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Isovector giant dipole resonance in hot rotating light nuclei in the calcium region

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The isovector giant dipole resonances in hot rotating light nuclei in the calcium region are studied using a rotating anisotropic harmonic oscillator potential and a separable dipole-dipole residual interaction. The influence of temperature on the isovector giant dipole resonance is assumed to occur through the change of deformation of the average field only. Calculations are performed for the three nuclei ^{40,42}Ca and ^{46}Ti which have spherical, oblate, and prolate ground states, respectively, to see how their shape transitions at higher excited states affect the isovector giant resonance frequencies built on them. It is seen that, while the width fluctuations present at $T=0$ vanish at $T=0.5$ MeV in ^{40,42}Ca, they persist up to $T=1.5$ MeV in the case of ^{46}Ti. This behavior brings out the role of temperature on shell effects which in turn affects the isovector giant dipole resonance widths.

I. INTRODUCTION

The isovector giant dipole resonance (IVGDR) is an out-of-phase small-amplitude collective oscillation of protons against neutrons with a dipole spatial nature. The study of such a resonance has been and still is a major topic of research in nuclear physics. But, until recently, only the giant dipole resonances built on the ground state of a nucleus were investigated. According to the hypothesis by Morinaga[1] and Brink,[2] not only the ground state but also the excited states should have giant dipole resonances built on them. On the experimental side, Newton *et al.*[3] first observed giant dipole resonances built on excited states in heavier nuclei while in light nuclei they have been studied in detail for the $A=28$ system by Dowell *et al.*[4] The study of giant dipole resonances in hot rotating systems is interesting because it gives us two additional degrees of freedom, namely, the temperature and rotation which can provide us with new information on the nuclear structure.

On the theoretical side, there are two types of calculations: (i) The harmonic oscillator model introduced by Brink[2] for the IVGDR built on the ground state which has been extended to the rotating case by several authors;[5–8] (ii) linear-response theory used by Egido and Ring[9] extended to finite temperatures.[10] In our previous investigation, we have used the first method to study the effect of rotation on IVGDR of certain calcium isotopes wherein the bulk of the angular momentum is of an aligned nature; the shape and deformation of the above nuclei at high spin were determined by the Mottelson-Nilsson method for rotating light nuclei, and the allowed angular velocities for these deformations were obtained by the Fermi liquid drop model (FLDM).

In the present paper, we have used the Mottelson-Nilsson method for hot rotating light nuclei[11] to obtain the shape and deformation of ^{40,42}Ca and ^{46}Ti as a function of temperature and spin. This method is an extension of the earlier Mottelson-Nilsson method for cold rotating light nuclei[12] and incorporates the effect of temperature also. The calculation of free energy and the total energy in this method is based on the Mottelson-Nilsson method[12] which has some marked advantages compared to the Strutinsky method for cold nuclei,[13] especially for light nuclei. In this method, unlike the Strutinksy method for hot rotating nuclei,[14] there is no need to renormalize the single-particle level densities at finite temperature.

The first step of our study is to determine the equilibrium deformations of nuclei at different spins and temperatures using the Mottelson-Nilsson method for hot rotating light nuclei.[11] The main advantage of this method is that the changes of surface diffuseness with spin are automatically taken into account in this method. For the study of the IVGDR which is mainly a surface effect,[15] this method is thus more suitable. The next step of our calculations is to find out the allowed angular velocities for these deformations. This was done in our previous work[8] using the Fermi liquid drop model.[16] But this model has some restrictions when one wants to consider prolate shapes and hence we have used the rotating liquid drop model (RLDM) in this study. It is known[17] that the angular velocities determined by RLDM are the same as those obtained by the FLDM. The disadvantage which can be pointed out in the present method is the noninclusion of the $l\cdot s$ and l^2 terms in the IVGDR frequency calculations. But this, we hope may not alter the results much for the case of light nuclei considered here, since the above terms may be very small or negligible for them.[13]

In Sec. II we give, for the sake of completeness, a concise account of the Mottelson-Nilsson method for hot rotating light nuclei[11] for determining the shapes and deformations as a function of temperature and rotation. Section III details the method of obtaining the IVGDR frequencies as a function of temperature and rotation. Finally the results are presented and discussed in Sec. IV.

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II. DETERMINATION OF SHAPES OF HOT ROTATING LIGHT NUCLEI

In a rotating nucleus without internal excitation, the nucleons move in a cranked Nilsson potential with deformation described by ε and γ. The cranking is performed around one of the principal axes, the Z axis, and the cranking frequency is given by ω_c. We diagonalize the Hamiltonian in the rotating system

$$h^{\omega_c}=h^0-\omega_c j_z \tag{1}$$

in cylindrical representation up to $N=6$ shells. This is in contrast to the Strutinsky method, wherein the diagonalization up to $N=11$ shells is required. Thus there is an enormous reduction in computation time in the present method. Our results are restricted to smaller deformation with $\varepsilon<0.6$ and spin $I<30$. The eigenvalues of the diagonalization are the energies in the rotating system, $e_i^{\omega_c}$. The total nuclear spin is identified with the component along the rotation axis

$$I=I(\varepsilon,\gamma,\omega_c)=\sum_i m_i;\quad m_i=\langle j_z\rangle_i\ , \tag{2}$$

where the sum is taken over the occupied orbitals. The single-particle energies in the fixed frame are calculated as expectation values of the static Hamiltonian h^0, and they are given by

$$e_i=e_i^{\omega_c}+\hbar\omega_c m_i\ . \tag{3}$$

The total energy is obtained as

$$E=E(\varepsilon,\gamma,\omega_c)=\sum_i e_i+E_c\ , \tag{4}$$

where E_c is the nuclear Coulomb energy which depends on the deformation. Since the effect of the pairing correlation is negligible in light nuclei, the latter can be taken to rotate with the rigid body moment of inertia. But this is true only if one uses the cranked harmonic oscillator or the Wood-Saxon potential. This is generally not possible in the case of the Nilsson potential because of the strange property due to the l^2 term. But the latter term is very small for the light nuclei considered in the present investigation. The renormalization of the rotational behavior is hence found to be unnecessary here. Furthermore, we do not renormalize the spin or the total energy and the latter will not lead to any problem since we are dealing with only energy differences but not their absolute values. In the present calculations where the energy dependence on spin is taken directly from the sum of single-particle energies, if the diffuseness changes with spin, this should be automatically accounted for. One can construct an energy surface for fixed I from Eqs. (2) and (4), and the minima in these surfaces then determine the shape and deformation of the given nuclei.

For systems at the finite temperature, the shape at high spins is determined by the lowest minimum of the free energy,

$$F(\{\beta_i\},T,I)=E(\{\beta_i\},I)-TS(\{\beta_i\},I)\ , \tag{5}$$

calculated for the grand partition function

$$Z=\mathrm{tr}\{\exp[-(h^{\omega_c}-\lambda N)/T]\}\ . \tag{6}$$

Here E, S, and T denote the potential energy, the entropy, and the temperature, respectively. h^{ω_c} is given by Eq. (1) and λ has a meaning of the chemical potential, introduced to conserve the particle number in the average. The independent particle energy E and the entropy S at high spins are defined as

$$E=\sum_{i=1}^{\infty} e_i n_i\ , \tag{7}$$

$$S=-\sum_{i=1}^{\infty}[n_i\ln n_i+(1-n_i)\ln(1-n_i)]\ , \tag{8}$$

where the single-particle energies e_i are given by Eq. (3). The occupation probability n_i of a single-particle state i is given by

$$n_i=1/\{1+\exp[(e_i^{\omega_c}-\lambda)/T]\}\ . \tag{9}$$

It is to be noted that unlike the Strutinsky method,[14] in this method there is no need to renormalize the single-particle level density at finite temperature.

III. EFFECT OF TEMPERATURE AND ROTATION ON ISOVECTOR GIANT DIPOLE RESONANCE FREQUENCIES

To study the properties of the IVGDR's in rapidly rotating light nuclei, one can use, for the average field of the nucleus, an oscillator potential with deformation parameters consistent with the angular momentum of the system. It is therefore essential to first track the rotation-induced changes of nuclear shapes. This can be done within the framework of either the rotating liquid drop model[18] or the Fermi liquid drop model. But RLDM is preferred to FLDM since the latter has some restrictions when one wants to consider prolate shapes. Further, it is known that the angular velocities determined by RLDM are the same as those obtained by FLDM. The magnitude of the angular velocity of nuclear rotation Ω can be simply estimated by considering the nuclear rotation as that of a rigid body. As is well known, this assumption seems to provide a reasonable picture of nuclear rotation. Thus we have the relation[18]

$$\Omega=\frac{\sqrt{I(I+1)}}{\mathcal{J}_{\mathrm{rig}}}=\frac{I}{\mathcal{J}_{\mathrm{rig}}}\ , \tag{10}$$

which relates the angular velocity Ω, angular momentum I, and the nuclear moment of inertia $\mathcal{J}_{\mathrm{rig}}$. Here $\mathcal{J}_{\mathrm{rig}}$ denotes a rigid-body moment of inertia for a nucleus of a given shape which may itself depend on angular momentum I. Looking for the lowest rotational states, one should employ the largest possible moment of inertia,[7]

$$\mathcal{J}/\hbar^2=\frac{A^{5/3}}{72}[1-\sqrt{5/4\pi}\,\varepsilon\cos(\gamma-2\pi/3)]\ \mathrm{MeV}^{-1}\ . \tag{11}$$

In order to obtain the IVGDR energies of the rotating nucleus, the average field of the nucleus has been taken to be an oscillator potential with deformation parameters consistent with the angular momentum of the system.

The rotation-induced changes of the shape of nuclei can be simulated by the average Hamiltonian of a triaxial harmonic oscillator

$$H_{\text{av}}(\Omega)=\sum_{\nu=1}^{A} h_\nu(\Omega)\,, \tag{12}$$

where

$$h(\Omega)=\frac{p^2}{2m}+\frac{m}{2}(\omega_x^2x^2+\omega_y^2y^2+\omega_z^2Z^2)-\Omega 1_z \tag{13}$$

and

$$L_z=\sum_{\nu=1}^{A} 1_z(\nu)$$

is the operator of rotation about the z axis. To generate the isovector dipole excitation mode, we add to the Hamiltonian (12) the effective dipole interaction

$$H_{\text{int}}=\eta\sum_{i=x,y,z}\frac{m\omega_i^2}{2A}\left[\sum_{\nu=1}^{A}\tau_3^{(\nu)}x_i(\nu)\right]^2\,, \tag{14}$$

where $\tau_3(\nu)$ is the third projection of the Pauli isospin matrix

$$\tau_3=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$$

and η is a parameter that characterizes the isovector component of the neutron or proton average field

$$V_{\substack{(n)\\(p)}}(\nu)=\frac{m}{2}\left[1\mp\eta\frac{N-Z}{A}\right]\sum_{i=x,y,z}\omega_i^2x_i^2(\nu)\,. \tag{15}$$

The frequencies of the giant dipole resonance in a rotating nucleus can be obtained by diagonalizing analytically the Hamiltonian (12) with the effective interaction (14) within the framework of the standard random-phase-approximation procedure.

Transforming to the laboratory system, we get the frequencies of the giant dipole resonance as

$$\begin{aligned}
\varpi_1&=(1+\eta)^{1/2}\omega_z\,,\\
\varpi_2\mp\Omega&=\left[(1+\eta)\frac{\omega_y^2+\omega_x^2}{2}+\Omega^2+\tfrac{1}{2}[(1+\eta)^2(\omega_y^2-\omega_x^2)^2+8\Omega^2(1+\eta)(\omega_y^2+\omega_x^2)]^{1/2}\right]^{1/2}\mp\Omega\,,\\
\varpi_3\mp\Omega&=\left[(1+\eta)\frac{\omega_y^2+\omega_x^2}{2}+\Omega^2-\tfrac{1}{2}[(1+\eta)^2(\omega_y^2-\omega_x^2)^2+8\Omega^2(1+\eta)(\omega_y^2+\omega_x^2)]^{1/2}\right]^{1/2}\mp\Omega\,.
\end{aligned} \tag{16}$$

Thus we obtain five frequencies, ϖ_1, $\varpi_2-\Omega$, $\varpi_2+\Omega$, $\varpi_3-\Omega$, and $\varpi_3+\Omega$, for the collectively rotating triaxial nuclei. For prolate nuclei ($\omega_x=\omega_y\neq\omega_z$) rotating about an axis perpendicular to its symmetry axis, all the above five frequencies will exist. But for oblate nuclei ($\omega_x=\omega_y\neq\omega_z$) rotating about its symmetry axis, as first shown by Hilton in Ref. 7, only two frequencies, namely, ϖ_1 and $\varpi_2-\Omega=\varpi_3+\Omega$ will exist and thus all effects due to rotation vanish and only those purely due to deformation will be left. For the spherical nuclei ($\omega_x=\omega_y=\omega_z$), which comes under the latter category, one gets only one frequency, namely $\varpi_1=\varpi_2-\Omega=\varpi_3+\Omega$.

Since the influence of temperature effects on IVGDR is assumed to occur through the change of deformation of the average field caused by temperature, the above expressions [Eq. (16)] themselves are used for the study of hot rotating nuclei also. The temperature decreasing the shell effects has an appreciable influence on the evolution of the shape of the fast rotating nuclei. Consequently, the influence of the temperature effects on the IVGDR occur through the change of the deformation parameters of the average field.

IV. RESULTS AND DISCUSSION

The aim of this work is to study the combined effects of rotation and temperature on the properties of IVGDR in the light nuclei ^{40,42}Ca and ^{46}Ti. We have considered the effect of rotation alone on the IVGDR in certain calcium isotopes in our earlier paper,[8] and in this work we include

TABLE I. Shape and deformation of ^{40}Ca with temperature and spin.

T	0 MeV		0.5 MeV		1.0 MeV		1.5 MeV	
I	γ	ε	γ	ε	γ	ε	γ	ε
0	−180°	0.0	−180°	0.0	−180°	0.0	−180°	0.0
4	−180°	0.0	−180°	0.0	−180°	0.0	−180°	0.0
8	−180°	0.0	−180°	0.1	−180°	0.1	−180°	0.1
12	−180°	0.2	−180°	0.1	−180°	0.1	−180°	0.1
16	−180°	0.1	−180°	0.1	−180°	0.1	−180°	0.1
20	−180°	0.1	−180°	0.4	−180°	0.4	−180°	0.3
24	−180°	0.4	−180°	0.5	−180°	0.5	−180°	0.4

TABLE II. Shape and deformation of ^{42}Ca with temperature and spin.

T / I	0 MeV γ	0 MeV ε	0.5 MeV γ	0.5 MeV ε	1.0 MeV γ	1.0 MeV ε	1.5 MeV γ	1.5 MeV ε
0	−180°	0.1	−180°	0.0	−180°	0.0	−180°	0.0
4	−180°	0.1	−180°	0.0	−180°	0.0	−180°	0.0
8	−180°	0.2	−180°	0.1	−180°	0.0	−180°	0.0
12	−180°	0.1	−180°	0.1	−180°	0.1	−180°	0.1
16	−180°	0.1	−180°	0.1	−180°	0.1	−180°	0.1
20	−180°	0.3	−180°	0.1	−180°	0.1	−180°	0.2
24	−180°	0.5	−180°	0.4	−180°	0.4	−180°	0.3

the effect of temperature also in the investigation of IVGDR properties. It has been found[19,20] that the heating of a nucleus essentially does not influence the characteristics of the IVGDR with or without rotation. However, the temperature decrease of the shell effects has an appreciable influence on the evolution of the shape of fast rotating nuclei.[21] Consequently, the influence of the temperature effects on the IVGDR occurs through the change of the deformation parameters of the average field determining the characteristics of the IVGDR. We have used this fact in our calculations reported here.

In a recent series of experiments at Seattle,[22] statistical giant dipole resonance decays in a wide range of excited nuclei from $A=24$ to 66 have been studied. In these experiments, most of the compound nuclei were formed at initial excitation energies between 35 and 52 MeV and spins in the range of $0-25\hbar$. These energies correspond to mean final-state temperatures ranging from ~ 1 MeV in heavy systems to ~ 2 MeV in the light systems. In order to fix the upper limit of temperature that should be considered for the above nuclei, we have first determined the equilibrium deformations of ^{46}Ti by the Mottelson-Nilsson model for hot rotating nucleus at different temperatures and compared them with those obtained by the RLDM. It has been found that ^{46}Ti, which remains prolate with $\varepsilon=0.2$ until about a spin of $I=12$ at $T=0$, becomes spherical between $I=0-12$ at $T=1.5$ MeV and this behavior at $T=1.5$ MeV closely coincides with that predicted by RLDM calculations. Thus it has been concluded that the shell effects persist up to a temperature of about 1.5 MeV in ^{46}Ti and so, in our calculations reported here, we varied the temperature from 0 to 1.5 MeV in steps of 0.5 MeV.

In Tables I–III we show the shape and deformation of ^{40,42}Ca and ^{46}Ti obtained from the Mottelson-Nilsson method for hot rotating light nuclei as a function of temperature and spin. These are used as input parameters to get the rotational velocities by the RLDM. Then the frequencies of the IVGDR as a function of temperature and rotation are obtained by using Eqs. (16).

We present our results for the case of ^{40,42}Ca and ^{46}Ti in Figs. 1(a)–1(d), 2(a)–2(d), and 3(a)–3(d), respectively. For spherical shape of a nucleus, there is only one IVGDR frequency in the intrinsic or the laboratory frame. But for prolate and oblate shapes, we have two frequencies in the nonrotating case, the splitting being caused by static deformation. When such nuclei start rotating, the two frequencies will divide into three in the intrinsic frame. These three modes observed in the intrinsic system divide, in the laboratory frame, into five frequencies for the prolate case while the transformation to the laboratory frame just brings the frequencies back to coincide with their original values at zero rotation for oblate system rotating about the symmetry axis. This expected behavior is clearly brought out in Figs. 1–3. In ^{40}Ca and ^{42}Ca (see Figs. 1 and 2) the frequency splitting has only one or two components since they have only spherical shape or oblate shape rotating about the symmetry axis. But this behavior changes in ^{46}Ti (see Fig. 3). For this nucleus, the splitting has five, one as well as two components depending upon the prolate shape rotating about a perpendicular axis, spherical or oblate shapes ro-

TABLE III. Shape and deformation of ^{46}Ti with temperature and spin.

T / I	0 MeV γ	0 MeV ε	0.5 MeV γ	0.5 MeV ε	1.0 MeV γ	1.0 MeV ε	1.5 MeV γ	1.5 MeV ε
0	−120°	0.2	−120°	0.2	−180°	0.0	−180°	0.0
4	−120°	0.2	−120°	0.2	−140°	0.0	−180°	0.0
8	−120°	0.2	−180°	0.2	−180°	0.0	−180°	0.0
12	−120°	0.1	−120°	0.1	−180°	0.0	−180°	0.0
16	−180°	0.2	−180°	0.1	−180°	0.1	−180°	0.1
20	−180°	0.1	−180°	0.1	−180°	0.1	−180°	0.2
24	−180°	0.2	−180°	0.2	−180°	0.2	−180°	0.1

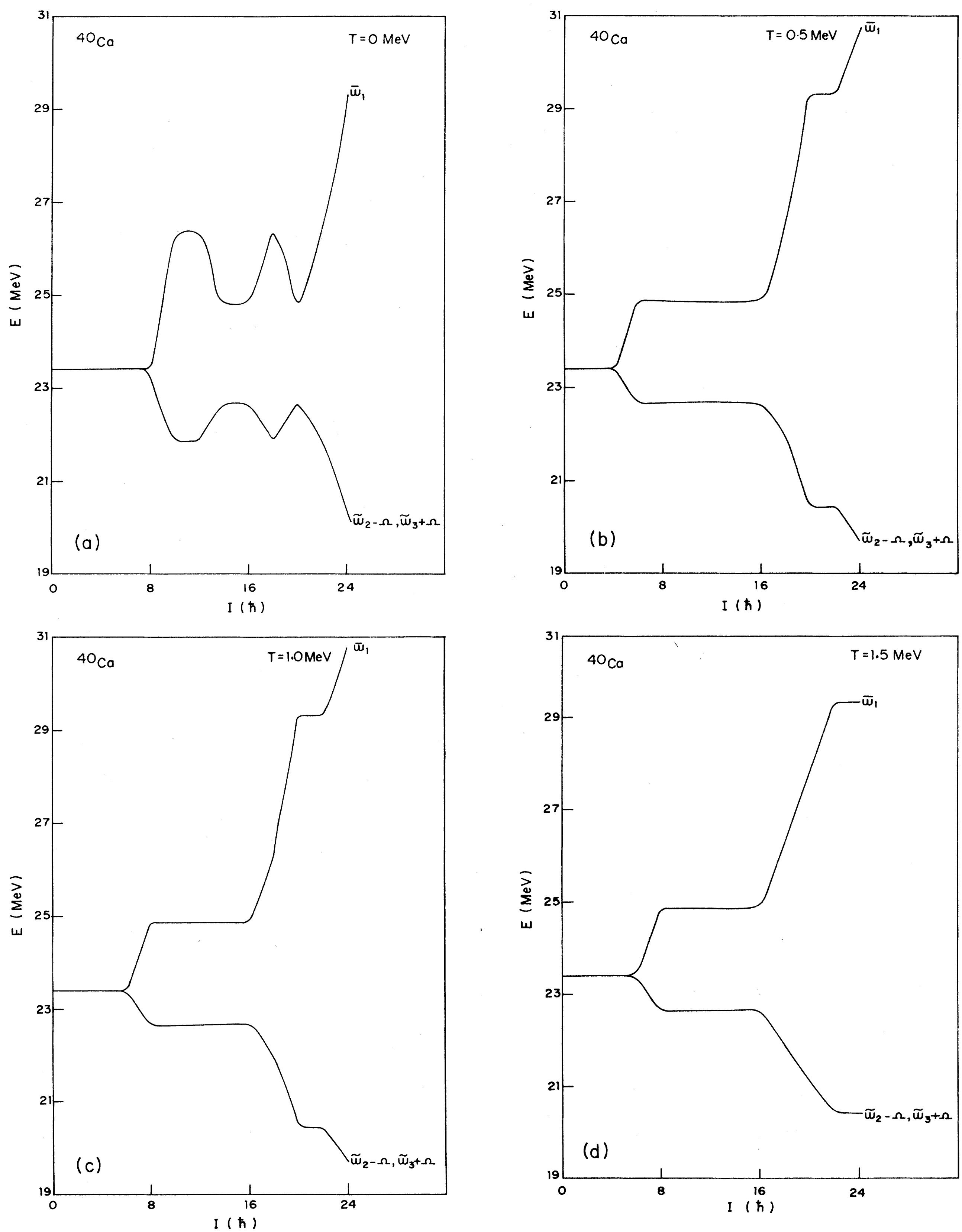

FIG. 1. (a)–(d) Dependence of the isovector giant dipole energy E on the angular momentum I for ^{40}Ca at different temperatures.

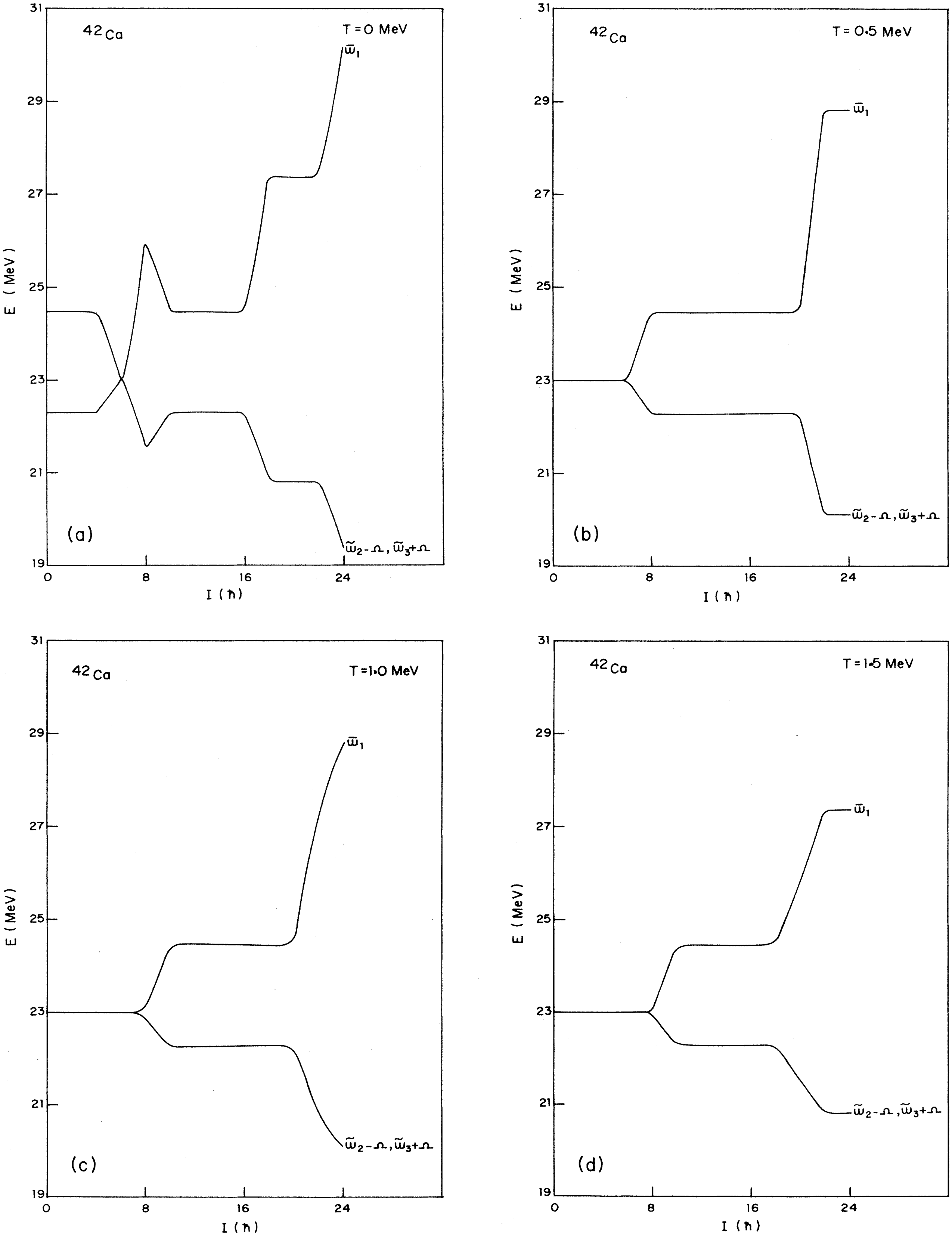

FIG. 2. (a)–(d) Dependence of the isovector giant dipole energy E on the angular momentum I for ^{42}Ca at different temperatures.

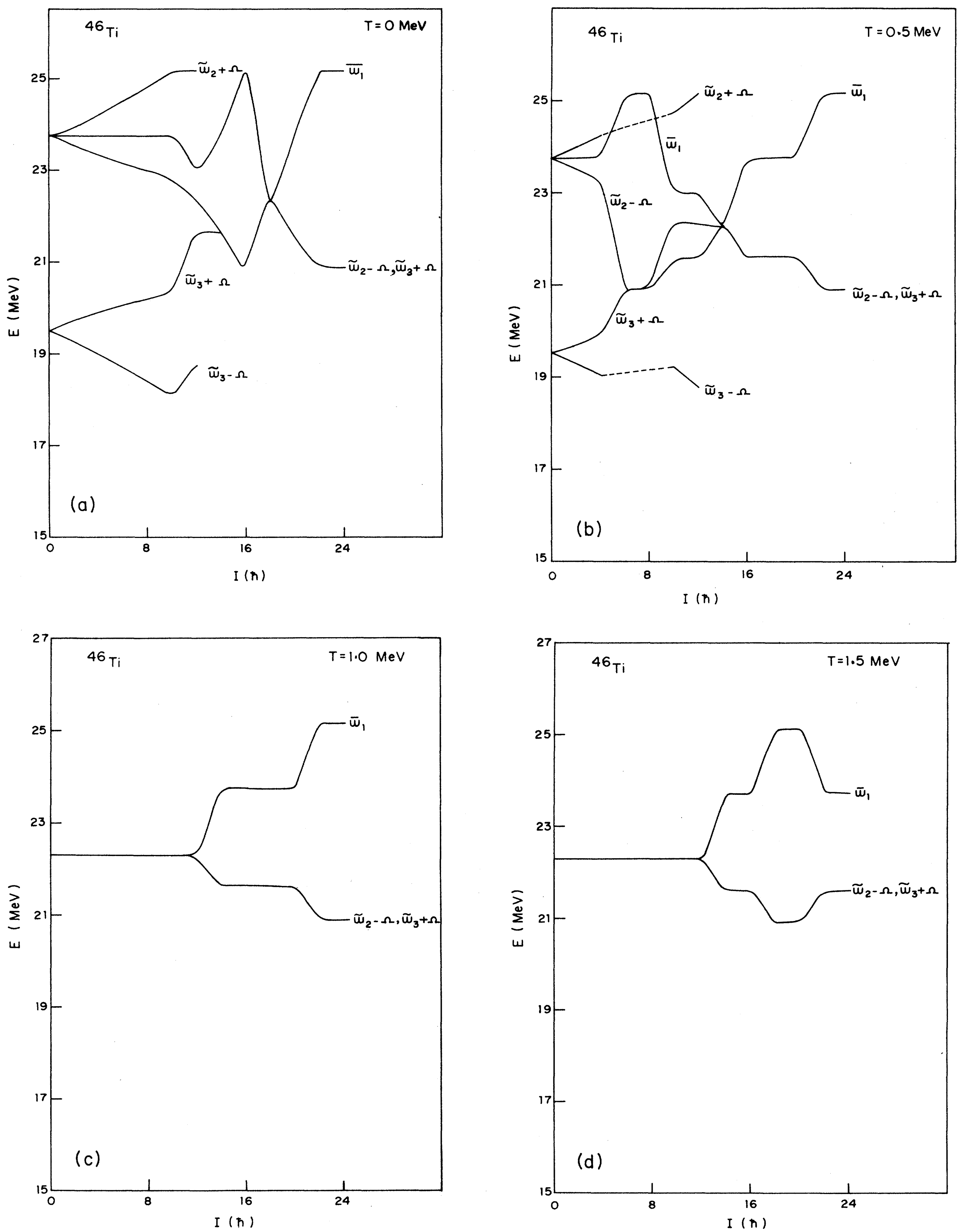

FIG. 3. (a)–(d) Dependence of the isovector giant dipole energy E on the angular momentum I for ^{46}Ti at different temperatures.

tating about the symmetry axis, respectively, at different spins and temperature. If we consider the effect of temperature alone, we see that the width fluctuations vanish at $T = 0.5$ MeV itself in the case of ^{40,42}Ca, but persist up to $T = 1.5$ MeV in the case of ^{46}Ti. But the general broadening of IVGDR widths with excitation is clearly seen in all the cases considered.

To sum up, in this work we have chosen three nuclei which are spherical, oblate, and prolate at the ground state. We have studied their shape transitions with temperature and rotation which in turn influence the IVGDR properties.

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