Coulomb instability of hot nuclei

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An equation of state for asymmetric nuclear matter is derived and used to investigate the instability of hot nuclei caused by the Coulomb force. This equation of state takes into consideration the effect of degeneracy of the Fermi system. The nucleus is treated as a liquid drop with a sharp edge and a surface tension. Both the liquid and the surrounding vapor are described by the derived equation of state. The equations describing the coexistence between the drop and the vapor are determined and solved. The instability of these hot liquid drops at temperatures above a certain limiting value is verified by the absence of any real solution of the coexistence equations at such temperatures. The value of the limiting temperature depends on the nucleus but is always less than the critical temperature of nuclear matter. It is found that high-order degeneracy effects as well as symmetry energy corrections must be included in the calculation if meaningful information about the bulk and surface properties of hot nuclear matter is to be extracted from any measurements of the limiting temperature.

I. INTRODUCTION

The occurrence of two distinct fluid phases, a dense liquid phase and a dilute gaseous phase, in nuclear matter has been the subject of several theoretical investigations.[1-5] These two distinct phases coexist at temperatures below the critical temperature T_c whereas only one fluid phase can exist at or above T_c. Associated with these two phases is the possibility[6-12] of observing phenomena related to the liquid-gas phase transition near or below the critical temperature. Such a transition may occur in the hot equilibrated compound nuclei produced in intermediate-energy heavy-ion collisions. However, it was recognized early on[3,13] that the finite size of these systems and the presence of the Coulomb force may lead to a substantial modification of this phase transition. In particular, hot charged nuclei become unstable above a temperature $T_{\rm lim}$ much lower than the critical temperature T_c of infinite nuclear matter, as demonstrated[14] by the inability of these hot nuclei (or liquid drops) to exist in equilibrium with the surrounding vapor of nucleons for temperatures $T > T_{\rm lim}$. This Coulomb instability was investigated in Ref. 14 with the use of a simple equation of state for nuclear matter that included only the first-order degeneracy correction and neglected symmetry energy effects.

In the present work a more elaborate equation of state is used that includes higher-order degeneracy corrections as well as the isospin effects associated with the symmetry energy in order to study the effect of these corrections on the stability of hot nuclei. As will be seen this will lead to a better understanding of the bulk and surface properties of hot nuclear matter.

In Sec. II the equation of state for asymmetric nuclear matter is derived. The equilibrium conditions between the liquid drop and the surrounding vapor are then studied in Sec. III. The resulting coexistence equations are solved in Sec. IV and the importance of each of the corrections introduced into the equation of state is investigated. Section V contains a discussion and concluding remarks about the present work.

II. THE NUCLEAR EQUATION OF STATE

The starting point in obtaining the desired nuclear equation of state is to assume an effective nucleon-nucleon interaction of the Skyrme type:[15,16]

$$v_{12} = -t_0(1+x_0P_\sigma)\delta(\mathbf{r}_1-\mathbf{r}_2) + \frac{t_3}{6}(1+x_3P_\sigma)\rho^\sigma\left[\frac{\mathbf{r}_1+\mathbf{r}_2}{2}\right]\delta(\mathbf{r}_1-\mathbf{r}_2)\,, \tag{2.1}$$

where P_σ is the spin-exchange operator, and the density dependence has been generalized as suggested by Ref. 3 to control the stiffness of the equation of state. The original Skyrme interaction is linear in the density ($\sigma=1$). For symmetric nuclear matter, in which protons and neutrons are treated equally and the Coulomb force is switched off, the equation of state does not depend on the values of x_0 and x_3 and is given by:[3]

$$\tilde{P}(T,\rho) = -a_0\rho^2 + a_3(1+\sigma)\rho^{2+\sigma} + T\rho\left[1+\sum_{n=1}^{\infty} b_n\left[\frac{\lambda_T^3\rho}{g}\right]^n\right] \tag{2.2a}$$

or

$$\tilde{\mu}(T,\rho) = -2a_0\rho + a_3(2+\sigma)\rho^{1+\sigma} + T\left[\ln\left[\frac{\lambda_T^3\rho}{g}\right] + \sum_{n=1}^{\infty}\frac{n+1}{n}b_n\left[\frac{\lambda_T^3\rho}{g}\right]^n\right], \tag{2.2b}$$

where ρ is the nuclear density, $\tilde{P}$ is the pressure, $\tilde{\mu}$ is the chemical potential, T is the temperature in energy units, g is the spin-isospin degeneracy ($g=4$), and λ_T is the

thermal wavelength of the nucleon,

$$\lambda_T = \left[\frac{2\pi\hbar^2}{mT}\right]^{1/2} . \tag{2.2c}$$

The parameters a_0 and a_3 in (2.2a) and (2.2b) are related to the parameters of the Skyrme interaction:

$$a_0 = \tfrac{3}{8}t_0, \quad a_3 = \tfrac{1}{16}t_3 , \tag{2.2d}$$

while the b_n's are the coefficients of the virial series for an ideal Fermi gas.[3] The first five coefficients are given by

$$\begin{aligned}
b_1 &= \frac{\sqrt{2}}{8} \approx 0.177 , \\
b_2 &= \frac{1}{8} - \frac{2\sqrt{3}}{27} \approx -3.30\times10^{-3} , \\
b_3 &= \frac{3}{32} + \frac{5\sqrt{2}}{64} - \frac{\sqrt{6}}{12} \approx 1.11\times10^{-4} , \\
b_4 &= \frac{317}{1728} + \frac{\sqrt{2}}{8} - \frac{\sqrt{3}}{6} - \frac{4\sqrt{5}}{125} \approx -3.54\times10^{-6} . \\
b_5 &= \frac{35}{128} + \frac{1687\sqrt{2}}{6912} - \frac{5\sqrt{3}}{72} - \frac{5\sqrt{6}}{36} - \frac{\sqrt{10}}{20} \\
&\approx 8.30\times10^{-8} .
\end{aligned} \tag{2.2e}$$

The quantity $(\lambda_T^3\rho/g)$ that occurs in Eqs. (2.2) is a measure of the degeneracy of the Fermi system so that these equations are expansions in the degree of degeneracy.[13] The equation of state used in Ref. 14 corresponds to Eqs. (2.2) but with the $n>1$ terms neglected, i.e., the effect of degeneracy is included to first order only. The values of a_0 and a_3 (or, equivalently, t_0 and t_3) and σ can be determined by the properties of the ground state of nuclear matter:[3]

$$\sigma = (K - 9E_B - E_K)/(9E_B + 3E_K) , \tag{2.3a}$$

$$a_0\rho_0 = [(1+\sigma)E_B + (\sigma+\tfrac{1}{3})E_K]/\sigma , \tag{2.3b}$$

$$a_3\rho_0^{1+\sigma} = (E_B + E_K/3)/\sigma , \tag{2.3c}$$

where E_B and E_K are the nuclear matter binding energy per particle and kinetic energy per particle, respectively, ρ_0 is the nuclear matter saturation density, and $K = 9\rho_0^2(\partial^2 E/\partial\rho^2)_{\rho_0}$ is the nuclear incompressibility. In addition x_3 is given the usual value[16] of 1 while x_0 is determined by the symmetry energy coefficient in nuclear matter:[16]

$$a_\tau = \frac{1}{3}\frac{\hbar^2k_F^2}{2m} + \tfrac{2}{3}(x_0+\tfrac{1}{2})a_0\rho_0 - a_3\rho_0^{1+\sigma} \approx 30 \text{ MeV} , \tag{2.4}$$

where k_F is the Fermi momentum.

Two sets of parameters so determined are given in Table I. The difference between the two sets stems from the difference in the values of K used in their evaluation which directly affects the values of σ. Apart from x_0 and x_3 (which do not affect the results for symmetric nuclear matter) these two sets are identical to the parameters used in Ref. 14. This is done in order to facilitate the comparison between the present results and those of Ref. 14. Also listed in Table I are the values of the critical temperature T_c of nuclear matter determined by the present equation of state, Eq. (2.2). The values of T_c are seen to differ very little from the corresponding values obtained in Ref. 14, also listed in the table. This is due to the fact that at such high temperatures the higher-order degeneracy corrections [$n>1$ in Eq. (2.2)] are negligible. On the other hand, for temperatures $T \lesssim T_{\text{lim}}$ which are much lower than T_c, these higher-order corrections become important and cannot be neglected as will become apparent in later sections.

If the proton number Z and neutron number N are not equal, the protons must be treated separately from the neutrons by assigning different energy levels and a different chemical potential for each species. The chemical potential for species q will then be given by[13]

$$\mu_q = \varepsilon_q + T\left[\ln\left[\frac{\lambda_T^3\rho_q}{g_s}\right] + \sum_{n=1}^{\infty}\frac{n+1}{n}b_n\left[\frac{\lambda_T^3\rho_q}{g_s}\right]^n\right] , \tag{2.5a}$$

where $q = p$ (or n) for a proton (or neutron), $g_s = 2$ is the spin-degeneracy of each species and ε_q is the single-particle potential energy for species q:

$$\begin{aligned}
\varepsilon_q = &-t_0\left[\left(1+\frac{x_0}{2}\right)\rho - (x_0+\tfrac{1}{2})\rho_q\right] \\
&+\tfrac{1}{4}t_3[\rho^{1+\sigma} - \sigma\rho^{\sigma-1}\rho_q^2 + (\sigma-1)\rho^\sigma\rho_q] + \delta_{qp}V_{\text{Coul}}(\rho)
\end{aligned} \tag{2.5b}$$

with $V_{\text{Coul}}(\rho)$ being the Coulomb potential energy for a uniformly charged spherical drop of radius R:

$$V_{\text{Coul}}(\rho) = \frac{6}{5}\frac{Ze^2}{R} = \frac{6}{5}Ze^2\left[\frac{4\pi\rho}{3A}\right]^{1/3} , \tag{2.5c}$$

where $A = N + Z$ is the total number of nucleons in the liquid drop. Strictly speaking, Eq. (2.5a) is correct only for short-range forces so that the inclusion of the Coulomb term in (2.5b) is a simplifying approximation. However, since the equation of state is mainly determined by the much stronger short-range nuclear force this approximation is not expected to have a drastic effect on the results of the calculation. The Coulomb term is included here rather than added later because it is the most important source for the asymmetry between the protons and neutrons and because Eq. (2.5a) with the Coulomb term will eventually only be used for a drop of liquid nuclear matter. Finally, it must be noted that if the Coulomb force is switched off and $N = Z$ so that $\rho_p = \rho_n = \rho/2$ then Eq. (2.5a) reduces to Eq. (2.2b).

Introducing the variables

$$x = \frac{\rho_p}{\rho} = \frac{Z}{A}, \quad y = \frac{\rho_n}{\rho} = \frac{N}{A}, \quad \alpha = y - x \tag{2.6a}$$

so that

$$\rho_n = \frac{\rho}{2}(1+\alpha), \quad \rho_p = \frac{\rho}{2}(1-\alpha) , \tag{2.6b}$$

and expanding Eq. (2.5a) up to second order in the asymmetry parameter α, leads to the chemical potential for

the neutrons:

$$\mu_n(T,\rho,\alpha)=\tilde{\mu}(T,\rho)+\mu_{n,\mathrm{sym}}(T,\rho,\alpha)\ , \tag{2.7a}$$

where $\tilde{\mu}(T,\rho)$ is the chemical potential for symmetric nuclear matter, Eq. (2.2b), and $\mu_{n,\mathrm{sym}}$ is the α-dependent correction:

$$\mu_{n,\mathrm{sym}}(T,\rho,\alpha)=\mu_1(T,\rho,\alpha)+\mu_2(T,\rho,\alpha) \tag{2.7b}$$

with

$$\mu_1(T,\rho,\alpha)=\left\{\tfrac{4}{3}(x_0+\tfrac{1}{2})a_0\rho-2a_3\rho^{\sigma+1}+T\left[1+\sum_{n=1}^{\infty}(n+1)b_n\left[\frac{\lambda_T^3\rho}{g}\right]^n\right]\right\}\alpha \tag{2.7c}$$

and

$$\mu_2(T,\rho,\alpha)=\left\{-\sigma a_3\rho^{\sigma+1}+T\left[-\frac{1}{2}+\sum_{n=1}^{\infty}\frac{(n^2-1)}{2}b_n\left[\frac{\lambda_T^3\rho}{g}\right]^n\right]\right\}\alpha^2\ . \tag{2.7d}$$

Note that μ_1 is linear in α while μ_2 is proportional to α^2. Similarly, the proton chemical potential is given by

$$\mu_p(T,\rho,\alpha)=\tilde{\mu}(T,\rho)+\mu_{p,\mathrm{sym}}(T,\rho,\alpha)+\mu_{\mathrm{Coul}}(\rho)\ , \tag{2.8a}$$

where

$$\mu_{p,\mathrm{sym}}(T,\rho,\alpha)=\mu_{n,\mathrm{sym}}(T,\rho,-\alpha)=-\mu_1(T,\rho,\alpha)+\mu_2(T,\rho,\alpha) \tag{2.8b}$$

and

$$\mu_{\mathrm{Coul}}(\rho)=V_{\mathrm{Coul}}(\rho)=\tfrac{6}{5}Ze^2\left[\frac{4\pi\rho}{3A}\right]^{1/3}\ . \tag{2.8c}$$

With the chemical potentials determined it is then possible to obtain the total pressure of the system by using the Gibbs-Duhem relation:

$$\frac{\partial P}{\partial\rho}=x\rho\frac{\partial\mu_p}{\partial\rho}+y\rho\frac{\partial\mu_n}{\partial\rho} \tag{2.9}$$

which yields

$$P(T,\rho,\alpha)=\tilde{P}(T,\rho)+P_{\mathrm{sym}}(T,\rho,\alpha)+P_{\mathrm{Coul}}(\rho)\ , \tag{2.10a}$$

where $\tilde{P}(T,\rho)$ is the pressure of symmetric nuclear matter, Eq. (2.2a), $P_{\mathrm{sym}}(T,\rho,\alpha)$ is the α-dependent correction:

$$P_{\mathrm{sym}}(T,\rho,\alpha)=\left[\tfrac{2}{3}(x_0+\tfrac{1}{2})a_0\rho^2-(1+\sigma)a_3\rho^{2+\sigma}+T\rho\sum_{n=1}^{\infty}\frac{n(n+1)}{2}b_n\left[\frac{\lambda_T^3\rho}{g}\right]^n\right]\alpha^2\ , \tag{2.10b}$$

and $P_{\mathrm{Coul}}(\rho)$ is the Coulomb contribution:

$$P_{\mathrm{Coul}}(\rho)=\left[\frac{4\pi\rho}{3A}\right]^{1/3}\frac{Z^2e^2}{5A}\rho\ . \tag{2.10c}$$

This expression for $P_{\mathrm{Coul}}(\rho)$ is the one used in Ref. 14 and differs from the expression actually calculated from the Gibbs-Duhem relation, Eq. (2.9), by a factor of $\frac{3}{2}$. However, since the inclusion of the Coulomb force is done in an approximate and simplified manner in both cases, it is not possible to prefer one expression over the other and so the expression used in Ref. 14 is maintained here also for ease of comparison. It must be pointed out though, that the expression (2.10c) is correct at $T=0$ (apart from a small exchange term). Finally, it is noted that the pressure depends quadratically on α whereas the chemical potentials contain terms linear as well as quadratic in α.

III. THE COEXISTENCE EQUATIONS

Following Ref. 14 the hot nucleus is considered as a spherical drop of uniformly distributed nuclear matter at constant temperature with a sharp edge. The bulk pressure inside this drop is governed by the equation of state (2.10). This drop is in thermal, mechanical, and chemical equilibrium with the surrounding vapor which is taken to be an electrically screened gas of nucleons that obeys the equation of state (2.10) without the Coulomb term. For the temperature dependence of the surface tension $\gamma(T)$ of the liquid drop, the formula suggested by Ref. 12 and used in Ref. 14 is also used here in order to facilitate the comparison:

$$\gamma(T)=1.14\ \mathrm{MeV\,fm^{-2}}\left[1+\frac{3}{2}\frac{T}{T_c}\right]\left[1-\frac{T}{T_c}\right]^{3/2}\ . \tag{3.1a}$$

This expression vanishes at the critical temperature T_c and has the correct quadratic decrease with T at low temperatures. The surface tension provides a further contribution to the pressure inside the liquid drop:

$$P_{\mathrm{surf}}=-2\gamma(T)/R=-2\gamma(T)\left[\frac{4\pi\rho}{3A}\right]^{1/3}\ , \tag{3.1b}$$

where R is the radius of the drop.

With all these ingredients it is now possible to write down the coexistence equations at equilibrium which are the standard requirements of the equality of the inside and outside temperatures, pressures, neutron chemical potentials and proton chemical potentials:

$$\tilde{P}(T,\rho_L)+P_{\mathrm{sym}}(T,\rho_L,\alpha_L)+P_{\mathrm{Coul}}(\rho_L)+P_{\mathrm{surf}}(T,\rho_L)=\tilde{P}(T,\rho_V)+P_{\mathrm{sym}}(T,\rho_V,\alpha_V)\ , \tag{3.2a}$$

$$\tilde{\mu}(T,\rho_L)+\mu_{n,\mathrm{sym}}(T,\rho_L,\alpha_L) = \tilde{\mu}(T,\rho_V)+\mu_{n,\mathrm{sym}}(T,\rho_V,\alpha_V)\ , \quad (3.2b)$$

$$\tilde{\mu}(T,\rho_L)+\mu_{p,\mathrm{sym}}(T,\rho_L,\alpha_L)+\mu_{\mathrm{Coul}}(\rho_L) = \tilde{\mu}(T,\rho_V)+\mu_{p,\mathrm{sym}}(T,\rho_V,\alpha_V)\ , \quad (3.2c)$$

where the subscript L refers to the liquid drop and V to the surrounding vapor, and where allowance has been made for the asymmetry parameter α_V of the vapor to be different from the asymmetry parameter α_L inside the hot nucleus. Equations (3.2) are thus a generalization of Eqs. (2.5) of Ref. 14 to the case where symmetry energy effects are taken into consideration and protons and neutrons are not treated as identical. The solution of the three simultaneous equations (3.2) to determine the three unknowns ρ_L, ρ_V, and α_V can in principle be much more difficult than the solution of the two simultaneous equations of Ref. 14. However, a major simplification occurs if Eqs. (3.2b) and (3.2c) are added and subtracted from each other and use is made of Eqs. (2.7b) and (2.8b). The resulting two equations together with Eq. (3.2a) form a new set of coexistence equations that are to be solved:

$$\tilde{P}(T,\rho_L)+P_{\mathrm{sym}}(T,\rho_L,\alpha_L)+P_{\mathrm{Coul}}(\rho_L)+P_{\mathrm{surf}}(T,\rho_L) = \tilde{P}(T,\rho_V)+P_{\mathrm{sym}}(T,\rho_V,\alpha_V)\ , \quad (3.3a)$$

$$\tilde{\mu}(T,\rho_L)+\tfrac{1}{2}\mu_{\mathrm{Coul}}(\rho_L)+\mu_2(T,\rho_L,\alpha_L) = \tilde{\mu}(T,\rho_V)+\mu_2(T,\rho_V,\alpha_V)\ , \quad (3.3b)$$

$$\mu_1(T,\rho_L,\alpha_L)-\tfrac{1}{2}\mu_{\mathrm{Coul}}(\rho_L)=\mu_1(T,\rho_V,\alpha_V)\ . \quad (3.3c)$$

The simplification in solving the coexistence equations (3.3) results from the fact that the left-hand side of (3.3c) is a very slowly varying function of ρ_L so that without knowing the exact value of ρ_L it is possible to use (3.3c) to determine α_V for a given ρ_V and the resulting α_V is then used to solve Eqs. (3.3a) and (3.3b). A further simplification results if the symmetry corrections are neglected for the vapor state in Eqs. (3.3a) and (3.3b). These corrections are both proportional to α_V^2. In this approximation Eqs. (3.3a) and (3.3b) become completely independent of α_V [which is now solely determined by Eq. (3.3c)] and are quite similar to the coexistence equations of Ref. 14 which in the present notation can be written as follows:

$$\bar{P}(T,\rho_L)+P_{\mathrm{Coul}}(\rho_L)+P_{\mathrm{surf}}(T,\rho_L)=\bar{P}(T,\rho_V)\ , \quad (3.4a)$$

$$\bar{\mu}(T,\rho_L)+\frac{Z}{A}\mu_{\mathrm{Coul}}(\rho_L)=\bar{\mu}(T,\rho_V)\ , \quad (3.4b)$$

where $\bar{P}$ and $\bar{\mu}$ are given by Eqs. (2.2) but with the $n>1$ terms neglected.

The similarities and differences between the present coexistence equations (3.3) and Eqs. (3.4) used in Ref. 14 are to be noted. The pressure equations (3.3a) and (3.4a) differ by the presence of the symmetry correction for the liquid drop as well as for the surrounding vapor. The chemical potential equations (3.3b) and (3.4b) differ not only by the symmetry correction terms but also by a modification of the Coulomb term. In addition Eqs. (3.3a) and (3.3b) must be solved simultaneously with Eq. (3.3c). Another difference between Eqs. (3.3) and (3.4) is the fact that Eqs. (3.4) contain only the first-order degeneracy correction whereas degeneracy is included in Eqs. (3.3) to any order desired. The inclusion of the higher-order degeneracy corrections must, however, be treated with care since the series that appear in Eqs. (2.2), (2.5), and (2.7) are asymptotic (or semiconvergent) series[13] that converge only in the limit of high T. The summation of these series must be carried out by employing the usual techniques of stopping the summation at the term of lowest magnitude and the averaging of successive terms.[17] Associated with this summation is an unavoidable computational error which tends to zero as the temperature increases. In the present calculations degeneracy is included to fifth order, i.e., terms with $n\leq 5$ are included in the various summations. The averaging of successive terms means effectively that only half of the last term ($n=5$) is included in the summation and this was used in the present work for all calculations involving temperatures ≤ 5 MeV since in such cases the temperature is low enough that the fifth-order term becomes comparable to the fourth-order term. In such a situation averaging of successive terms is necessary to reduce the computational error. For temperatures ≈ 5.5 MeV averaging was found to have a very small effect ($\lesssim 1\%$) and so it was not used in calculations involving higher temperatures.

IV. SOLUTION OF THE COEXISTENCE EQUATIONS AND THE COULOMB INSTABILITY

In the present section the coexistence equations (3.3) are solved first by making some simplifying approximations and then exactly. The results of the various solutions are compared with each other and with the results of Ref. 14 in order to investigate the importance of each of the corrections introduced in the present work. As a first step the effect of the higher-order degeneracy corrections alone is investigated without including the symmetry effects. For this purpose Eqs. (3.4) are solved for the case where $\bar{P}$ and $\bar{\mu}$ are replaced by $\tilde{P}$ and $\tilde{\mu}$ [see Eq. (2.2)], and the results are compared with those of Ref. 14 which are carried out with $\bar{P}$ and $\bar{\mu}$. One such calculation is shown in Figs. 1 and 2 for the case of a ^{109}Ag nucleus at a temperature of 5 MeV. The equation of state used corresponds to the first force given in Table I ($\sigma=\frac{1}{4}$). Figure 1 shows two 5-MeV isotherms for this nucleus: one is calculated with the first-order degeneracy correction only (i.e., the same as in Ref. 14) and the other one is calculated to fifth order in the degeneracy ($n\leq 5$). The corresponding bulk isotherms are also shown in the figure. The drop isotherms differ from the bulk isotherms by the inclusion of the same Coulomb and surface contributions to the pressure as used in Ref. 14 so that the difference between the two drop isotherms is due entirely to the higher-order degeneracy corrections. The inclusion of these corrections is seen to be important at liquid densities since their contribution is comparable to the Coulomb and surface terms. Figure 2 shows the dependence of the chemical potential on the pressure at $T=5$ MeV for both the bulk and liquid-drop situations.

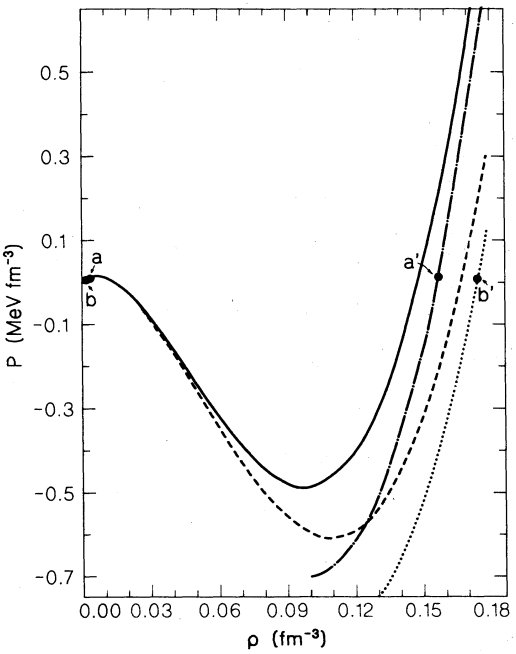

FIG. 1. Bulk and drop isotherms at $T=5$ MeV calculated with the $\sigma=\frac{1}{4}$ force. The solid curve is the bulk isotherm with only the first-order ($n=1$) degeneracy correction included while the dashed curve is the bulk isotherm when the higher-order ($n\leq 5$) degeneracy corrections are also included. The dotted-dashed and dotted curves are the drop isotherms with the first-order and higher-order degeneracy corrections, respectively. The drop isotherms are obtained from the corresponding bulk isotherms by adding the same Coulomb and surface terms appropriate for a ^{109}Ag nucleus. Points a and a' give the vapor-drop coexistence points for the $n=1$ case while b and b' are the corresponding points for the $n\leq 5$ case. None of these curves includes symmetry effects.

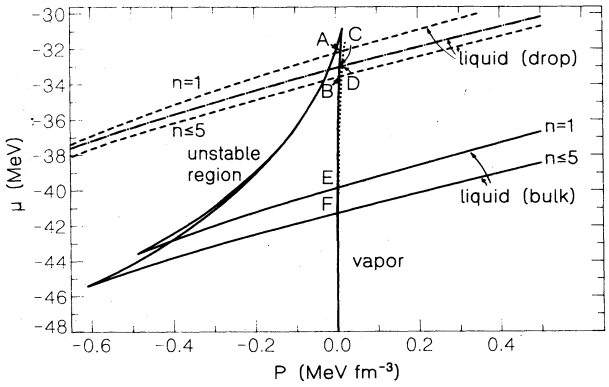

FIG. 2. Dependence of the chemical potential on the pressure at $T=5$ MeV. Solid curves give the dependence for bulk quantities with the first-order ($n=1$) and higher ($n\leq 5$) degeneracy corrections and points E and F give the corresponding bulk coexistence points. The dashed curves give the corresponding dependence for a ^{109}Ag nucleus for the two cases ($n=1$) and ($n\leq 5$) but without any symmetry corrections, and points A and B give the corresponding drop-vapor coexistence points. Point A corresponds to points a and a' in Fig. 1 while B corresponds to b and b'. The dotted-dashed curve includes both the degeneracy ($n\leq 5$) and symmetry corrections for the liquid drop. The dotted curve corresponds to the vapor with the symmetry corrections included. Point D corresponds to the exact solution of Eqs. (3.3). Each of the points A,B,C,D is specified in Table II by the row bearing the same letter.

Two bulk curves are shown, one giving $\bar{\mu}$ vs $\bar{P}$ which include the first-order degeneracy correction ($n=1$) while the second curve gives $\tilde{\mu}$ vs $\tilde{P}$ which include the higher-order corrections ($n\leq 5$). Each bulk curve consists of three parts corresponding to the vapor, unstable, and liquid regions of the equation of state. The intersection of the bulk liquid and vapor curves gives the bulk coexistence point and it is seen from Fig. 2 that the inclusion of the higher-order degeneracy corrections has a significant effect on the location of the bulk coexistence point which moves from point E to point F in the figure. Figure 2 also shows that a similar shift (from point A to point B) occurs in connection with the vapor-drop coexistence point, i.e., the solution of the coexistence equations, which is the intersection of the vapor part of the bulk curve with the (dashed) liquid-drop curves obtained from the corresponding bulk curves by adding the Coulomb and surface contributions. The effect of the higher-order degeneracy corrections on the location of the drop-vapor coexistence point is seen to be large as is also evident by comparing rows A and B of Table II which gives the equilibrium values of the vapor and drop densities as well as the equilibrium pressure and chemical potential as calculated by the various approximations. All quantities, especially ρ_L, ρ_V, and P, are drastically affected by the degeneracy corrections.

After investigating the importance of the degeneracy

TABLE I. The two sets of parameters of the Skyrme interaction used in the nuclear equation of state utilized in the present calculations together with the value of the nuclear compressibility K used in determining them. Both sets correspond to $E_B=16$ MeV, $E_K=24$ MeV, $a_\tau=30$ MeV, and $\rho_0=0.17$ fm^{-3}. Also listed are the values of the corresponding critical temperature T_c of symmetric nuclear matter as determined by the present calculation and also by Ref. 14.

σ	$a_0\rho_0$ (MeV)	$a_3\rho_0^{1+\sigma}$ (MeV)	x_0	K (MeV)	T_c (MeV)	T_c (MeV) (Ref. 14)
$\frac{1}{4}$	136	96	0.75	222	17.34	17.22
1	64	24	0.47	384	22.98	22.90

TABLE II. Equilibrium values of the vapor and liquid-drop densities, pressure and chemical potential for a ^{109}Ag nucleus at $T=5$ MeV. All calculations are carried out with the first force listed in Table I. The type of calculation refers to the various approximations used in solving the coexistence equations: *A*, degeneracy is included to first order only, symmetry corrections are ignored, i.e., the same as Ref. 14; *B*, higher-order ($n \leq 5$) degeneracy corrections are included but no symmetry corrections; *C*, higher-order degeneracy corrections are included, symmetry corrections are included for the drop but not for the vapor; *D*, exact solution of Eqs. (3.3), i.e., with the inclusion of all corrections for the drop as well as vapor. The last column gives the values of the asymmetry parameter of the vapor surrounding the nucleus as calculated by solutions *C* and *D*.

Type of calculation	ρ_V (fm^{-3})	ρ_L (fm^{-3})	P (MeV fm^{-3})	μ (MeV)	α_V
A	2.75×10^{-3}	0.158	0.010	−32.2	
B	1.75×10^{-3}	0.176	0.007	−33.6	
C	2.0×10^{-3}	0.176	0.008	−33.1	0.98
D	3.9×10^{-3}	0.176	0.017	−33.1	0.79

corrections alone, the next step is to include, in addition, the symmetry correction terms for the liquid drop and neglect them for the surrounding vapor. This amounts to dropping the vapor symmetry terms on the right-hand side of Eqs. (3.3a) and (3.3b) which thus become decoupled from Eqs. (3.3c) and can be solved like Eqs. (3.4) to obtain the equilibrium values of ρ_L and ρ_V. The dependence of the drop chemical potential on the pressure with the symmetry effects included is also shown in Fig. 2 where it is given by the dotted-dashed curve. It is seen that the inclusion of the symmetry corrections for the drop partially counteracts the effect of the degeneracy corrections as the coexistence point moves from point *B* to point *C*. The new location of the drop-vapor coexistence point is listed in row *C* of Table II which also gives the value of the asymmetry parameter α_V of the vapor surrounding the hot ^{109}Ag nucleus. This value of α_V is determined by solving Eq. (3.3c) with the equilibrium values of ρ_L and ρ_V, and it is found that α_V is rather large and close to the maximum value of 1 indicating that the surrounding vapor consists mainly of neutrons. Although this value of α_V is somewhat too large (see below), yet the preponderance of the neutrons is qualitatively understood in terms of the Coulomb barrier which hinders the evaporation of the protons.[18] It is also in qualitative agreement with the results of finite-temperature Hartree-Fock calculations.[19] This point will be discussed further below.

The last step is to include the symmetry terms for the vapor especially since α_V, as has just been observed, turns out to be quite large which means that the symmetry correction contribution to the vapor may not be neglected. This means that the three simultaneous equations (3.3) are to be solved exactly. As mentioned in Sec. III, advantage is taken of the fact that the left-hand side (lhs) of Eq. (3.3c) is a very slowly varying function of ρ_L. As an example, for a ^{109}Ag nucleus at $T=5$ MeV the lhs of Eq. (3.3c) decreases from a value of 7.197 MeV at $\rho_L=0.17$ fm^{-3} to 7.161 MeV at $\rho_L=0.18$ fm^{-3}. Since the equilibrium value of ρ_L is expected from the previous approximate results to lie in the interval $0.17<\rho_L<0.18$ fm^{-3} (see Table II), the error involved in determining α_V in this case will be $<1\%$ whatever value is used between the two limits just quoted. In the calculation the value 7.18 was used for the lhs of Eq. (3.3c). The resulting α_V, which is now a function of ρ_V, is then inserted in the right-hand side of Eqs. (3.3a) and (3.3b) and the resulting vapor pressure and chemical potential are given by the dotted curve shown in Fig. 2. This curve intersects the (dotted-dashed) liquid-drop curve at point *D* which thus corresponds to the exact coexistence point determined by Eqs. (3.3). The corresponding equilibrium quantities are given in row *D* of Table II, where it is seen that the inclusion of the symmetry corrections for the vapor has a very noticeable effect on the equilibrium values of the pressure and the vapor density but a negligible effect on ρ_L and μ. The resulting value of $\alpha_V=0.79$ is lower than the approximate value of 0.98 obtained by neglecting the vapor symmetry terms (row *C* of Table II). Nevertheless, it still indicates that the vapor consists mainly of neutrons. This lower value of α_V is also more reliable since terms $O(\alpha^3)$ have been neglected in deriving the symmetry contribution, which means that the lower α_V is more acceptable. It must be noted, however, that whereas the drop curve (the dotted-dashed curve of Fig. 2) corresponds to a fixed value of α_L ($\alpha_L=\frac{15}{109}$ for ^{109}Ag) the dotted vapor curve does not correspond to a fixed value of α_V since for every point on this curve the value of α_V is determined by Eq. (3.3c).

A general feature of the solutions of Eqs. (3.3) is that as the temperature is increased the liquid-drop μ-p curve moves upward with respect to the corresponding vapor curve so that at the limiting temperature $T_{\rm lim}$ the coexistence point coincides with the upper boundary of the vapor phase. For $T>T_{\rm lim}$ the liquid and vapor curves do not intersect at all and Eqs. (3.3) do not have a solution. The meaning of $T_{\rm lim}$ is thus obvious: below $T_{\rm lim}$ the nucleus can exist in equilibrium with the surrounding vapor whereas above $T_{\rm lim}$ the nucleus is unstable and will fragment or emit charged particles. This is the Coulomb instability investigated in Ref. 14 and mentioned in the introduction in the present work. The details of what actually happens to the unstable hot nucleus are, however, beyond the scope of this work. Figure 3 illustrates the existence of $T_{\rm lim}$ by plotting the p-μ curves for the case of a ^{109}Ag nucleus at three successive temperatures of 6, 6.1,

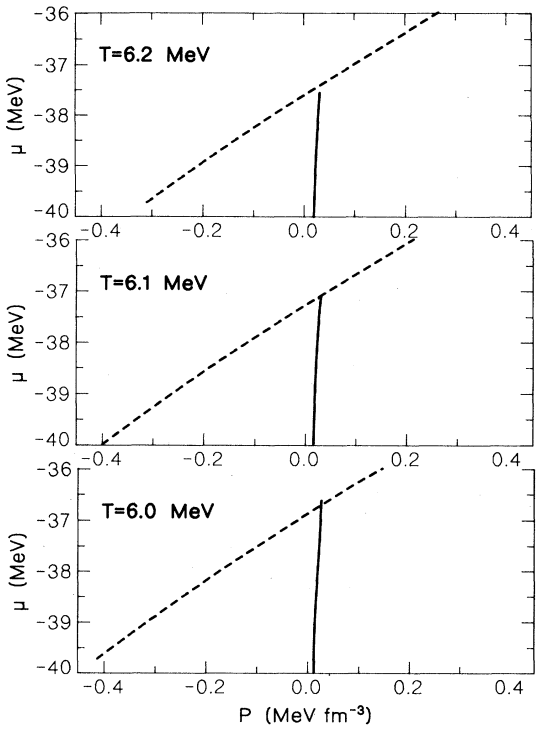

FIG. 3. The chemical potential versus the pressure below, at, and above the limiting temperature. The solid curve describes the vapor phase while the dashed curve describes a drop representing ^{109}Ag. Above $T_{\rm lim}$ the two curves do not intersect and no drop-vapor coexistence is possible. Both the vapor and drop curves contain the degeneracy ($n \leq 5$) and symmetry terms.

and 6.2 MeV. The calculations are carried out with the first force of Table I ($\sigma = \frac{1}{4}$) and include the symmetry as well as degeneracy corrections. For this case $T_{\rm lim} = 6.1$ MeV and for higher temperatures the drop and vapor curves fail to intersect.

The value of $T_{\rm lim}$ depends on the approximation used in solving the coexistence equations and this in turn reflects the importance of each of the corrections introduced in the present work. Table III shows the values of $T_{\rm lim}$ for the two nuclei ^{109}Ag and ^{208}Pb calculated with the two forces listed in Table I. The results indicate the importance of including the higher-order degeneracy corrections as well as symmetry effects since these corrections are similar in magnitude to those brought about by changing the force used in the bulk equation of state or changing the temperature dependence of the surface tension. Changing the force used in the equation of state can change the limiting temperature by 2–3 MeV, as can be seen in Table III, with the softer equation of state ($\sigma = \frac{1}{4}$) leading consistently to lower values of $T_{\rm lim}$. Changing the expression used for the surface tension was found in Ref. 14 to change $T_{\rm lim}$ by ~ 1 MeV. On the other hand, the inclusion of both the degeneracy and symmetry corrections lowers $T_{\rm lim}$ by 0.5–2.2 MeV for the cases listed in Table III. These corrections must therefore be included if measurements of $T_{\rm lim}$ are to be used, as suggested by Ref. 14, to study the bulk and surface characteristics of hot nuclear matter.

TABLE III. Values of the limiting temperature for the nuclei ^{109}Ag and ^{208}Pb calculated with the two forces of Table I. The force is identified by the value of σ. The letters A,B,C,D refer to the various approximations used in the calculation (see caption for Table II). The last column gives the values of the asymmetry parameter of the vapor surrounding the nucleus as calculated by the exact solution of the coexistence equations (i.e., solution D) at $T = T_{\rm lim}$.

		$T_{\rm lim}$ (MeV)				
Nucleus	σ	A	B	C	D	α_V
^{109}Ag	$\frac{1}{4}$	6.57	7.31	6.80	6.10	0.45
^{109}Ag	1	9.20	9.72	9.22	8.28	0.41
^{208}Pb	$\frac{1}{4}$	5.40	6.27	5.49	4.05	0.77
^{208}Pb	1	7.81	8.51	7.60	5.56	0.76

The dependence of the limiting temperature on x_0 [see Eq. (2.1)] can also be investigated. Although in principle the value of x_0 is determined from the symmetry energy coefficient a_τ of nuclear matter through Eq. (2.4), yet it is useful to treat x_0 as a parameter and see how it affects the value of $T_{\rm lim}$, especially since there is some uncertainty in the value of a_τ and the other quantities that enter Eq. (2.4). For this purpose $T_{\rm lim}$ was determined for a ^{109}Ag nucleus with the $\sigma = \frac{1}{4}$ force of Table I but with different values of x_0. It was found that $T_{\rm lim}$ dropped from 6.1 to 5.1 MeV when x_0 was increased from 0.75 to 1, and that $T_{\rm lim}$ was raised to 6.4 MeV when x_0 was decreased to 0.5. This indicates that $T_{\rm lim}$ is sensitive to the value of x_0 being used.

Finally, it is of interest to investigate the variation with temperature of the asymmetry parameter α_V of the vapor. In order to be able to compare results obtained with the use of different forces for the same nucleus or the results for different nuclei, values of α_V at the limiting temperature and at 1.1 MeV below $T_{\rm lim}$ are calculated. This is more meaningful than comparing values of α_V at the same absolute temperature since different forces lead to different limiting temperatures even for the same system. Values of α_V at the limiting temperature are given in the last column of Table III and it is seen that for a ^{109}Ag nucleus the $\sigma = \frac{1}{4}$ force gives $\alpha_V = 0.45$ while the force with $\sigma = 1$ leads to a slightly lower value $\alpha_V = 0.41$. The corresponding values at 1.1 MeV below $T_{\rm lim}$ (i.e., at 5 and 7.2 MeV) are 0.75 and 0.60, respectively, indicating that the abundance of the neutrons in the vapor increases as the temperature is lowered. As mentioned earlier, this can be qualitatively understood in terms of the Coulomb barrier which is more effective at hindering the evaporation of the protons at lower temperatures. The difference between the values of α_V predicted by the two forces can also be understood in terms of the Coulomb barrier since the limiting temperature is higher for the $\sigma = 1$ force; at such higher temperatures the protons are more successful at overcoming the barrier which leads to a reduction in α_V. Also, it must be noted that the height of the Coulomb barrier itself depends on the nuclear force used.

The values of α_V for ^{208}Pb at the limiting temperature are almost equal (0.77 and 0.76) for both forces. These values are larger than for the ^{109}Ag case and this is due to the fact that the Coulomb barrier for ^{208}Pb is almost twice as high as for ^{109}Ag whereas the limiting temperature is lower than for ^{109}Ag. At $T=T_{\rm lim}-1.1$ MeV the solution of Eqs. (3.3) for ^{208}Pb leads to values of $\alpha_V>1$; e.g., the force with $\sigma=1$ gives $\alpha_V=1.18$ at $T=4.45$ MeV. Such values for α_V are not physically acceptable since, by definition, $|\alpha_V|$ must be ≤ 1. The occurrence of values of $\alpha_V>1$ is due to the fact that terms $O(\alpha_V^3)$ have been neglected in deriving Eqs. (3.3) [see Eqs. (2.6)–(2.8)]. Such terms would be important when $|\alpha_V|$ is ≈ 1. In any case the present results indicate that α_V is large (≈ 1) so that the vapor consists almost entirely of neutrons at such low temperatures.

V. DISCUSSION AND CONCLUSION

In the present work the effect of the symmetry corrections as well as the higher-order degeneracy contributions to the equation of state of nuclear matter has been investigated. The results indicate that these terms must be taken into consideration when investigating the stability of hot nuclei. In particular, they may not be neglected if measurements of the limiting temperature are to be used to study the equation of state of nuclear matter or the temperature dependence of its surface properties. This is the main difference between the present work and Ref. 14 whose results nevertheless remain qualitatively correct especially as regards the existence of the limiting temperature.

In carrying out the present calculations many of the simplifying features used in Ref. 14 were also used here. This was done for the purpose of facilitating the comparison between the two sets of results. Such simplifying features included the assumption of a uniform density distribution inside the nucleus and also for the vapor, the neglect of the temperature dependence of the Coulomb contribution and the treatment of the vapor as being completely screened. One other feature is that the equations of state used here as well as in Ref. 14 do not include the density dependence of the effective mass. This is due to the fact that the effective nucleon-nucleon interaction given by Eq. (2.1) does not contain the finite-range terms that are present in the original, and more general, Skyrme interaction. It is these finite-range terms that lead to an effective mass $m^*(\rho)\neq m$. The role of the density dependence of the effective mass will be investigated in a future work.

Finally, the approximation of neglecting terms $O(\alpha^3)$ must be examined, especially in light of the fact that the asymmetry parameter for the vapor α_V turns out to be close to unity in certain cases. This entails that in such a situation the higher-order symmetry terms must be included in the calculation or that the pressure and chemical potentials must be evaluated exactly and the expansion in terms of powers of α_V given up altogether. This, however, means that the simplicity gained by transforming the present coexistence equations to the form given in Eqs. (3.3) will be lost and the system of three simultaneous coexistence equations has to be solved numerically.

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