Intrinsic basis function in the Dyson boson mapping

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This Brief Report deals with the solution of the shell-model Hamiltonian in the boson representation. In the present approach, the fermion Hamiltonian is exactly mapped to the boson space using the Dyson boson mapping. The problem of mapping the basis states is resolved by employing the intrinsic boson basis. The method has been applied to a model Hamiltonian consisting of a monopole pairing and a deformation force. It is observed that only *s* and *d* bosons are dominant in the intrinsic wave function for the model case studied. This is synonymous to the interacting boson model approach.

Recently there has been extensive use[1–3] of the intrinsic states in the interacting boson model (IBM).[4] The essential reason for this is to reduce the dimensionality of the problem. This dimensionality problem becomes quite severe on the inclusion of the higher angular momentum (J) bosons in the IBM, for example, on including the $J=4$ (g) bosons in the IBM usually referred to as *sdg* IBM. But the problem of fitting the large number of parameters still persists. For example, the *sdg* IBM Hamiltonian contains 35 parameters. These parameters are obtained in a least-squares fit to the observed data. In this fitting a large number of different parameter sets are obtained which give almost identical results. Sometimes it becomes quite difficult to fix upon a parameter set.

The parameters of a boson Hamiltonian can be fixed by making use of a suitable mapping procedure.[5–15] The mapping transforms the fermion operators in the shell-model configuration space to a physical subspace of the bosons. In this way the parameters of the boson Hamiltonian are fixed from the fermion space. These studies are also essential in establishing a microscopic basis of the IBM or *sdg* IBM. Various mappings have been discussed in the literature.[5] In this work we employ the Dyson boson mapping (DBM). This mapping has a finite character and therefore any fermion operator written in terms of bifermion operators will have a finite number of terms in the boson representation. For the evaluation of the various required matrix elements one requires a set of basis vectors. In principle, the basis vectors can also be obtained through the mapping. But this approach has been found to be quite cumbersome, and probably in some cases impossible. In this Brief Report, we try to examine the problem by making use of the intrinsic basis function. This intrinsic basis function approach has been applied to a model Hamiltonian with monopole pairing interaction.[6,8] But with a monopole pairing Hamiltonian the admixture between states is severely restricted. In fact, in the case of a two-level model with monopole interaction it has been shown[8] that the unphysical (spurious) subspace is completely decoupled from the physical subspace. Therefore, calculations with the intrinsic basis in this two-level model case lead to exact results. In this work we study a more realistic Hamiltonian which, in principle, admits mixing between different states. We study an example of a single j shell with the model Hamiltonian consisting of a monopole pairing and a deformation force,

$$H=\kappa\sum_{Jm}\langle m|Y_{20}|m\rangle\begin{bmatrix} j & j & J\\ m & -m & 0\end{bmatrix}(-1)^{j+m}(C_j^\dagger\tilde{C}_j)_{J0}-GA^{+}_{00}(jj)A_{00}(jj)\ , \tag{1}$$

with

$$A^\dagger_{JM}(jj')=(c_j^\dagger c_{j'}^\dagger)_{JM}=\sum_{mm'}\begin{bmatrix} j & j' & J\\ m & m' & M\end{bmatrix}C^\dagger_{jm}C^\dagger_{j'm'}\ , \tag{2}$$

$$\tilde{C}_{Jm}=(-1)^{j-m}C_{j-m}\ , \tag{3}$$

and the square brackets [ ] denotes a Clebsch-Gordan coefficient. The operators $C^\dagger_{jm}$ ($C_{j'm'}$) in Eqs. (1) and (2) are the single-particle fermion creation (annihilation) operators.

For making the transformation to the boson space we employ the DBM for a single j shell. This transformation is given by

$$A^{\dagger}_{JM} \rightarrow b^{\dagger}_{JM} - 2 \sum_{J_1 J_2 J_3 J_4} \begin{Bmatrix} j & j & J_1 \\ j & j & J_2 \\ J_3 & j & J_4 \end{Bmatrix} \hat{J}_1 \hat{J}_2 \hat{J}_3 \hat{J}_4 [(b^{\dagger}_{J_1} b^{\dagger}_{J_2})_{J_4} \otimes \tilde{b}_3]_{JM} , \tag{4a}$$

$$A_{JM} \rightarrow b_{JM} , \tag{4b}$$

and

$$(C^{\dagger}_j \tilde{C}_j)_{JM} \rightarrow 2 \sum_{J_1 J_2} \hat{J}_1 \hat{J}_2 (-1)^{J+J_2} \begin{Bmatrix} j & j & J_1 \\ J_2 & J & j \end{Bmatrix} (b^{\dagger}_{J_1} \tilde{b}_{J_2})_{JM} , \tag{4c}$$

where $b^{\dagger}_{JM}$ ($b_{J'M'}$) are the boson creation (annihilation) operators. In Eqs. (4a)–(4c) and hereafter the quantum number j in the operators $A^{\dagger}$ and $b^{\dagger}$ is disregarded. It is to be mentioned that the mapping [Eqs. (4a)–(4c)] is nonunitary. This nonunitary problem does not enter in this work, since we are mainly concerned here with the calculation of the ground states. Moreover, this problem of nonunitarity in the DBM has already been resolved.[16] Using the mapping, Eqs. (4a)–(4c) the model Hamiltonian, Eq. (1) in the boson representation is

$$H^B = H^B_D + H^B_P , \tag{5}$$

with

$$H^B_D = \frac{8}{\sqrt{5}} \kappa \cdot \hat{j} \begin{bmatrix} j & 2 & J \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix} \sum_{J_1 J_2} \hat{J}_1 \hat{J}_2 (-1)^{J_2} \begin{Bmatrix} J_1 & J_2 & 2 \\ j & j & j \end{Bmatrix} (b^{\dagger}_{J_1} \tilde{b}_{J_2})_{20} \tag{6}$$

and

$$H^B_P = -\frac{G}{2} \hat{j}^2 \left[s^{\dagger} s + 2 \sum_{J_1 J_2 J_3} (-1)^{J_2 + J_3} \begin{Bmatrix} J_1 & J_2 & J_3 \\ j & j & j \end{Bmatrix} \frac{\hat{J}_1 \hat{J}_2 \hat{J}_3}{\hat{j}} [(b^{\dagger}_{J_1} b^{\dagger}_{J_2})_{J_3} \otimes \tilde{b}_{J_3}]_{00} s \right] , \tag{7}$$

where

$$\hat{J} = \sqrt{2J+1} , \tag{8}$$

$$s = b_{00} . \tag{9}$$

For the calculation of the energy, we make use of the intrinsic function defined through

$$|\Phi\rangle = \frac{b_0^{\dagger p}}{\sqrt{p!}} |0\rangle , \tag{10}$$

where

$$b_0^{\dagger} = \sum_J \alpha_{J0} b^{\dagger}_{J0} , \tag{11}$$

TABLE I. Ground-state energies obtained in this work for the various values of the deformation strength κ, compared with the exact results (Ref. 17).

No. of pairs	Exact	*sd*	*sdg*	Exact	*sd*	*sdg*	Exact	*sd*	*sdg*
		$\kappa=0.0$			$\kappa=0.5$			$\kappa=1.0$	
1	−6.0	−6.0	−6.0	−6.12	−6.12	−6.12	−6.44	−6.42	−6.44
2	−10.0	−10.0	−10.0	−10.20	−10.21	−10.21	−10.75	−10.76	−10.78
3	−12.0	−12.0	−12.0	−12.23	−12.28	−12.28	−12.9	−13.0	−13.06
		$\kappa=1.5$			$\kappa=2.0$			$\kappa=2.4$	
1	−6.90	−6.84	−6.90	−7.47	−7.33	−7.46	−7.99	−7.76	−7.85
2	−11.58	−11.57	−11.63	−12.62	−12.53	−12.7	−13.6	−13.38	−13.6
3	−13.96	−14.19	−14.25	−15.3	−15.60	−15.76	−16.57	−16.86	−17.16

TABLE II. Parameters α_0 and α_2 [appearing in Eq. (11)] and λ (Lagrangian multiplier).

No. of pairs	α_0	α_2	λ	α_0	α_2	λ	α_0	α_2	λ
		$\kappa=0.0$			$\kappa=0.5$			$\kappa=1.0$	
1	1.0	0.0	−6.0	0.99	−0.132	−6.12	0.97	−0.23	−6.42
2	1.0	0.0	−8.0	0.99	−0.12	−8.18	0.98	−0.22	−8.70
3	1.0	0.0	−6.0	0.99	−0.11	−6.22	0.98	−0.20	−6.85
		$\kappa=1.5$			$\kappa=2.0$			$\kappa=2.4$	
1	0.95	−0.30	−6.84	0.94	−0.35	−7.34	0.9	−0.37	−9.36
2	0.96	−0.29	−9.45	0.93	−0.346	−10.39	0.92	−0.38	−11.24
3	0.96	−0.28	−7.87	0.94	−0.34	−9.19	0.93	−0.38	−10.42

and p denotes the number of pairs. The weight factors α_{J0}'s in Eq. (11) depend on the dynamics of the system. These are obtained through the variational procedure

$$\delta\frac{\langle\Phi|H^B|\Phi\rangle}{\langle\Phi|\Phi\rangle}=0 , \qquad (12)$$

with the constraint

$$\sum_J \alpha_{J0}^2=1 , \qquad (13)$$

which is incorporated in Eq. (12) through the Lagrangian multiplier. In Eq. (12), the energy matrix is given by

$$\langle\Phi|H^B|\Phi\rangle=-\frac{G}{2}\hat{j}^2 p\left|\alpha_0^2+2(p-1)\sum_{J_1J_2J_3}(-1)^{J_2}\frac{\hat{J}_1\hat{J}_2}{\hat{j}}\begin{bmatrix}J_1 & J_2 & J_3\\ 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}J_1 & J_2 & J_3\\ j & j & j\end{Bmatrix}\alpha_0\alpha_{J_3}\alpha_{J_1}\alpha_{J_2}\right|$$
$$-\frac{8}{\sqrt{5}}\cdot p\cdot\kappa\cdot\hat{j}^3\sum_{J_1J_2}(-1)^{J_2}\begin{bmatrix}j & j & J_2\\ \frac{1}{2} & -\frac{1}{2} & 0\end{bmatrix}\begin{Bmatrix}j & j & J_1\\ \frac{1}{2} & -\frac{1}{2} & 0\end{Bmatrix} . \qquad (14)$$

Explicit numerical calculations have been carried out for a single j shell with $j=\frac{11}{2}$. In this case the possible values of J in Eq. (11) are $0,2,\ldots,10$. Only even values of J are permitted, due to the antisymmetry requirement. Two sets of calculations have been performed. In one case we allowed only $J=0$ and 2 in Eq. (11), which is analogous to the IBM, and in the other case we included $J=4$ bosons too. The results of this work are compared with the exact results[17] denoted by "Exact" in Table I. The calculations have been performed for various values of the deformation strength, κ. For $\kappa=0$, i.e., for the case of pure pairing, the results of these calculations are exact. This is due to the fact that for only $J=0$ the intrinsic state is exactly the same as the physical state (the state obtained through mapping). As is evident from Table I, for the deformation values greater than zero, our results are quite close to the exact results. It turns out that only $J=0$ and 2 bosons are important in our model study. This is clearly seen from the comparison of the energies with $J=0$ and 2 bosons (sd) with those where $J=4$ bosons were also included (sdg). It is to be pointed out that in some cases our results overestimate the ground-state correlations. This can be mainly attributed to the overcompleteness of the intrinsic basis.[1] This overestimation also appears in the random-phase approximation.[5] The overcompleteness is related to the fact that the intrinsic basis contains unphysical (spurious) subspace in addition to the physical subspace. The inclusion of g bosons worsens the results in some cases, especially in the middle of the shell. This is due to the fact that the addition of more basis such as g bosons admits more spurious components. But, as is evident from Table I, this problem does not seem to be very serious.

The obtained state parameters α_0 and α_2 are listed in Table II. As expected, the parameter α_2 monotonically increases with the deformation strength κ. The parameters for a fixed κ are almost the same for all the particle numbers. This is due to the fact that in our model Hamiltonian we have a quadrupole mean field instead of the quadrupole-quadrupole interaction. In the case of the quadrupole-quadrupole interaction, the parameter α_2 changes with the particle number.[18]

In summary, we note that this approach is an alternative way of performing the nuclear structure calculations in the boson representation. The parameters of the boson Hamiltonian are fixed from the fermion space. In this formalism the higher angular momentum bosons can be easily included in the calculations. However, in the model case studied, only $J=0$ and 2 bosons seem to be important in the wave function. This is synonymous to the IBM approach.

The method of intrinsic basis functions will be extended to the variational estimate of the ground state in the multi-j-shell calculations with more realistic forces.

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