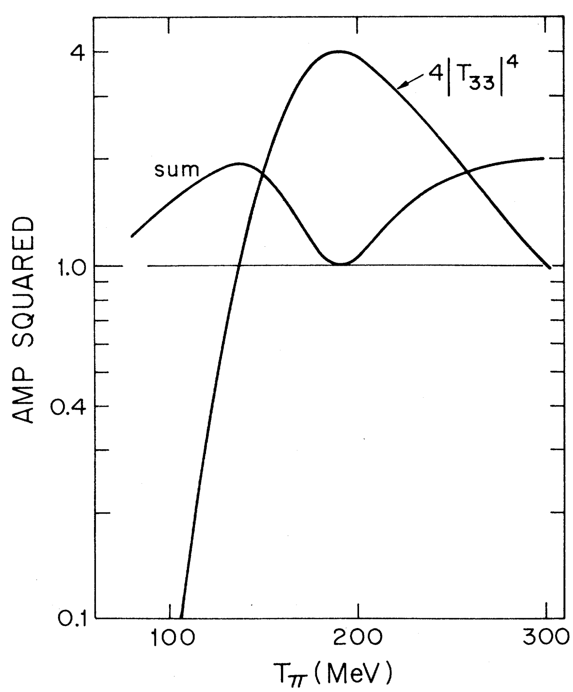

FIG. 3. Cross sections for two simple amplitudes—one independent of energy, the other varying as $2|T_{33}|^2e^{2i\phi}$, and the cross section for the sum of the two amplitudes.

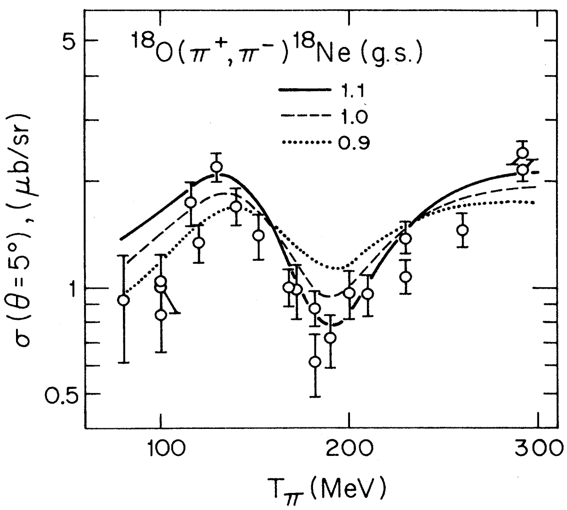

FIG. 4. The ^{18}O data, together with two-amplitude calculations for relative magnitudes, at 300 MeV, of 1.1, 1.0, and 0.9.

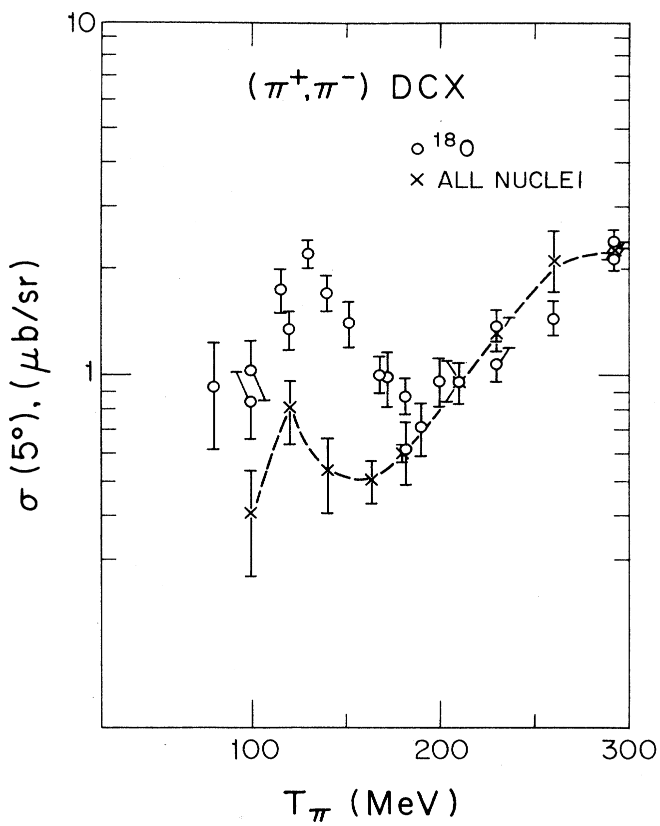

FIG. 5. The ^{18}O data are plotted again as open circles. The ×'s are the quantity $f(E)$ from the fit (Ref. 1) to 5° cross sections of $\sigma(A,E)=\{[(N-Z)(N-Z-1)]/2\}/(18/A)^{10/3}f(E)$. The dashed curve serves only to connect the ×'s.

Note that we have inserted no additional energy dependence—no powers of the momentum k. Furthermore, neither of the two interfering amplitudes is similar to that calculated with the code PIESDEX (Ref. 7) (Fig. 1).

What process can give rise to a cross section that is nearly independent of energy (in the range 100–300 MeV), rather than having the dramatic rise calculated for lowest-order DCX? One candidate might be the total reaction cross section, which is nearly constant in energy for several nuclei. From Table I of Ref. 13, we note that for light nuclei, the quantity $\sigma_{\mathrm{reac}} \equiv \sigma_{\mathrm{tot}} - \sigma_{\mathrm{el}}$ varies by less than 40% in the energy range 100–300 MeV. If the DCX cross section from the DIAT amplitude alone, as a function of energy, is roughly a constant fraction of the total reaction cross section, then little energy dependence would result.

This naive model that appears to work so well for ^{18}O cannot be easily tested for other nuclei because the data are much sparser. We can, however, compare the ^{18}O data with a "composite" excitation function for several nuclei.

At all energies between 100 and 292 MeV, when all DIAS 5° cross sections are included in a fit[1] to the form

$$\sigma(A,E) = \frac{(N-Z)(N-Z-1)}{2}\left[\frac{A}{18}\right]^{-\alpha} f(E) ,$$

the best-fit value of α is very close to $\frac{10}{3}$. We can then use the fitted values of $f(E)$ to plot a composite excitation function, as is done in Fig. 5. Here we have included all data in the fits of Ref. 1.

From 180 to 292 MeV, the two excitation functions are very similar, but at lower energies the bump that is so pronounced in the ^{18}O data is barely apparent in the composite. Is ^{18}O different from all other nuclei, as regards DCX? Is it an accident that a naive model fits the ^{18}O data so well? Better data, at more energies, might help answer the first question. The answer to the second probably awaits a more serious theoretical approach. It may be, however, that our simple result implies that an important feature is being omitted from standard lowest-order calculations of DCX.

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