PHYSICAL REVIEW C VOLUME 39, NUMBER 5 MAY 1989

Use of form factors in electromagnetic interactions

H. W. L. Naus
National Institute of Nuclear Physics and High-Energy Physics (NIKHEF, section K), 1009 DB Amsterdam, The Netherlands

J. H. Koch
National Institute of Nuclear Physics and High-Energy Physics (NIKHEF, section K), 1009 DB Amsterdam, The Netherlands and Institute for Theoretical Physics, University of Amsterdam, 1018 XE Amsterdam, The Netherlands
(Received 2 November 1988)

We comment on the description of electromagnetic reactions involving hadrons, when the internal structure of the hadrons is taken into account. General off-shell vertex operators, only constrained by Lorentz and gauge invariance, are used. The electromagnetic production of pions on a nucleon is discussed as an example. Commonly used *ad hoc* recipes involving phenomenological form factors are discussed in the framework of an exact formulation.

I. INTRODUCTION

When describing the electromagnetic interaction with extended objects, one has to take into account the internal structure of the target. This is often done in a compact way by the use of (phenomenological) form factors for the particles (nucleons, pions, Δ's, etc.). It is clear that in such reactions—like electroproduction of pions from a nucleon—electromagnetic and strong form factors enter into the description. However, this has to be done in such a way that gauge invariance, as expressed by the generalized Ward-Takahashi identity,[1] is maintained. The fact that the particles involved can be off their mass shells must also be taken into account. It is the purpose of this article to discuss this in detail, provide the general framework, and then to put commonly used or proposed recipes into perspective. In this paper we will mainly focus on the electromagnetic production of pions from a nucleon as an instructive example.

The most commonly used approach to describe pion photoproduction and electroproduction is to calculate the Born diagrams to first order in the electromagnetic and strong couplings. By using the Ward identities, Kroll and Ruderman[2] showed that at threshold the (γ,π) amplitude to lowest order in the photon energy is indeed equivalent to the result obtained in lowest order Born approximation using the renormalized masses, and coupling constants. This "Kroll-Ruderman theorem" is often seen as a justification for extending the Born diagram approach to higher energies. The only indication of the nucleon's internal structure in these Born terms is the anomalous magnetic moment, which yields sizable contributions. As we will see in the following, other terms due to the internal structure of the particles have to be included in a consistent description as well.

In applying the Born term approach to pion electroproduction and by using electromagnetic form factors for the hadrons, it is well known that one does not get a gauge invariant amplitude (see, e.g., Ref. 3). A similar problem is encountered when calculating the meson exchange currents in electron scattering. One way to remedy this situation is to assume that pion and nucleon electromagnetic form factors are the same. If one also includes a strong pion-nucleon form factor, it also has to be taken the same to maintain gauge invariance. This is in contrast with the experimental information. It has also been argued that one could simply use different (phenomenological) on-shell form factors, but then add *ad hoc* gauge terms which, while not contributing to the physical amplitude, would ensure gauge invariance. These approaches are summarized in the paper by Dressler.[4] Recently, Gross and Riska[5] developed another description in the context of meson exchange current operators, where it is allowable to use different form factors. We examine this method below.

The problem assumes a new dimension by the fact that, even in photoproduction and electroproduction on a *free* nucleon, the vertex operators and (on-shell) form factors for the free particles are not sufficient for a general description. One has to extend the on-shell form factors off the mass shell and include additional off-shell operators with off-shell form factors. This was already realized, for instance, by Berends and West.[6] By choosing the additional electromagnetic off-shell form factors in a certain way, they obtain a conserved amplitude.

Since the above recipes were introduced in an *ad hoc* fashion, we will in this paper provide a general theoretical framework to deal with the problem of internal structure and gauge invariance. As we will see, a consistent gauge invariant description of electromagnetic processes with hadrons requires a microscopic model for the origin of their structure. Our discussion is mainly based on the work of Kazes,[7] who takes the full off-shell structure of all vertices into account. We will put the above recipes into perspective and outline how the problem must be attacked in general. While we will focus on pion production—in particular the (γ,π^+) reaction—and use the language of pions and nucleons only, our comments are quite general and can easily be extended to other cases.

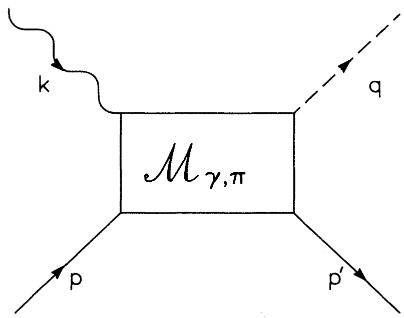

FIG. 1. Electromagnetic production of a pion from a nucleon.

II. GAUGE INVARIANCE IN PION PRODUCTION

We consider photoproduction and electroproduction of a pion from a nucleon (see Fig. 1)

$$\gamma(k)+N(p)\rightarrow N'(p')+\pi(q) \ . \tag{2.1}$$

The photon is virtual in the case of electroproduction. We denote the operator for this process by $M_\mu(p',q;p,k)$. By imposing gauge invariance one can derive, for example, for π^+ production (for details see Ref. 7)

$$k^\mu M_\mu(p',q;p,k)=e[\Delta'^{-1}(q)\Delta'(q-k)\Lambda_5(p',p) - \Lambda_5(p',p+k)S'(p+k)S'^{-1}(p)] \ . \tag{2.2}$$

Here Λ_5 is the full irreducible π-N vertex operator, and S' and Δ' are the dressed nucleon and pion propagators, respectively. Similar relations hold for π^- production and π^0 production. In general, one can divide all diagrams contributing to this process into two classes as follows (see Fig. 2).[8]

The first we will call class A, which can be put together from, or reduced to, the building blocks of dressed meson and nucleon vertices and propagators. This part of the operator can be written as

$$\begin{aligned}M_\mu^A(p',q;p,k)=&\Lambda_5(p',p+k)S'(p+k)\Gamma_\mu^p(p+k,p)\\&+\Gamma_\mu^\pi(q,q-k)\Delta'(q-k)\Lambda_5(p',p)\\&+\Gamma_\mu^n(p',p'-k)S'(p'-k)\Lambda_5(p'-k,p) \ .\end{aligned} \tag{2.3}$$

Γ_μ^p and Γ_μ^n are the irreducible general proton and neutron electromagnetic vertices.

The second class of diagrams, class B, cannot be reduced. It contains internal insertions of the photon into the dressed vertices.

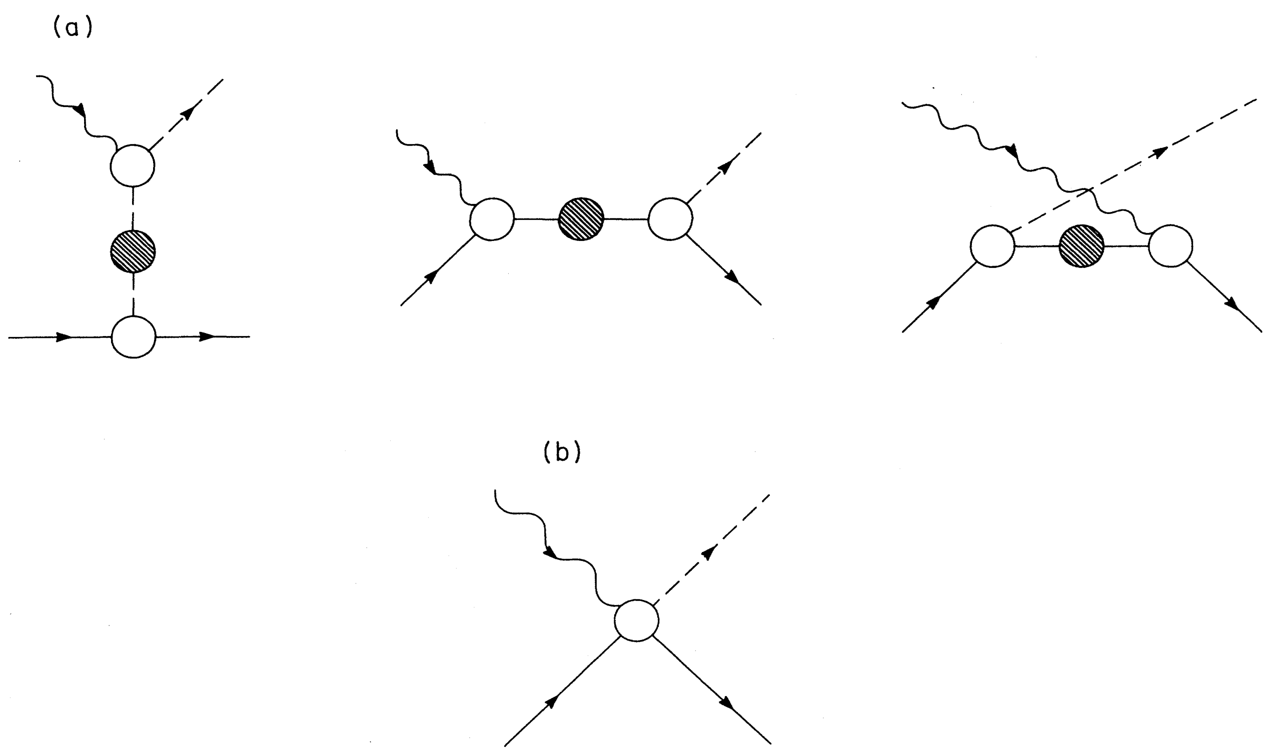

FIG. 2. (a) Class A diagrams contributing to the pion production process. The open circles indicate dressed vertices, the hatched circles self-energy insertions. (b) Class B diagrams.

By using the Ward-Takahashi identities for the nucleons and pion,

$$(p'-p)^{\mu}\Gamma_{\mu}^{p,n}(p',p)=e^{p,n}[S'(p')^{-1}-S'(p)^{-1}] , \quad (2.4)$$

$$(q'-q)^{\mu}\Gamma_{\mu}^{\pi}(q',q)=e^{\pi}[\Delta'(q')^{-1}-\Delta'(q)^{-1}] , \quad (2.5)$$

where e^i are the charges in units of e, one easily gets from Eqs. (2.2) and (2.3)

$$k^{\mu}M_{\mu}^{B}(p',q;p,k)=e[\Lambda_5(p',p)-\Lambda_5(p',p+k)] . \quad (2.6)$$

This holds in general for the (renormalizable) pseudoscalar case. However, for pseudovector pion-nucleon coupling, the above results only hold in lowest order. In this case, the point coupling contact term, which is not reducible, also belongs to class B. Without a strong form factor it satisfies Eq. (2.6); if one uses a form factor in the pseudovector vertex more M_{μ}^{B} terms are necessary.

As an illustration, we consider a model where the pion-nucleon interaction is pseudoscalar, coupling constant $g_{\pi NN}$, and the nucleon is dressed by a neutral (pseudoscalar-) scalar meson M, which couples with strength G according to

$$L_{MNN}=G\bar{\psi}\Gamma\psi\phi_M , \quad \Gamma=1 \text{ (or } i\gamma_5) . \quad (2.7)$$

All diagrams which one obtains to order G^2 and to first order in $g_{\pi NN}$ are shown in Fig. 3. [Note that the last diagram in Fig. 3(a) does not contribute.] It is easily verified that condition (2.6), which is a consequence of the requirement of gauge invariance, is satisfied by the dressed πNN vertex, which in this model is

$$\Lambda_5(p',p)=g_{\pi NN}i\gamma_5+g_{\pi NN}G^2\Gamma \times \int d_4t\, \Delta_M(t)S(p'-t)i\gamma_5S(p-t)\Gamma , \quad (2.8)$$

where Δ_M, and S are the bare propagators. Also, one can prove that the diagrams together satisfy Eq. (2.2). For this to be true, it is essential to include the self-energy integral for the nucleon. [As can be seen by power counting, these diagrams contain divergences, and a renormalization procedure is necessary. The above relations then hold for the renormalized quantities. If one chooses the dimensional regularization procedure,[9] which preserves the local gauge symmetry, the above equations apply directly to the (4-ϵ)-dimensional expressions.] Note that in this simple model the neutron electromagnetic vertex does not get dressed and we have $\Gamma_{\mu}^{n}=0$.

The class B diagrams subject to condition (2.6) are necessary to ensure gauge invariance if the particles have structure. Standard models for pion production based on the Born diagram concept do not include this type of diagrams (except for the contact term for pseudovector coupling), but nevertheless include, e.g., an anomalous magnetic moment coupling which is due to internal structure. In the language of class A and B diagrams, Born terms—except the just mentioned contact term—are class A diagrams with on-shell vertices and free propagators. In nonrelativistic potential scattering, Heller[10] showed that in the case of nonlocal potentials current conservation requires the existence of "interaction currents." These currents, which are due to internal insertion into potentials, are similar to the class B diagrams. They play a crucial role in the quantitative description[11] of πN bremsstrahlung and the extraction of a magnetic moment of the Δ.

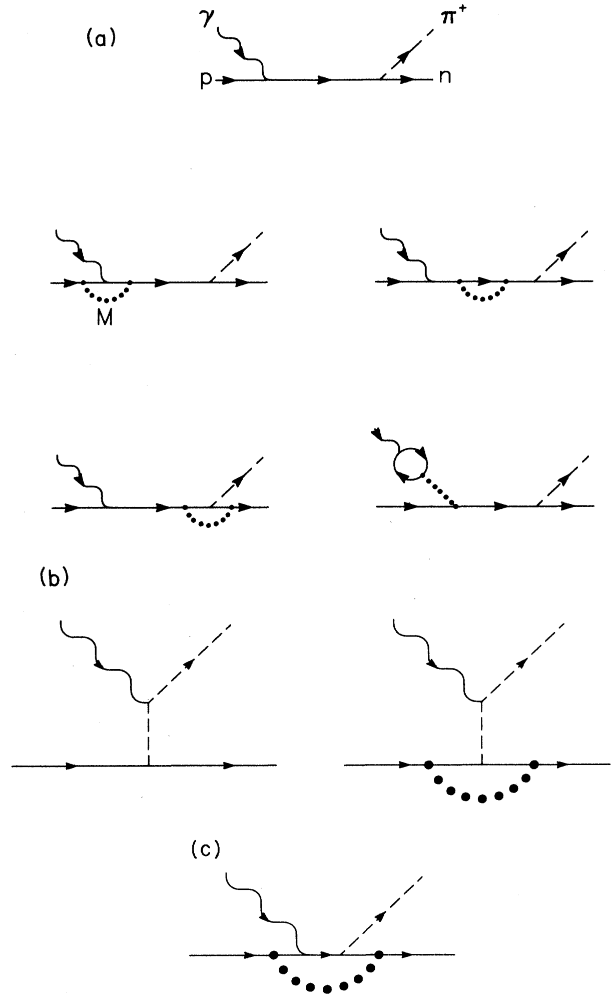

FIG. 3. Feynman diagrams for pion production contributing to order G^2 and $g_{\pi NN}$ in the model described in the text. (a) Nucleon pole contributions, class A; (b) pion pole contributions, class A; (c) class B diagram.

It is also important to realize that in general the vertices appearing in the amplitude M_{μ}^{A} are half off shell. When the outgoing nucleon is on shell, $p'^2=M^2$, the reducible half-off shell electromagnetic vertex, $\Gamma_{\mu}^{\rm red}$, has the general form (see, e.g., Ref. 12 or 13)

$$\bar{u}(p')\Gamma_\mu^{\text{red}}=e\bar{u}(p')\left[\left[\gamma_\mu f_1^{(+)}+\frac{i\sigma_{\mu\nu}}{2M}k^\nu f_2^{(+)}+k_\mu f_3^{(+)}\right]\Lambda_+ +\left[\gamma_\mu f_1^{(-)}+\frac{i\sigma_{\mu\nu}}{2M}k^\nu f_2^{(-)}+k_\mu f_3^{(-)}\right]\Lambda_-\right] , \tag{2.9}$$

where $f_i^{(\pm)}=f_i^{(\pm)}(k^2,p^2,M^2)$. Spin and isospin labels are omitted. We define $W^2=p^2$, where we take $W>0$. The projection operators $\Lambda_\pm$ are defined as

$$\Lambda_\pm=(\pm p\cdot\gamma+W)/2W . \tag{2.10}$$

A similar form holds in the case where the initial nucleon is on shell. The relation between the reducible and the irreducible vertex is

$$\bar{u}(p')\Gamma_\mu^{\text{red}}(p',p)=\bar{u}(p')\Gamma^\mu(p',p)S'(p)S^{-1}(p) , \tag{2.11}$$

where $S(p)$ is the bare nucleon propagator. Applying the Ward-Takahashi identity yields[12,13] a relation between $f_1^{(\pm)}$ and $f_3^{(\pm)}$. Therefore the (gauge invariant) vertex involves four independent form factors, which depend on three scalar variables. On shell, it reduces to the usual form

$$\Gamma_\mu(k^2)=\gamma_\mu F_1(k^2)+F_2(k^2)\frac{i\sigma_{\mu\nu}k^\nu}{2M} . \tag{2.12}$$

At the photon point, $k^2=0$, $f_1^{(+)}$ and $f_1^{(-)}$ in Eq. (2.9) are subject to the condition

$$f_1^{(\pm)}(0,p^2,M^2)=e^{p,n} . \tag{2.12a}$$

The most general electromagnetic vertex for the pion is

$$\Gamma_\mu^\pi(q',q)=e[(q+q')_\mu h^+(k^2,q'^2,q^2)+(q'-q)_\mu h^-(k^2,q'^2,q^2)] , \tag{2.13}$$

where $k_\mu=(q'-q)_\mu$. On shell, $h^+(k^2,m_\pi^2,m_\pi^2)$ is $F_\pi(k^2)$, the electromagnetic form factor of a free pion. Similarly, the general off-shell πNN vertex has the structure

$$\begin{aligned}\Lambda_5(p',p)=&i\gamma_5 g_1(p^2,p'^2,(p-p')^2)+i\gamma_5(p\cdot\gamma-M)g_2(p^2,p'^2,(p-p')^2)/M\\&+(p'\cdot\gamma-M)i\gamma_5 g_3(p^2,p'^2,(p-p')^2)/M+(p'\cdot\gamma-M)i\gamma_5(p\cdot\gamma-M)g_4(p^2,p'^2,(p-p')^2)/M^2 .\end{aligned} \tag{2.14}$$

For on-shell nucleons, what remains is the free πNN form factor

$$F_{\pi NN}((p-p')^2)=g_1(M^2,M^2,(p-p')^2) . \tag{2.15}$$

To gain further insight into the problem of gauge invariance in pion production, we consider for simplicity the photoproduction of pions near threshold. We expand the full (γ,π^+) production amplitude to order m_π/M at threshold, $\mathbf{q}=0$. In this case, the pion pole diagram does not contribute. From Eq. (2.6), we can obtain a Taylor expansion of the operator M_B up to terms linear in k,

$$M_\mu^B(p',q;p,k)=-e\left[\frac{\partial\Lambda_5(p',p)}{\partial p^\mu}+\tfrac{1}{2}k_\lambda\frac{\partial^2\Lambda_5(p',p)}{\partial p^\lambda\partial p^\mu}\right]+S_\mu , \tag{2.16}$$

where S_μ is an undetermined term of the order k, which is separately gauge invariant. Using Eq. (2.14) and defining $P=(M,\mathbf{0})$, we obtain for the matrix element in this order

$$\begin{aligned}\epsilon^\mu\bar{u}(p')M_\mu^B u(p)=&-(e/M)\epsilon^\mu\bar{u}(P)[i\gamma_5\gamma_\mu g_2+Mk_0 i\gamma_5\gamma_\mu g_2'-i\gamma_5\gamma_\mu\mathbf{k}\cdot\boldsymbol{\gamma}g_2/(2M)]u(P)\\&+\epsilon^\mu\bar{u}(P)S_\mu u(P)+O((k/M)^2) ,\end{aligned} \tag{2.17}$$

where the g_i and g_i' are evaluated at their on-shell values. Expanding the matrix element of the class A operator Eq. (2.3) consistently up to terms linear in the photon momentum or, equivalently, in powers of m_π/M, one gets in the two-body c.m. frame

$$\begin{aligned}\epsilon^\mu\bar{u}(p')M_\mu^A u(p)=e\epsilon^\mu\bar{u}(P)[&(-g_1/2M+g_2/M)i\gamma_5\gamma_\mu+(-g_1'+2g_2')k_0 i\gamma_5\gamma_\mu-k_0 g_1 i\gamma_5\gamma_0\gamma_\mu/(4M^2)\\&+(g_1-2g_2)i\gamma_5\Sigma_\mu\kappa_p^{(-)}/M+(g_1-2g_2)i\gamma_5\gamma_\mu\mathbf{k}\cdot\boldsymbol{\gamma}/(4M^2)+\Sigma_\mu\kappa_n^- i\gamma_5(g_1-2g_3)/M]u(P) ,\end{aligned} \tag{2.18}$$

where

$$\Sigma_\mu = i\sigma_{\mu\nu}k^\nu/(2M)$$

and

$$\kappa^-_{p,n} = f_2^{p,n(-)}(0,M^2,M^2) \ .$$

Both Eqs. (2.17) and (2.18) depend on the off-shell dynamics through form factors such as g_2 or $f_2^{(-)}$, which are not directly observable and model dependent. If we only keep terms of order $(k/M)^0$ in the threshold amplitude, Eqs. (2.17) and (2.18) yield

$$\begin{aligned}\epsilon^\mu \bar{u}(p')M^A_\mu u(p) \\ &= e\epsilon^\mu \bar{u}(P)[(-g_1/2M + g_2/M)i\gamma_5\gamma_\mu]u(P) \ , \quad (2.19\text{a})\end{aligned}$$

$$\begin{aligned}\epsilon^\mu \bar{u}(p')M^B_\mu u(p) \\ &= e\epsilon^\mu \bar{u}(P)[(-g_2/M)i\gamma_5\gamma_\mu]u(P) \ , \quad (2.19\text{b})\end{aligned}$$

which together result in the total threshold matrix element

$$\begin{aligned}\bar{u}(p')\epsilon^\mu M_\mu u(p) &= \bar{u}(P)\epsilon^\mu (M^A + M^B)_\mu u(P) \\ &= -e\bar{u}(P)g_1/(2M)\epsilon^\mu i\gamma_5\gamma_\mu u(P) \ . \quad (2.20)\end{aligned}$$

The sum of the two contributions only depends on on-shell (renormalized) quantities. The terms involving the model-dependent g_2 have disappeared. This is the result of Kroll and Ruderman,[2] who show that this threshold result is also obtained by evaluating the lowest order Born terms with renormalized coupling constants and masses in this limit. This "Kroll-Ruderman theorem" does not justify using the Born terms for the higher order terms or away from threshold into the resonance region as is commonly done. It is important to stress that the result, Eq. (2.20), is obtained by a cancellation of the g_2 terms from class A and B diagrams. In the higher terms in k/M, the (model-dependent) internal structure aspects enter, requiring knowledge of the half-off-shell form factors, e.g., $f_2^{(-)}$, for the description. Then also the up-to-now unspecified term S_μ in Eq. (2.16) can contribute. This will be discussed in a forthcoming paper, which also discusses π^0 photoproduction and the restrictions imposed on these terms by partial conservation of axial vector current (PCAC).

The above considerations point out the necessity of consistently including the internal structure of hadrons in electromagnetic reactions such as pion photoproduction. Including only an on-shell anomalous magnetic moment coupling in the Born terms is not sufficient. A full treatment necessarily requires a microscopic model for the origin of the form factors and a prediction of their off-shell form. It is also clear that such a model is needed to calculate the "type-B" diagrams, which are crucial to obtain a gauge invariant description. Without further assumptions, only the term of order k^0 of the photopion amplitude is determined model independently. For low-energy Compton scattering, also the two lowest terms in k are determined in a general way,[14,8] since the term analogous to S_μ in Eq. (2.16) is of order k^2, and terms depending on off-shell form factors cancel in this case.

III. RECIPES TO RESTORE GAUGE INVARIANCE

It was realized early on that the renormalized Born terms describing pion electroproduction lead to a nonconserved current when on-shell (phenomenological) electromagnetic form factors for the pion and nucleon were used. One way to resolve this problem is to simply take $F_\pi(k^2) = F_1(k^2)$, which, however, is not in agreement with experiment. If also strong form factors, e.g., at the πNN vertex, are used, this recipe also requires this form factor to be the same as the electromagnetic ones. Another recipe is to construct the total current, J^B_μ, from the standard Born terms with on-shell form factors at the vertices. One then subtracts an *ad hoc* term proportional to k_μ,

$$J^B_\mu \to J^B_\mu - k_\mu k\cdot J^B/k^2 \ , \quad (3.1)$$

which is chosen to restore gauge invariance. This extra term does not contribute to the physical amplitude when contracted with the electron current j^e, since $kj^e = 0$. These approaches are reviewed in the recent paper by Dressler.[4] In the context of electron scattering, Tokarev[15] has used a similar subtraction method. He demands that the general one-body electromagnetic nucleon vertex *operator*, Γ, satisfies

$$k\cdot\Gamma = 0 \ , \quad (3.2)$$

rather than the Ward-Takahashi identity. He uses an operator of the most general form and then performs a subtraction procedure analogous to (3.1) to achieve this.

Clearly, such procedures are not satisfactory, and several authors have studied this problem. Berends and West[6] discuss the electroproduction of a pion by considering the standard pole graphs. They use the general electromagnetic vertices for the pion and the half-off-shell nucleon. Since a point coupling at the πNN vertex is assumed, Berends and West do not have to consider the internal insertions in the strong vertex (class B diagrams). By imposing the Ward-Takahashi identity, they derive relations among the electromagnetic form factors analogous to the procedure in Ref. 12. They take all form factors at their on-shell value, i.e., with $p^2 = p'^2 = M^2$ and $q^2 = q'^2 = m_\pi^2$, and include no nucleon and pion self-energies. This still leaves two model-dependent nucleon form factors in the production amplitude for which no on-shell information is available. The authors suggest choosing these "undetermined" nucleon form factors—$f_1^{(-)}(k^2,M^2,M^2)$ and $f_2^{(-)}(k^2,M^2,M^2)$ in the notation of Eq. (2.9)—as follows:

$$f_1^{(+)}(k^2,M^2,M^2) = f_1^{(-)}(k^2,M^2,M^2) \ , \quad (3.3\text{a})$$

$$f_2^{(+)}(k^2,M^2,M^2) = f_2^{(-)}(k^2,M^2,M^2) \ . \quad (3.3\text{b})$$

This keeps the amplitude gauge invariant. In fact, this choice generates precisely the gauge terms one would obtain by the *ad hoc* subtraction procedure of Eq. (3.1).

Without a microscopic model, the assumptions (3.3a) and (3.3b) about the form factors cannot be justified. However, at the photon point, $k^2 = 0$, Eq. (3.3a) is exact, since gauge invariance requires that[12]

$$f_1^{(+)}(0,M^2,M^2)=f_1^{(-)}(0,M^2,M^2)=e^{p,n}\ . \qquad (3.4)$$

Therefore, assumption (3.3a) may be a good approximation for small k^2. An example for the behavior of the form factors $f_1^{(+)}$ and $f_1^{(-)}$ at larger k^2 in a one-pion loop model can be found in Ref. 13. In contrast, assumption (3.3b) about the form factors $f_2^{(\pm)}$ is more arbitrary. There is no constraint from the Ward-Takahashi identity on these magnetic form factors. In fact, the estimates based on dispersion relations by Nyman[16] suggest

$$f_2^{(-)}(0,M^2,M^2)\approx\frac{2M}{m_\pi}f_2^{(+)}(0,M^2,M^2)\ . \qquad (3.5)$$

Also the one-pion loop model[13] yields an $f_2^{(-)}$ which is much larger than $f_2^{(+)}$. It is important to realize that

$$f_2^{(+)}(0,M^2,M^2)=f_2^{(-)}(0,M^2,M^2) \qquad (3.6)$$

is assumed implicitly in the standard Born approximation for, e.g., the photoproduction of pions on nucleons. Also in the previously mentioned recent work of Tokarev[15] such an assumption is made. It allows one to eliminate, in Eq. (2.9), the model-dependent $f_2^{(-)}(0,M^2,M^2)$ and to use the experimental anomalous magnetic moment, $f_2^{(+)}(0,M^2,M^2)=\kappa$.

Gross and Riska[5] also realize the importance of the Ward-Takahashi identity when dealing with extended particles, such as mesons and nucleons. Their main goal is to construct a very general meson-exchange current operator with arbitrary phenomenological electromagnetic form factors for nucleons and mesons and with strong form factors at the meson-nucleon vertex. Their discussion is, of course, closely connected to the electroproduction of pions. The starting point is Eq. (2.5), which relates the pion electromagnetic vertex, Eq. (2.13), to the full pion propagator. This propagator can be written as

$$\Delta'(q^2)=\left[q^2-m_\pi^2+\Pi(q^2)\right]^{-1} \qquad (3.7)$$

with $\Pi(q^2)$ the self-energy. This allows one to express the general vertex, Eq. (2.13), as

$$\Gamma_\mu^\pi(q',q)=h^+(k^2,q'^2,q^2)\left[Q_\mu-\frac{k\cdot Q}{k^2}k_\mu\right] +e_\pi[\Delta'^{-1}(q'^2)-\Delta'^{-1}(q^2)]k_\mu/k^2\ , \qquad (3.8)$$

where $Q_\mu=(q+q')_\mu$. To make contact with the formalism of Gross and Riska, one has to rewrite h^+ in Eq. (3.8) as

$$h^+(k^2,q'^2,q^2)=F_\pi(q^2)-C(k^2,q'^2,q^2)\left[1-\frac{e_\pi[\Delta'^{-1}(q'^2)-\Delta'^{-1}(q^2)]}{q'^2-q^2}\right]\ , \qquad (3.9)$$

which yields Eq. (6.13) of Ref. 5. However, $C(k^2,q'^2,q^2)$ is then replaced by an undetermined function F_0 which *only depends on* k^2. Whether or not this is a meaningful approximation is *a priori* impossible to assess.

An important further ingredient in their recipe for meson-exchange currents is the treatment of the strong πNN vertex and the pion self-energy, which is needed in Eq. (3.8). They assume that the πNN vertex is characterized by a form factor $f_\pi(q^2)$ and suggest shifting this q^2 dependence into the self-energy of the pion, resulting in

$$\Delta'(q^2)=\frac{f_\pi^2(q^2)}{q^2-m_\pi^2}\ . \qquad (3.10)$$

This cannot be possible in general, since the strong vertex involved is off shell and will have a more complicated structure as in Eq. (2.14). The dependence on the nucleon off-shell momenta will preclude a shift of the form factors into a pion self-energy, which can only depend on q^2. For similar reasons, one cannot expect in general a recipe of this type to apply to photoproduction or electroproduction of pions on a nucleon.

For the γNN vertex, Gross and Riska suggest the form

$$\Gamma_\mu(p',p)=\tfrac{1}{2}\{F_1^s(k^2)-1+[F_1^v(k^2)-1]\tau_3\} \times\left[\gamma_\mu-\frac{k_\mu(k\cdot\gamma)}{k^2}\right]+\frac{1+\tau_3}{2}\gamma_\mu +\frac{i\sigma_{\mu\nu}}{2M}k^\nu\mathbf{F}_2(k^2)\ . \qquad (3.11)$$

This form ensures that all form factors drop out from the Ward-Takahashi identity and that the resulting current is conserved. However, for the initial and final nucleon off shell, the Ward-Takahashi identity is only satisfied by this vertex if the nucleon self-energy is assumed to be zero, which is only true for a free nucleon. Also for the half-off-shell electromagnetic nucleon vertex discussed in Sec. II, the above recipe cannot be true in general. The prescription in Eq. (3.11) corresponds to choosing

$$f_{1,2}^{(+)}=f_{1,2}^{(-)} \qquad (3.12)$$

in Eq. (2.9), just as in the recipe of Berends and West, Eqs. (3.3a) and (3.3b). The same remarks of caution, therefore, also apply in this case to this part of the recipe proposed in Ref. 5.

IV. CONCLUDING REMARKS

We have discussed the description of electromagnetic reactions on particles with internal structure, using the production of pions on nucleons as an example. Two classes of amplitudes can be distinguished. One class can be described in terms of diagrams which are separated from the rest by one propagator of the participating particles. The other class cannot be reduced in this fashion and corresponds to internal insertions of the photon into the dressed vertices. In pion electroproduction and photoproduction, this insertion is done in the strong pion-nucleon vertex. In Compton scattering, for example, the insertion occurs in the dressed photon-nucleon vertex. Both classes are crucial to obtain a gauge invariant amplitude. Since production processes (or other two-step reactions such as Compton scattering) involve intermediate off-shell propagation, also the off-shell form of the vertices is needed. To obtain the class B diagrams and the off-shell vertices, one clearly needs a microscopic model for the hadron structure. This excludes in general a description of these reactions in terms of (phenomenological) on-shell properties like the anomalous magnetic moment or the form factors of the free nucleon and pion. The validity of the existing recipes which we discussed here can *a priori* not be judged or justified without first examining the problem on a microscopic level—which has not been done yet. Estimates for, e.g., the electromagnetic form factors of half-off-shell nucleons certainly raise some doubts about the commonly made assumptions.

The most widely used description of the electromagnetic production of pions on a nucleon is based on the lowest order Born diagrams with on-shell vertices and free propagators and without the class B internal insertion diagrams. This approach is only at threshold supported by the "Kroll-Ruderman theorem," which states that then the photoproduction amplitude to zeroth order in (m_π/M) can be obtained from the first-order Born diagrams with the renormalized masses and coupling constant. In deriving the Kroll-Ruderman theorem, however, one needs both classes of diagrams. The theorem is therefore not a justification that this Born approach is appropriate for the higher orders in (m_π/M) at threshold or for the amplitude at higher energies. Without further assumptions—such as PCAC—model-dependent terms will appear in the higher order correction terms at threshold. This will be discussed in a separate publication.

As electromagnetic reaction mechanisms at intermediate energies will be investigated in detail with the new generation of electron accelerators, a consistent treatment of the internal structure of the hadrons—pions, nucleons, Δ's, etc.—in the framework outlined in this paper is clearly necessary.

This work is part of the research program of the National Institute for Nuclear Physics and High-Energy Physics (NIKHEF, section K), made possible by financial support from the Foundation for Fundamental Research on Matter (FOM) and the Netherlands' Organization for Scientific Research (NWO).

[1]J. C. Ward, Phys. Rev. **78**, 182 (1950); Y. Takahashi, Nuovo Cimento **6**, 371 (1957).

[2]N. M. Kroll and M. A. Ruderman, Phys. Rev. **93**, 233 (1954).

[3]J. D. Walecka and P. A. Zucker, Phys. Rev. **167**, 1479 (1967); J. L. Friar and S. Fallieros, Phys. Rev. C **13**, 2571 (1976).

[4]E. T. Dressler, Can. J. Phys. **66**, 279 (1988).

[5]F. Gross and D. O. Riska, Phys. Rev. C **36**, 1928 (1987).

[6]F. A. Berends and G. B. West, Phys. Rev. **188**, 2538 (1969).

[7]E. Kazes, Nuovo Cimento **13**, 1226 (1959).

[8]M. Gell-Mann and M. L. Goldberger, Phys. Rev. **96**, 1433 (1954).

[9]G. 't Hooft and M. Veltman, Nucl. Phys. **B44**, 189 (1972).

[10]L. Heller, in *The Two Body Force in Nuclei,* edited by S. M. Austin and G. M. Crawley (Plenum, New York, 1971).

[11]L. Heller, S. Kumano, J. C. Martinez, and E. J. Moniz, Phys. Rev. C **35**, 718 (1987). These authors propose a gauge invariant recipe, but in the actual calculations drop a term analogous to S_μ in Eq. (2.16) (M. Weyrauch, private communication).

[12]A. M. Bincer, Phys. Rev. **118**, 855 (1960).

[13]H. W. L. Naus and J. H. Koch, Phys. Rev. C **36**, 2459 (1987). Note that in this reference a different notation for irreducible and reducible vertices is used.

[14]F. E. Low, Phys. Rev. **96**, 1428 (1954).

[15]M. V. Tokarev, Few-Body Systems **4**, 133 (1988).

[16]E. M. Nyman, Nucl. Phys. **A154**, 97 (1970).