Dirac equation analysis of intermediate-energy ^{3}He-nucleus scattering

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(Received 17 October 1988)

A one-body Dirac equation is used to model ^{3}He+^{40}Ca elastic scattering at 197 and 217 MeV. A relativistic analogue of the Watanabe model is employed to constrain the real parts of the ^{3}He-nucleus potentials. This procedure results in an accurate description of the experimental differential cross-section data, as well as a prediction of the elastic spin observables. Calculations for triton-nucleus scattering are also performed. Implications for relativistic models of nuclear scattering are discussed.

The relativistic approach has proven to be very successful in describing intermediate-energy proton-nucleus scattering.[1–2] This has stimulated interest in using relativistic wave equations to model other nuclear probes. Light ions are particularly intriguing projectiles in this regard. One can investigate, for example, the validity of using a one-body relativistic wave equation to represent the scattering of a composite particle (as is done for proton-nucleus scattering). Furthermore, if the models are based on the Dirac description of nucleon-nucleus scattering, the consistency of the relativistic approach to nuclear physics can be tested. Finally, if the above program is successful, it may be possible to gain information concerning the neutron-nucleus interaction.

Already investigations in this direction have been done for deuteron-nucleus scattering.[3–6] The small binding energy of the deuteron makes it likely that its constituents interact independently with the target nucleus, and therefore the process can mainly be described in terms of free nucleon-nucleus scattering. Indeed, a good parameter-free fit has been obtained in this manner to intermediate-energy deuteron-nucleus data, especially the spin observables, using the relativistic Kemmer-Duffin-Petiau formalism.[6] Although such success is encouraging, there are some difficulties associated with the deuteron, due to its spin-one nature. One is the fact that several different relativistic spin-one formulations exist, resulting in an ambiguity as to which one to choose.[7] Another is the occurrence of tensor potentials in relativistic formalisms[3–6] which may be important, but are difficult to include in a practical calculation.

With the above in mind, it is natural to consider the next simplest composite nuclei, ^{3}H and ^{3}He. Although these nuclei are more tightly bound than the deuteron, meaning a physical description of mass-3 scattering should be more complicated than for deuteron-nucleus scattering, the fact that they are spin-$\frac{1}{2}$ does yield some advantages. First of all, there exists the possibility of calculating and measuring elastic spin observables unlike for, say, alpha-particle scattering. There is also a clear choice regarding the one-body relativistic wave equation to be used, in contrast to the situation for deuteron-nucleus scattering. Furthermore, the spin dependence of the potentials is relatively simple and can be handled in existing computer codes. Also, a comparison of the scattering of the two mirror nuclei, ^{3}H and ^{3}He, can lead to insight into the nature of the neutron-nucleus potential, as discussed above. Although one may question the use of the Dirac equation for such extended objects, a study by Bleszynski, Bleszynski, and Jaroszewicz[8] has suggested the validity of such a procedure.

In the present study, a one-body Dirac equation will be used to model ^{3}He-nucleus scattering at intermediate energies.[9] Specifically, differential cross-section data for ^{3}He+^{40}Ca scattering at both 197 MeV (Ref. 10) and 217 MeV (Ref. 11) will be fit using Dirac scalar and vector potentials. A relativistic version of the Watanabe model[12] will be used as a basis for constructing these potentials. Since nucleon-nucleus optical potentials are required as input to the Watanabe model, this will allow for a check on the consistency of the Dirac approach to nuclear scattering, as alluded to earlier. An important byproduct of this procedure is that, because the Dirac equation correlates the effective central and spin-orbit potentials in a specific way, a fit to the differential cross section will yield a prediction of the elastic spin observables. The ^{3}He-nucleus potentials will also be employed to obtain predictions for triton-nucleus scattering.

The basic equation to be used is the one-body Dirac equation

$$[\boldsymbol{\alpha}\cdot\mathbf{p}+\beta(m+S_{hA})]\psi=(E-V_{hA}-V_c)\psi \ , \qquad (1)$$

where S_{hA} and V_{hA} are spherically symmetric ^{3}He-nucleus Dirac scalar and vector potentials, respectively, and V_c represents the Coulomb potential. By using a relativistic analogue of the Watanabe model,[12] one can construct, for example, the scalar ^{3}He-nucleus potential for an incident kinetic energy K by taking the appropriate nucleon-nucleus scalar potentials at one-third the kinetic energy and folding them with the ^{3}He density. If isospin dependence is suppressed, the above statement can be expressed in equation form as

$$S_{hA}(\mathbf{r};K)=\int d\mathbf{r}'\rho_h(\mathbf{r}')S_{nA}\left(|\mathbf{r}-\mathbf{r}'|;\frac{K}{3}\right) , \qquad (2)$$

where S_{nA} is the Dirac scalar nucleon-nucleus potential and ρ_h is the ^{3}He density. A similar procedure holds for constructing the vector potential. Of course, the Watanabe model is not expected to give a perfect description of light-ion scattering. On the other hand, it can provide a parameter-free first approximation to experimental data. It can furthermore be used as a foundation upon which to build a phenomenological optical potential. In the present study, calculations will be performed both for the parameter-free Watanabe model and an optical model potential obtained by fitting differential cross-section data. In each case comparisons will be made with experiment and predictions of the vector polarization will be presented. The Watanabe model will be important to the phenomenology because it will be used to constrain some of the potential parameters, particularly the real geometries.

In order to determine the ^{3}He-nucleus Watanabe model potentials, nucleon-nucleus potentials at the appropriate energies are required. Due to the lack of relevant data, the neutron-nucleus optical potential is assumed to be equal to the proton-nucleus optical potential; however, the Coulomb potential is included, as indicated in Eq. (1), by taking the projectile to be a point charge interacting with the static charge distribution of the target nucleus. Although Dirac global optical potentials are available for $p+{}^{40}$Ca scattering,[13] they were not used since they contain surface absorption, and it was desired to fit the ^{3}He-nucleus data using only volume absorption, as was the case with nonrelativistic analyses.[10,11] The Dirac nucleon-nucleus potentials used in this study were instead obtained by fitting $p+{}^{40}$Ca data at 65 MeV (Ref. 14) and 80 MeV.[15] The four independent potentials, scalar and vector, real and imaginary, were parametrized, in standard Woods-Saxon form, as

$$U(r)=W/(1+e^{(r-A^{1/3}C)/Z}) , \quad (3)$$

where W is in MeV, C and Z are in fermis, and A is the atomic mass number of the target. The 80 MeV potentials were previously available.[15] The 65 MeV potentials were determined using E. D. Cooper's RUNT program.[16] In light of the reservations expressed in Ref. 13 concerning the absolute normalization of this data, the normalization was at first searched on as a free parameter. However, since this resulted in a normalization parameter very close to one, this parameter was ultimately held fixed at unity. The parameters for the 65 and 80 MeV optical potentials are listed in Table I.

Having obtained the necessary nucleon-nucleus potentials, the Watanabe model ^{3}He-nucleus potentials can now be determined. In order to retain the convenience of using analytic Woods-Saxon form factors, the folding, as exhibited in Eq. (2), was done in an approximate manner. Both the real and imaginary strengths were multiplied by three, the radius parameters were left unchanged, and the diffusenesses were adjusted such that, to second order in the quantity $y=(Z/A^{1/3}C)$, the approximate potential would have the same mean-square radius as the exact folded potential. The volume integral obtained in this procedure is exact to first order in y. This approximation

TABLE I. The input nucleon-nucleus potentials. The symbols in the left-hand column have the following meaning: The first letter indicates the type of parameter, as shown in Eq. (3); the second letter indicates scalar or vector; the third letter indicates real or imaginary.

	65 MeV	80 MeV
WVR	384.88	325.00
CVR	1.0453	1.0513
ZVR	0.7197	0.5634
WVI	−33.27	−30.88
CVI	0.8095	1.2064
ZVI	0.8329	0.5577
WSR	−444.31	−422.04
CSR	1.0615	1.0378
ZSR	0.7223	0.6002
WSI	24.76	24.32
CSI	0.7993	1.1570
ZSI	0.4690	0.4410

is in accord with empirical observations made in the study of light-ion scattering, which suggest that the primary effect of folding is to multiply the strength and increase the diffuseness of the potential.[17] For the ^{3}He$+{}^{40}$Ca potentials at 197 MeV, the proton-nucleus potentials at 65 MeV, which corresponds to the correct value for the Watanabe model, were used as input. The 80 MeV $p+{}^{40}$Ca potentials were used to construct the ^{3}He potentials at 217 MeV. Although, for this case, the nucleon-nucleus potentials should be taken at a slightly lower energy, no data exists at the appropriate energy, and the error made in representing the Watanabe model potentials in this manner should be small. However, as will be seen below, this discrepancy will affect the way in which the 217 MeV data set is fit.

As the parameter-free Watanabe model does not describe the data well, it is necessary to utilize phenomenology in order to obtain agreement with experiment. However, the Watanabe model will still be used to fix some of the parameters of the phenomenological potentials. This serves to reduce the number of free parameters in the model and has the additional advantage in that a connection is retained between the Dirac descriptions of proton scattering and ^{3}He scattering. Since the Watanabe model neglects effects arising from the breakup of the projectile, which are expected mainly to increase absorption, it is natural to expect that the imaginary potentials would have to be changed to account for this. Therefore, in fitting the 197 MeV data, the real potentials were taken from the Watanabe model and the imaginary potentials were searched on, resulting in a good six-parameter description of the experimental data. For the 217 MeV case, this procedure failed to produce satisfactory results, most likely due to the fact that the Watanabe potentials were built up from nucleon-nucleus potentials at the wrong energy. Thus, an alternate method of obtaining a six-parameter fit to the data was adopted. While the imaginary geometries were searched on, the imaginary strengths were held fixed at the Watanabe model values, and instead, the real strengths were allowed to vary. This can be justified by observing that the real strengths have a greater energy dependence than the imaginary strengths and thus would be more affected by using input potentials at a different energy, and noting that in the previous fit, the imaginary strengths were found to be very close to the Watanabe model values. The number of free parameters in both of these fits is the same as the number used in nonrelativistic analyses.[10,11] These parameter searches were also done with the RUNT code[16] and the best-fit potentials are exhibited in Table II.

An inspection of Figs. 1 and 2 shows that the phenomenological potentials yield a good description of the differential cross sections. On the other hand, the parameter-free Watanabe model is only reliable at forward angles (less than about 12 deg), as the predicted cross sections flatten out and are well above the data for larger angles. As mentioned above, an advantage of working with a Dirac formalism is that a definite prediction is obtained for the spin observables. The calculated polarizations are also displayed in Figs. 1 and 2. It is seen that the predictions of the Watanabe model potentials and the phenomenological potentials are qualitatively similar. However, the phenomenology produces polarizations which are more oscillatory at larger angles, in line with the oscillations of the cross sections. In Table III, the predicted reaction cross sections from the relativistic Watanabe and phenomenological models are compared with each other, and with predictions obtained using the nonrelativistic optical model parametrization of Refs. 10 and 11. The relativistic and nonrelativistic optical models produce values that agree with each other to within five percent, indicating their probable accuracy. The relativistic Watanabe model gives predictions that are within ten percent of the above numbers, suggesting that it can be used to obtain dependable estimates for the reaction cross section, despite its failure to describe the differential cross section. It should be noted that a geometric black-disk model yields a reaction cross section of 127 fm^2, which is in good agreement with results obtained from the above models, providing a radius of 6.35 fm is used. As this radius is consistent with the sum of

TABLE II. The best-fit ^{3}He-nucleus potentials. The symbols have the same meaning as in Table I.

	197 MeV	217 MeV
WVR	1154.6	1002.6
CVR	1.0453	1.0513
ZVR	0.8527	0.7257
WVI	−64.47	−92.64
CVI	1.3591	1.3226
ZVI	0.6227	0.6828
WSR	−1332.9	−1228.9
CSR	1.0615	1.0378
ZSR	0.8549	0.7546
WSI	71.03	72.96
CSI	1.1851	1.3237
ZSI	0.4788	0.5839

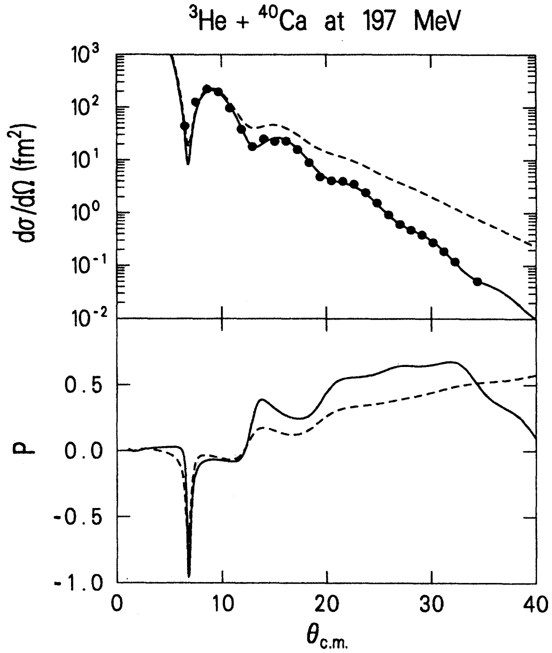

FIG. 1. Differential cross section and polarization for $^3He+^{40}Ca$ scattering at 197 MeV. The experimental data are from Ref. 10. The solid lines have been calculated using the best-fit potentials of Table II, while the dashed lines are obtained from the relativistic Watanabe model.

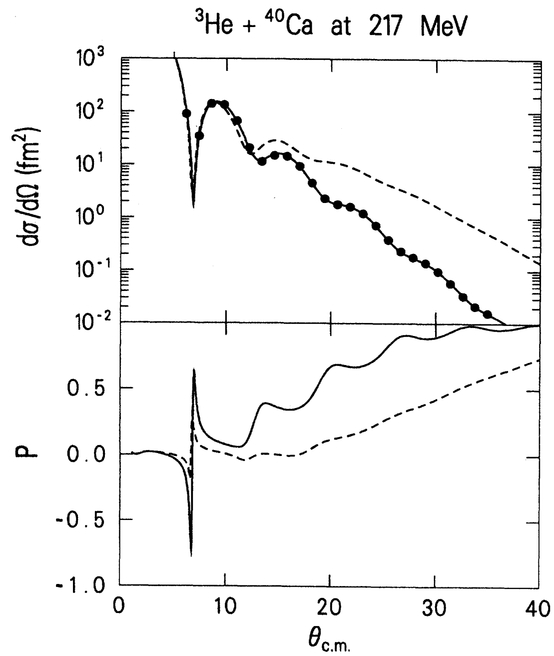

FIG. 2. Differential cross section and polarization for $^3He+^{40}Ca$ scattering at 217 MeV. The experimental data are from Ref. 11, and the solid and dashed lines have the same meaning as in the previous figure.

TABLE III. Predicted reaction cross sections in fm^2 for ^{3}He-nucleus scattering. The first line is obtained using the nonrelativistic optical potentials of Refs. 10 and 11; the second line is calculated using the best-fit relativistic potentials given in Table II; the third line is obtained with the relativistic Watanabe model potentials.

	197 MeV	217 MeV
NROM	129	130
ROM	124	124
REL WAT	117	121

two black-disk radii that might be appropriate for ^{3}He and ^{40}Ca, this suggests that the reaction cross section can be largely understood in terms of the geometrical properties of the two colliding nuclei.

It is also of interest to discuss the Dirac ^{3}He-nucleus potentials, as presented in Table II. As is the case with the Dirac description of proton-nucleus scattering, large scalar and vector potentials "cancel" each other, producing effective potentials of reasonable magnitude. The strengths of the best-fit potentials are very similar to the Watanabe model values. In addition, it was found adequate to let the parameters describing the real geometries remain at their Watanabe model values. However, the imaginary potential geometries were substantially different from the Watanabe model. In both cases, the phenomenological potentials were of much longer range. This characteristic is also found in nonrelativistic analyses of light-ion scattering and is to be expected on physical grounds.[18]

The ^{3}He-nucleus potentials that have been obtained here can be used with some confidence to predict triton-nucleus scattering observables as the target that has been considered is a self-conjugate nucleus. Naturally, the triton will experience a different Coulomb potential. However, an additional benefit of the Dirac approach is that it automatically includes a Coulomb correction term,[19] which is also affected by the charge of the projectile. The results of a calculation for ^{3}H$+^{40}$Ca scattering at 197 MeV are shown in Fig. 3. The differential cross section and polarization are very similar to the corresponding ^{3}He case, except that the minima near eight degrees are shallower and the oscillatory structure is slightly less pronounced at larger angles. A reaction cross section of 127 fm^2 is predicted for the 197 MeV tritons.

Of course, the Dirac potentials for ^{3}He-nucleus scattering can be used as input for reaction calculations. A particular instance would be for the (^{3}He,t) process. One can use a relativistic generalization of the Lane equations, as has been done for (p,n) reactions.[20] However, the utility of the potentials is not only restricted to relativistic models. The potentials here can be employed in a nonrelativistic calculation simply by converting them to effective Schrodinger-equivalent form.[1] An advantage of using the Dirac potentials, as opposed to the nonrelativistic-based potentials of Refs. 10 and 11 is that they have a realistic spin structure.

The results of this study are sufficiently promising to

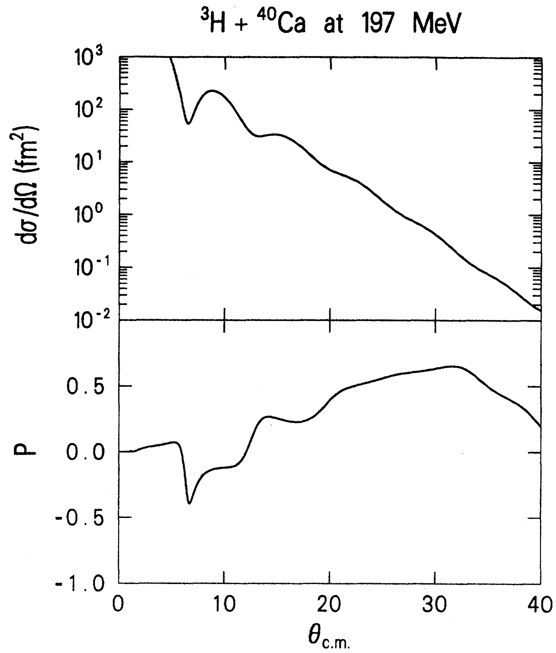

FIG. 3. Differential cross section and polarization for ^{3}H$+^{40}$Ca scattering at 197 MeV. The solid line was obtained using the ^{3}He-nucleus potentials of Table II together with an appropriate Coulomb potential.

encourage studies of the scattering of other composite particles. Research concerning the deuteron has already been cited, and some work on alpha-nucleus scattering has also been done.[21] Perhaps a more microscopic approach, such as double-folding, might be fruitful. Ultimately, the foundations for such models lie in a study of the complexities of the relativistic few-body problem. This approach has been pursued for relativistic deuteron-nucleus scattering.[3–5] However, it should be noted that theoretical justifications of light-ion scattering models are nontrivial even in the nonrelativistic case, and a more pragmatic approach might be to continue the use of one-body wave equations. It is reasonable to expect that such models should provide at least a "first-order" description of the underlying physics. A particular advantage in working with relativistic wave equations is that a specific spin dependence is imposed on the scattering potentials, which appears to result in an improved description of the spin observables. As far as the present model is concerned, further constraints would be provided by polarization measurements and/or triton-nucleus data. An investigation is underway concerning the scattering of mass-3 nuclei at low energies, where spin measurements have been made. It would also be interesting to learn if a description of ^{3}He-nucleus scattering would require "wine-bottle" shaped potentials at incident energies of about 200 MeV per nucleon, as seems to be the case for both proton-nucleus and deuteron-nucleus scattering.

In summary, it has been shown that a one-body Dirac equation, based on the Watanabe model, can be used to fit intermediate-energy ^{3}He-nucleus data. This success has provided evidence in support of the consistency of the relativistic approach to nuclear scattering and the validity of using the Dirac equation for a composite system. From a practical standpoint, the model has yielded predictions for the elastic spin observables as well as producing both effective central and spin-orbit ^{3}He-nucleus potentials. It thus appears worthwhile to continue the study of relativistic models of light-ion scattering.

I wish to thank S. L. Tabor for providing a tabulated version of his data and B. C. Clark for providing the 80-MeV proton-nucleus optical potentials. I also thank B. C. Clark, S. Hama, D. G. Madland, J. P. McDermott, and V. K. Mishra for helpful discussions.

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