Knockon exchange contribution in the resonating-group study of the nucleus-nucleus interaction

M. LeMere
Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California 91109

Y. C. Tang
School of Physics, University of Minnesota, Minneapolis, Minnesota 55455
(Received 19 December 1988)

The properties of the type-c nucleon-exchange term in a resonating-group formulation that takes into account the clustering nature of the nuclei involved are investigated. The results indicate that these properties are weakly affected by the degree of clustering, and the one-exchange term makes the dominant contribution among all class-A exchange terms. The equivalent type-1c, or knockon, exchange potential is found to have a characteristic range shorter than the direct potential and a characteristic energy that has a large value around 50 MeV/nucleon.

I. INTRODUCTION

One of the most interesting and important aspects of resonating-group investigations is the study of antisymmetrization or nucleon-exchange effects. In a two-cluster system that consists of clusters A and B with nucleon numbers N_A and N_B ($N_A > N_B$), and described, respectively, by translationally-invariant shell-model functions of the lowest configurations in harmonic-oscillator wells having width parameters α_A and α_B, it has been found[1-4] that nucleon-exchange terms are represented by integral kernel functions and can be conveniently classified into class-A and class-B terms which, in the Born approximation, yield Wigner-type and Majorana-type contributions to the internuclear interaction, respectively. Additionally, it was shown that among class-A terms the one-exchange type-c, or knockon, term (sometimes also referred to as the knockout exchange term) has a dominant influence, while for class-B terms the core-exchange type-a and type-d terms are particularly important (see Ref. 2 for the definition of exchange types).

Recently, we have extended the consideration of exchange effects to the more general and practical case where one of the nuclei involved exhibits properties of appreciable clustering. This was not a trivial extension and we have partially accomplished our purpose by analyzing the exponential parts of the comparatively simple exchange-normalization, or type-a, kernel function in a three-cluster resonating-group formulation (Ref. 5, hereafter referred to as LT). The results indicated that the characteristics of the important core-exchange type-a term are considerably affected by the clustering nature of the nuclei involved. When the degree of clustering is strong, the equivalent local interaction potential between the nuclei can acquire a rather long range compared to the direct potential, but its influence also decreases fairly rapidly with increasing energy.

With the completion of the above-mentioned investigation of the type-a kernel function, one can consider the study of the basic properties of class-B terms to be essentially finished. Our next task is then to examine the characteristics of class-A terms in the general case involving target clustering. For this purpose, we shall carefully analyze the type-c exchange term, with special emphasis paid on the particularly important one-exchange term. Here again, what will be done is to derive the general expression for the exponential part of the corresponding kernel function in a three-cluster $(A_1+A_2)+B$ resonating-group formulation where the nucleus A is now considered to possess an (A_1+A_2) cluster structure. This derivation is quite complicated, but, fortunately, can still be accomplished by utilizing the complex-generator-coordinate technique previously described.[6] Using this general expression, one can then proceed to determine the characteristic range and characteristic energy of the corresponding equivalent local potential and, thereby, learn the influence of target clustering on the essential features of this particular exchange term.

In Sec. II, we give a brief description of the type-c exchange term and of the way in which this exchange term is analyzed. Numerical studies with $\alpha+{}^6\mathrm{Li}$ and $\alpha+{}^{20}\mathrm{Ne}$ systems as illustrative examples are presented in Sec. III. Finally, in Sec. IV, we summarize our findings and make some concluding remarks.

II. TYPE-c EXCHANGE TERM

For a satisfactory understanding of exchange effects, it is necessary to examine the exchange-Hamiltonian kernel

$$\tilde{H}_E(\mathbf{R}',\mathbf{R}'')=\langle[\phi_1\phi_2 G(\boldsymbol{\rho})]\phi_3\delta(\mathbf{R}-\mathbf{R}')Z|H|\mathcal{A}''\{\mathcal{A}_{12}[\hat{\phi}_1\hat{\phi}_2 G(\boldsymbol{\rho})]\hat{\phi}_3\delta(\mathbf{R}-\mathbf{R}'')Z\}\rangle\ , \tag{1}$$

where H is the Hamiltonian operator of the system, and all other quantities appearing on the right-hand side of the above equation are defined in Sec. II of LT. By choosing all spatial functions to have Gaussian forms multiplied by polynomial functions, one finds that, after integrating over all nucleon coordinates, $\tilde{H}_E$ is given by

$$\tilde{H}_E(\mathbf{R}',\mathbf{R}'')=\sum_x \sum_q \sum_\nu [P^q_{x\nu}\exp(-A^q_{x\nu}\mathbf{R}'^2-C^q_{x\nu}\mathbf{R}'\cdot\mathbf{R}''-B^q_{x\nu}\mathbf{R}''^2)+\text{H.c.}] , \tag{2}$$

where $P^q_{x\nu}$ is a polynomial in $\mathbf{R}'^2$, $\mathbf{R}'\cdot\mathbf{R}''$, and $\mathbf{R}''^2$, and x represents the number of nucleons interchanged between nuclei A and B. The indices q and ν denote exchange types and modes, respectively. As was explained in Ref. 2, there are four exchange types with $q=a$, b, c, and d in the case considered here where all clusters are specified by the same oscillator width parameter. From the extensive investigations carried out in the two-cluster case,[2,4] it has been learned that the type-b term is generally of minor significance and, thus, does not need any additional attention. The type-a and type-d terms are important for understanding the main features of class-B contributions; they also will not be further considered, since we feel that these features are already reasonably understood as a result of our study reported in LT.

The type-c term is important with regard to class-A contributions and, therefore, will be carefully examined here. As was discussed in LT, there exist, for a fixed value of x, many modes of nucleon exchanges among the three clusters, with each mode specified by the index ν. In general, it is very difficult, even with the complex-generator-coordinate technique, to derive analytical expressions for the quantities $A^c_{x\nu}$, $B^c_{x\nu}$, and $C^c_{x\nu}$ appearing in the exponential factor of the kernel function. Fortunately, however, our experience gained from the investigation described in LT indicated to us that the essential features of the type-c exchange term can already be learned by studying only a special mode $\tilde{\alpha}$ which is characterized by the fact that all nucleon exchanges take place entirely between two clusters, say, clusters 1 and 3. For this particular mode, we have obtained the general expressions for these three quantities, denoted now as $A^c_{x\alpha}$, $B^c_{x\alpha}$, and $C^c_{x\alpha}$. These expressions are, however, extremely lengthy, and their meaning cannot be directly and easily comprehended; therefore, we choose not to show them here, but will make them available to any interested reader upon request.

The next step in the analysis is to construct equivalent local exchange potentials between nuclei A and B within the framework of the Born approximation.[7] These potentials are characterized by characteristic wave numbers $k^c_{x\alpha}$ and characteristic ranges $R^c_{x\alpha}$, given by

$$k^c_{x\alpha}=\left[\frac{4A^c_{x\alpha}B^c_{x\alpha}-(C^c_{x\alpha})^2}{A^c_{x\alpha}+B^c_{x\alpha}-|C^c_{x\alpha}|}\right]^{1/2} , \tag{3}$$

$$R^c_{x\alpha}=\left[\frac{2|C^c_{x\alpha}|}{4A^c_{x\alpha}B^c_{x\alpha}-(C^c_{x\alpha})^2}\right]^{1/2} . \tag{4}$$

Depending upon whether $C^c_{x\alpha}$ is smaller or larger than zero, these equivalent potentials will have either a Wigner (i.e., class-A) or a Majorana (i.e., class-B) character. Additionally, it is also useful to define two alternative characteristic quantities; these are the characteristic energy $E^c_{x\alpha}$ defined as

$$E^c_{x\alpha}=\frac{\hbar^2}{2M\mu_0}(k^c_{x\alpha})^2 , \tag{5}$$

with M being the nucleon mass and μ_0 being the reduced nucleon number of the $A+B$ system, and the characteristic weight $\zeta^c_{x\alpha}$ defined as

$$\zeta^c_{x\alpha}=(k^c_{x\alpha}R^c_{x\alpha})^2 . \tag{6}$$

This latter quantity (i.e., $\zeta^c_{x\alpha}$) is especially useful in qualitatively assessing the relative importance of the nucleon-exchange terms with respect to the quantity x; that is, for exchange terms belonging to the same class, the one with the largest value of $\zeta^c_{x\alpha}$ will generally be more important.

The characteristic range $R^c_{x\alpha}$ will be compared with the range parameter R_{D1} of the direct interaction, obtained by the use of a nucleon-nucleon potential that has a spatial dependence $\exp(-\kappa r^2)$ and interacts between nucleons in clusters 1 and 3. This range parameter has the form

$$R_{D1}=\left[\frac{(N_1+N_2)[N_1N_3-(N_1+N_3)\lambda]y+N_2N_3\lambda}{N_1N_3(N_1+N_2)\lambda y\alpha}\right]^{1/2} , \tag{7}$$

where y is the clustering parameter defined by Eq. (5) of LT and λ is given by

$$\lambda=\frac{\kappa}{\alpha+2\kappa} , \tag{8}$$

with α being the common oscillator width parameter of the three clusters. We feel that it is more appropriate to study the behavior of $R^c_{x\alpha}/R_{D1}$ rather than the behavior of $R^c_{x\alpha}$ itself, because, as y becomes smaller (i.e., stronger degree of clustering), R_{D1} becomes larger and, thus, it is more interesting to know whether the equivalent exchange potential has a range significantly different from that of the direct potential or not.

III. NUMERICAL STUDIES

Because of the complicated structures of $A^c_{x\alpha}$, $B^c_{x\alpha}$, and $C^c_{x\alpha}$, an analytical study of the characteristic quantities $k^c_{x\alpha}$, $R^c_{x\alpha}$, $E^c_{x\alpha}$, and $\zeta^c_{x\alpha}$ would be extremely difficult. Thus, the procedure that we adopt to learn the systematic features of these quantities is to perform a large number of numerical investigations involving many different choices of N_1, N_2, N_3, α, and y (κ is chosen to have a realistic value of 0.46 fm^{-2}). In this section, we discuss the results obtained in the $\alpha+{}^6$Li case with $(N_1,N_2,N_3)=(4,2,4)$ and $\alpha=0.367$ fm^{-2}, and in the

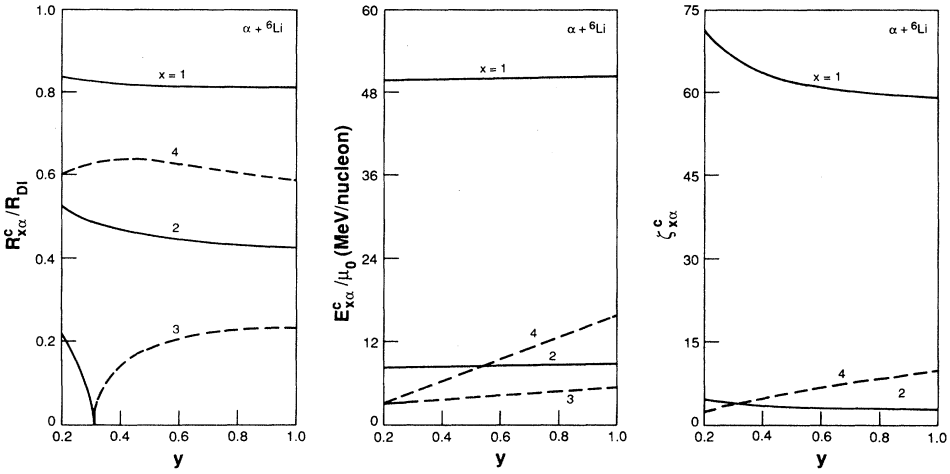

FIG. 1. $R^c_{x\alpha}/R_{D1}$, $E^c_{x\alpha}/\mu_0$, and $\zeta^c_{x\alpha}$ as a function of the clustering parameter y in the $\alpha+{}^6$Li case. The solid and dashed curves represent results for class-A and class-B exchange terms, respectively. Values for $\zeta^c_{x\alpha}$ with $x=3$ are very small and, hence, are not shown.

$\alpha+{}^{20}$Ne case with $(N_1,N_2,N_3)=(4,16,4)$ and $\alpha=0.32$ fm^{-2}. These two illustrative cases, one with small and the other with large nucleon-number differences, typify our findings concerning the general trends of the characteristic quantities as a function of the clustering parameter y.

Calculated values for $R^c_{x\alpha}/R_{D1}$, $E^c_{x\alpha}/\mu_0$, and $\zeta^c_{x\alpha}$ in the $\alpha+{}^6$Li and $\alpha+{}^{20}$Ne cases are shown in Figs. 1 and 2, respectively. In these figures, the solid and dashed curves represent, respectively, the results for class-A and class-B contributions. Since type-c exchange terms do not dominantly contribute in the class-B case, we shall concentrate only on the solid curves in the following discussion.

From Figs. 1 and 2, one finds that, in the whole range of y studied ($y=0.2$ corresponds to an unusually strong degree of clustering), $\zeta^c_{x\alpha}$ has by far the largest value when $x=1$. Therefore, this indicates that, even when target clustering is present, the one-exchange type-c, or knockon, term has the dominant influence among all class-A terms. For this particular exchange term, these two figures show additionally the following notable features.

(i) $R^c_{1\alpha}/R_{D1}$ is nearly y independent. In fact, we have found that, in the realistic range of y between 0.5 and 1, $R^c_{1\alpha}/R_{D1}$ has almost a constant value between 0.80 and 0.85 in every one of the large number of nuclear systems that we have examined. This shows that the important class-A, type-1c exchange potential generally has a range that is significantly shorter than that of the direct potential.

(ii) The characteristic energy per nucleon $E^c_{1\alpha}/\mu_0$ is almost system independent and has a large value around 50 MeV/nucleon for all values of y studied here.

(iii) The rapid rise of $\zeta^c_{1\alpha}$ in the small-y region merely

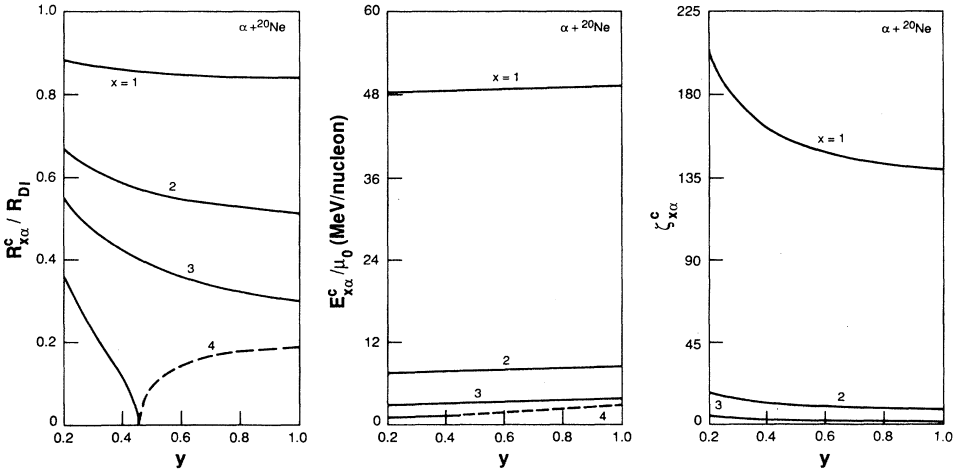

FIG. 2. Same as Fig. 1, except that the case under consideration is $\alpha+{}^{20}$Ne. Values for $\zeta^c_{x\alpha}$ with $x=4$ are very small and, hence, are not shown.

reflects the fact that $R^{c}_{1\alpha}$ increases when y decreases, and thus has no major significance.

The above findings show that the general properties of the type-1c exchange term depend weakly on the clustering nature of the nuclei involved. Therefore, all the conclusions about this term[1-4] reached previously in the two-cluster limiting case are valid and can be used to guide one's thinking regarding the construction of effective local potentials in complex nuclear systems.

Finding (i) has important implications that merit further discussion. Since investigations in many specific nuclear systems have always indicated that, in the low-energy region, class-A exchange potentials are overall attractive and have an appreciable influence,[6,8] it follows that the effective internuclear potential (i.e., the real central part of the empirical optical-model potential) must be somewhat shorter ranged than the direct potential. This is a significant fact that has the unfortunate consequence of complicating certain types of simple and appealing experimental analyses. For instance, it has often been asserted (see, e.g., Refs. 9 and 10) that one can use a folding-type approach to extract information about the neutron distribution from scattering experiments involving strongly interacting fermion systems. From our viewpoint, such an analysis could yield reliable results only if one has some means to determine the approximate magnitudes of the class-A exchange potentials at the energy under consideration.

IV. CONCLUSION

In this investigation, we study the properties of the type-c nucleon-exchange term in a resonating-group formulation that takes into account the clustering nature of the nuclei involved. The results indicate that these properties are only weakly affected by the degree of clustering, and that the one-exchange term still makes the dominant contribution among all class-A exchange terms.

For the equivalent type-1c, or knockon, exchange potential, we find that, regardless of the degree of clustering, it has a characteristic range somewhat smaller than the range of the direct potential and a characteristic energy that has a large value equal to about 50 MeV/nucleon. This latter feature is especially important, since it means that the knockon exchange contribution must always be properly considered in all aspects of low-energy nuclear physics.

Based on the results of our long series of investigations on nucleon-exchange effects, we are of the opinion that, for a macroscopic description of the internuclear interaction in terms of a local model, one must, at least, adopt a potential $\tilde{V}$ of the form

$$\tilde{V}(\mathbf{R}) = V_D(R) + V_a(R) + V_b(R)P^R , \tag{9}$$

where V_D is a direct potential obtained by a double-folding procedure, and P^R is a Majorana space-exchange operator that interchanges the position coordinates of the nuclei A and B, now treated as structureless point particles. The potential V_a is introduced to represent the effects of class-A exchange terms (mainly the type-1c exchange term), while the potential V_b is introduced to represent the effects of class-B exchange terms (mainly the core-exchange terms). Furthermore, it is important to note that V_a has been found to be attractive in all nuclear systems that we have examined, and should have a shorter range than the direct potential. As for the potential V_b, much is also known about its properties. It has an appreciable depth only when the nucleon-number difference of the interacting nuclei is small and can have a range either longer or shorter than the direct potential, depending again on the nucleon-number difference. In fact, even its sign has recently been determined by Baye,[11] using considerations based on the diagonal elements of the parity-projected norm kernel in the GCM (generator-coordinate method) formalism.

Effective potentials incorporating features contained in Eq. (9) have been adopted to analyze experimental results. For example, Satchler and Love[12] have explicitly included the V_a term in their folding-model study of heavy-ion interactions, while Ohkubo[13] has utilized the V_bP^R term in his optical-model analysis of like-ion systems, such as $^{12}C+^{13}C$, $^{15}N+^{16}O$, and $^{18}O+^{19}F$ scattering.

Since our initial investigation of nucleon-exchange effects in 1978,[1] much has been learned about the general consequences of antisymmetrization. For the equivalent local exchange potential, we now have rather satisfactory knowledge regarding the spatial range, the energy dependence, the exchange character, the dependence on parity and nucleon-number difference, and also whether it is attractive or repulsive. The major remaining problem is that there exists just partial knowledge concerning the depth of the exchange potential in the general case of $A+B$ interaction. The only information that we presently possess is from calculations in specific nuclear systems. Although it is clearly desirable to obtain a detailed general understanding of the depth problem, we believe that the task will be quite difficult. Indeed, it is our feeling that to accomplish this purpose would require not only long-term systematic investigations but also new ways of thinking which we are attempting to formulate at the present moment.

This work was supported in part by the U.S. Department of Energy under Contract No. DOE/DE-AC-02-79 ER 10364.

[1] M. LeMere and Y. C. Tang, Phys. Rev. C **19**, 391 (1979).
[2] M. LeMere, D. J. Stubeda, H. Horiuchi, and Y. C. Tang, Nucl. Phys. **A320**, 449 (1979).
[3] K. Aoki and H. Horiuchi, Prog. Theor. Phys. **66**, 1508 (1981); **66**, 1903 (1981); **67**, 1236 (1982); **68**, 1658 (1982); **68**, 2028 (1982); **69**, 857 (1983).
[4] M. LeMere, Y. Fujiwara, Y. C. Tang, and Q. K. K. Liu, Phys. Rev. C **26**, 1847 (1982).

[5]M. LeMere and Y. C. Tang, Phys. Rev. C **37**, 1369 (1988).
[6]Y. C. Tang, M. LeMere, and D. R. Thompson, Phys. Rep. **47**, 167 (1978); Y. C. Tang, in *Microscopic Description of the Nuclear Cluster Theory,* Vol. 145 of *Lecture Notes in Physics* (Springer-Verlag, Berlin, 1981).
[7]G. W. Greenlees and Y. C. Tang, Phys. Lett. **34B**, 359 (1971).
[8]Y. C. Tang, in *Proceedings of the International Symposium on Clustering Phenomena in Nuclei, Tübingen, 1981,* edited by P. Kramer and R. Schultheis (Attempto Verlag Tübingen GmbH, Tübingen, 1981).
[9]G. W. Greenlees, V. Hnizdo, O. Karban, J. Lowe, and W. Makofke, Phys. Rev. C **2**, 1063 (1970).
[10]H. J. Gils and H. Rebel, Phys. Rev. C **13**, 2159 (1976).
[11]D. Baye, Nucl. Phys. **A460**, 581 (1986).
[12]G. R. Satchler and W. G. Love, Phys. Rep. **55**, 183 (1979).
[13]S. Ohkubo, Phys. Rev. C **25**, 2498 (1982).