Nuclear liquid-gas phase transition with finite range force

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The possibilities of the occurrence of liquid-gas phase transition in nuclear matter and finite nuclei are investigated in the framework of temperature-dependent Hartree-Fock theory employing the finite-range Brink-Boeker effective interaction. The equation of state at subnuclear density is obtained. It is found that for infinite nuclear matter critical temperature for the phase transition is substantially reduced because of the finite-range nature of the force as compared to those obtained using zero-range forces. When finite-size effect and Coulomb force are taken into account, critical temperature is found to be around 8 MeV.

I. INTRODUCTION

The possibility of the occurrence of a liquid-gas phase transition in the intermediate-energy heavy-ion collision has evoked great interest in the last couple of years. It is still in the realm of expectation, as it has not been possible to establish that such a phenomenon really occurs in the experiment. Nevertheless, this topic has fascinated many, and literature[1-3] abounds with such speculation, mainly for the following two reasons. Firstly, the liquid-like behavior of nuclei has been well established. This has been amply proven by the success of the liquid drop model in describing many nuclear properties such as nuclear fission and other phenomena involving collective degrees of freedom. Secondly, the nucleon-nucleon potential with a hard core followed by an attractive part resembles very much the potential which gives rise to a van der Waals equation of state, and so should exhibit a liquid-gas phase transition. Although the above argument looks pretty convincing, still an element of scepticism cannot be ruled out, knowing the multifarious nature of nuclear dynamics. Several indications, both theoretical and experimental in nature, have been found over the last few years which point towards the possible existence of a phase transition.

At the experimental level, the mass yield distribution[4] observed in $p+\mathrm{Kr}$ and $p+\mathrm{Xe}$ reactions, with proton energy 80 to 300 GeV, follows the law $\sigma(A_f)\sim A_f^{-\tau}$, where $\sigma(A_f)$ is the cross section for the fragment with mass number A_f and τ is a parameter having the value 2.65. This is quite close to the prediction of Fisher's droplet model[5] of condensation. Thus these data had been taken as evidence for the occurrence of a liquid-gas phase transition in nuclei. However, such an interpretation of data has been questioned on several grounds. Firstly, many models,[6-11] without having explicitly the mechanism of a liquid-gas phase transition, can describe these data equally well. Secondly, it is difficult to understand why experimentally it is always possible to strike the critical point, even though it might exist.

Theoretically, one of us with Gross *et al.*[12] had first found the possible existence of a universal transition temperature at 5 MeV in the calculations of the multifragmentation process in the framework of a statistical model. At this temperature, the mass yield distributions are generally of "U-formed" shape. Above this temperature, the cross section departs from this shape and acquires exponential shape, because of the increase in the production of smaller fragments. Recently, Gross and Massman[13] in their more extensive calculations, treating Monte Carlo sampling, have also found the same. Similarly, Bondorf *et al.*[6] have also developed a model of statistical fragmentation, in which they find that the nucleus abruptly switches from a single entity to many fragments when temperature exceeds 5–6 MeV, which they call crack temperature. However, they find a second transition at a temperature of 11 MeV, which marks the increase of light clusters with $A\leq 4$. This second transition they speculate to be related to the liquid-gas phase transition supposed to exist in nuclear matter. It is not clear in which way the transitions predicted by Gross *et al.*[12,13] and Bondorf *et al.*[6] are related to the liquid-gas phase transition in the nuclear system. It must be understood that these calculations are for finite nuclei, where the concept of phase transition should be considered as approximate.

Jaqaman *et al.*[14,15] have investigated the occurrence of a liquid-gas phase transition in the case of both the infinite nuclear matter and finite nuclei in the framework of temperature-dependent Hartree-Fock (HF) theory with the zero-range Skyrme interaction. They find, for infinite nuclear matter, the critical temperature to be 15–20 MeV. Su *et al.*,[16] employing the real-time Green's-function method, have calculated the critical temperature for nuclear matter using various Skyrme interactions. They have also found similar results for critical temperature, as above. Jaqaman *et al.*[15] have found that this critical temperature decreases by 7 MeV when finite-size and Coulomb effects are taken into account. Thus they find the critical temperature for finite nuclei to

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be around 10 MeV.

As yet, no investigation has been performed with finite-range force. It is expected[15] that, with the increase of the range of the force, the system will become more dilute, leading to the decrease of critical temperature. If the critical temperature decreases by a couple of MeV when finite-range force is used, then the value of T_c for finite nuclei would be around 7–8 MeV, which is close to the transition temperature found by Gross *et al.*[12] and Bondorf *et al.*[6] It must be understood that the nature of the transition, which has been investigated microscopically, using the nucleon-nucleon interaction, refers to a change of state from nuclear liquid to a gas phase consisting of nucleons only. This seems to be different from the transition, in which nucleonic gas condenses into droplets of various sizes. Thus the situation is quite complex. At the moment, we are far from getting a coherent picture of the physics of the liquid-gas phase transition emerging from various studies.

Therefore, in this paper, we are interested in investigating this phenomenon extensively using a finite-range Brink-Boeker[17] force. We adopt the temperature-dependent HF theory like Jaqaman *et al.*[14,15] In Sec. II, we discuss the main features of our calculation of equation of state with this force and present our results on infinite nuclear matter. Section III contains our calculations of critical temperature and critical density for a finite system. Here the effect of a Coulomb interaction has also been considered. A discussion and conclusion are presented in Sec. IV.

II. EQUATION OF STATE FOR INFINITE NUCLEAR MATTER

We follow the procedure adopted by Jaqaman *et al.*[14] to calculate the equation of state for a system of interacting formions in the framework of temperature-dependent HF theory. They have calculated the equation of state with the Skyrme interaction, which is a zero-range force. Here we do the calculation with the Brink-Boeker[17] effective interaction, which has a finite range. This phenomenological nucleon-nucleon interaction is given by

$$\begin{aligned} U(\gamma) &= S_1(1-m_1+m_1P_M)e^{-\gamma^2/\mu_1^2} \\ &\quad + S_2(1-m_2+m_2P_M)e^{-\gamma^2/\mu_2^2} \\ &= V_1(\gamma)+V_2(\gamma) , \end{aligned} \tag{2.1}$$

where S_1, S_2, m_1, m_2, μ_1, and μ_2 are the parameters. The values of these parameters have been determined to reproduce the binding energy and density of nuclear matter and the ground-state energies of various light nuclei in the HF approximation. The values of these parameters are

$$S_1=-140.6 \text{ MeV}, \quad m_1=0.486, \quad \mu_1=1.4 \text{ fm} ,$$
$$S_2=389.5 \text{ MeV}, \quad m_2=-0.529, \quad \mu_2=0.7 \text{ fm} .$$

For an interacting Fermi gas, the single-particle energy in momentum space under HF approximation is given by

$$\epsilon_k=\frac{\hbar^2k^2}{2m}+\sum_j \frac{g_j}{4}\Gamma_j(k) , \tag{2.2}$$

where

$$\Gamma_j(k)=\sum_{k'}[\langle kk'|U|kk'\rangle+P_j\langle kk'|U|k'k\rangle] . \tag{2.3}$$

Here, the index j labels the four parts of the interaction, singlet even (SE), singlet odd (SO), triplet even (TE), triplet odd (TO), g_j is $(2S+1)\,(2T+1)$ the spin-isospin statistical weight, and P_j represents the parity of the state with values $+1$ and -1 for $+ve$ and $-ve$ parity, respectively.

For the temperature T, the HF single-particle energy ϵ_k is obtained as

$$\begin{aligned} \epsilon_k=\frac{\hbar^2k^2}{2m}+\sum_j \frac{g_j}{4}\Bigg\{ \frac{U(0)}{(2\pi)^3}\int d^3k'\eta_{k'}+P_j\int \Bigg[& V_1(0)\sum_{l=0}^{\infty}\frac{(-1)^l}{l!}\left[\frac{\mu_1}{2}(\mathbf{k}-\mathbf{k}')\right]^{2l} \\ & +V_2(0)\sum_{l=0}^{\infty}\frac{(-1)^l}{l!}\left[\frac{\mu_2}{2}(\mathbf{k}-\mathbf{k}')\right]^{2l}\Bigg] d^3k'\eta_{k'}\Bigg\} , \end{aligned} \tag{2.4}$$

where $V(0)=\int d^3\gamma V(\gamma)$ and $\eta_{k'}$ is the usual Fermi distribution function.

In the HF single-particle energy spectrum for the Brink-Boeker force given above, the expansion in powers of k^2 should in principle be taken to all orders. However, for computational feasibility, it has to be truncated. In the case of the Skyrme interaction, which is a zero-range force, the expansion only up to first order was considered to be adequate.[14] For the finite-range Brink-Boeker force, terms only up to k^2 are utterly inadequate. To determine the order of power of k^2 in Eq. (2.4), up to which the expansion must be taken, we undertake the study of the saturation property of nuclear matter at $T=0$, with this force.

The energy per particle of nuclear matter is given in the HF approximation by

$$\frac{E}{A}=\frac{3}{10}\frac{\hbar^2k_F^2}{m}+\sum_j \omega_j(k_F) , \tag{2.5}$$

where the potential-energy contribution is

$$\omega_j(k_F)=\frac{g_j}{4\sqrt{\pi}}S_1(1-m_1+m_1P_M)\left\{\frac{\mu_j^3}{12}+P_j\left[\left[\frac{1}{2\mu_j}-\frac{1}{\mu_j^3}\right]e^{-\mu_j^2}-\frac{3}{2\mu_j}+\frac{1}{\mu_j^3}+\frac{\sqrt{\pi}}{2}\mathrm{erf}(\mu_j)\right]\right\}$$
$$+\frac{g_j}{4\sqrt{\pi}}S_2(1-m_2+m_2P_M)\left\{\frac{\nu_j^3}{12}+P_j\left[\left[\frac{1}{2\nu_j}-\frac{1}{\nu_j^3}\right]e^{-\nu_j^2}-\frac{3}{2\nu_j}+\frac{1}{\nu_j^3}+\frac{\sqrt{\pi}}{2}\mathrm{erf}(\nu_j)\right]\right\}, \quad (2.6)$$

with

$$\mu_j=k_F\mu_1,\quad \nu_j=k_F\mu_2$$

and the error function

$$\mathrm{erf}(x)=\frac{2}{\sqrt{\pi}}\int_0^x \exp(-t^2)dt\ .$$

Here, k_F is the Fermi momentum which is related to density through the relation $\rho=2k_F^3/3\pi^2$. We have calculated the above expression as a function of Fermi momentum and plotted it in Fig. 1 as a solid curve. As expected, the minimum of the curve gives the saturation point $E/A=15.84$ MeV , $k_F=1.45$ fm^{-1} used by Brink and Boeker.[17] To determine the order of k_F in the above expression, which should be adequate to describe the saturation property, we calculated E/A vs k_F curves for the orders 9, 13, 19, and 29, which are shown as the dashed lines in the same Fig. 1 marked a, b, c, and d, respectively. It can be seen that expansion up to k_F^{30}, i.e., fifteenth order in k_F^2, is adequate. Therefore, in all our calculations we take the summation of l up to 15. As usual the effective mass m^* is obtained by fitting ϵ_k with an expression $\hbar^2k^2/2m^*$. m^* plays an important role in the dynamics of the system. In the case of the zero-range Skyrme interaction, m^* does not depend upon the temperature of the system. It also does not depend upon the momentum. However, in the finite-range force, such as that of Brink and Boeker, m^* depends upon both. For a fixed temperature and density, the dependence of m^*/m on k is not very significant. It practically remains constant with variation of k. However, the dependence of m^* on the temperature is very substantial, and hence it plays a rather important role in the determination of the equation of states of the system. In Fig. 2 we have presented the variation of m^*/m with temperature for two values of density, 0.17 fm^{-3} and 0.06 fm^{-3}.

The results of the calculation of pressure versus density for various temperatures ranging between $T=10$ to $T=19$ are presented in Fig. 3(a). All the isotherms below $T=14$ show the coexistence of a liquid and gas phase. The isotherms $T=16$ and 19 have the characteristics of only the gas phase. The critical temperature and critical density can be obtained from the observation of these isotherms. The critical point is defined by the condition

$$\frac{\partial P}{\partial \rho}=0=\frac{\partial^2 P}{\partial \rho^2}\ .$$

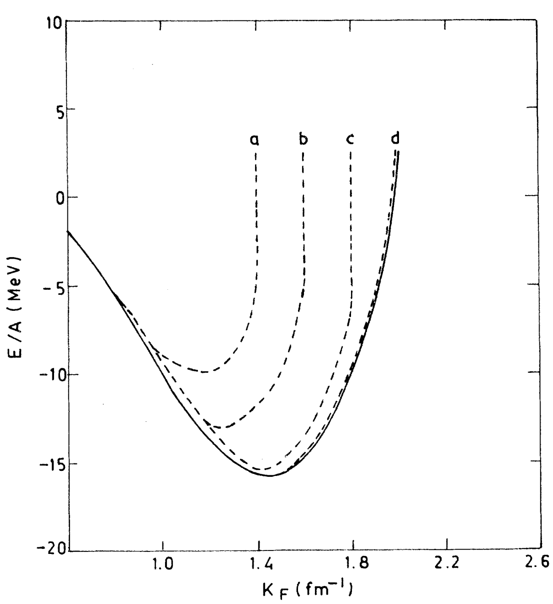

FIG. 1. Total energy per particle of nuclear matter E/A vs k_F the Fermi momentum. The solid curve is the result obtained with the Hartree-Fock expression for E/A [Eq. (2.5)]. The dashed curves a, b, c, and d are obtained when the exponentials in Eq. (2.5) are expanded to various orders in k_F and truncated at k_F^9, k_F^{13}, k_F^{19}, and k_F^{29}, respectively.

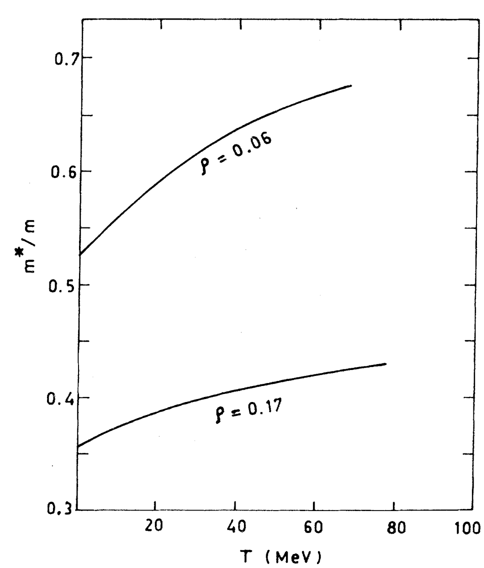

FIG. 2. The temperature dependence of effective mass for the Brink-Boeker force is shown.

Thus the point of inflection (which embodies the above condition) is contained in the isotherm $T=14$ MeV at $\rho=0.06\ \mathrm{fm}^{-6}$. Therefore, the critical temperature is $T_c=14$ MeV and critical density is $\rho_c=0.06\ \mathrm{fm}^{-3}$. As expected, the critical density is about 35% the normal nuclear matter density. In order to find out how the finite range of the force affects the phase-transition property of the system, we have calculated the pressure-versus-density curves for the same set of temperatures with the Skyrme forces used by Jaqaman *et al.*[14] In Figs. 3(b) and 3(c) these results are presented for the forces with the density dependence ρ^{σ}, with $\sigma=1$ and 0.1, respectively. For the case with $\sigma=1$ the critical temperature and density are, respectively, 19 MeV and 0.07 fm^{-3},

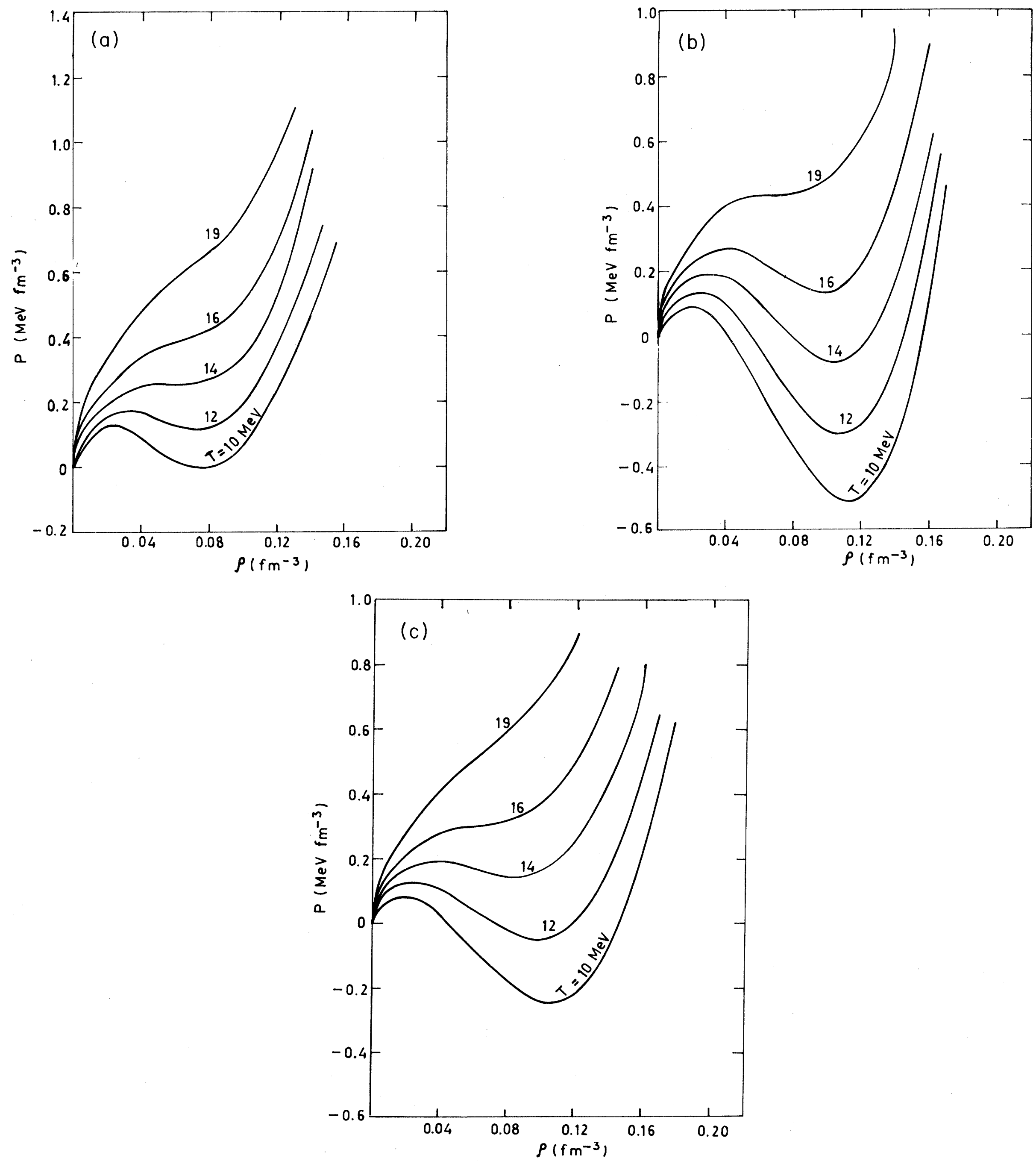

FIG. 3. (a) Nuclear matter isotherms, pressure P vs density for various temperatures T calculated with the Brink-Boeker force. (b) Same as (a) except for the Skyrme force with density dependence ρ^{σ}, with $\sigma=1$. (c) Same as (a) except for the Skyrme force with density dependence ρ^{σ}, with $\sigma=0.1$.

while the corresponding values are 15.5 MeV and 0.062 fm^{-3} for the case with $\sigma=0.1$. Thus the critical temperature is lower compared to the zero-range Skyrme interaction. Comparison of the three sets of curves [Figs. 3(a)–3(c)] clearly shows that, for a given temperature and pressure, the Brink-Boeker force gives a value of density significantly lower than those of the two Skyrme interactions. The increase of range will favor a less compact dilute configuration for the system, which will affect the transition to the gaseous phase at lower temperature. Thus the lowering of the critical temperature due to the finite range of the interaction is expected.

Jaqaman *et al.*[14] have compared their equation of state calculated with the Skyrme interaction with the van der Waals equation of state, by writing in the corresponding state form. They have found that these two qualitatively agree. We feel that it is worthwhile to see how much the nuclear system resembles the van der Waals gas with the finite-range Brink-Boeker force, so we use the corresponding state form by changing the variables in the equation of state to the dimensionless quantities

$$t=\frac{T}{T_c},\quad \vartheta=\frac{V}{V_c}=\frac{\rho_c}{\rho},\quad p=\frac{P}{P_c}\ .$$

The equation of state for the van der Waals gas becomes

$$p=\frac{8t}{3\vartheta-1}-\frac{3}{\vartheta^2}\ . \tag{2.7}$$

In Fig. 4 we have plotted p vs ϑ for the van der Waals gas (dashed lines) and for the nuclear system with Brink-Boeker force (solid lines). From the figure it can be seen that, above the critical temperature, they agree quite well. And for temperature below the critical point, the agreement is qualitative. The feature that the nuclear system behaves like a liquid below a certain temperature, like a gas above that temperature, and also remains in a mixed phase for certain range of temperature is being reaffirmed with finite-range force.

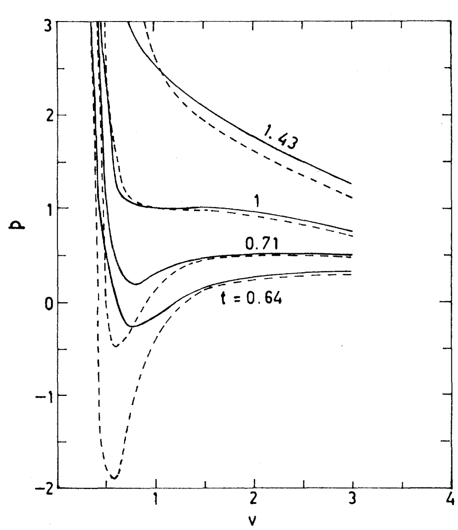

FIG. 4. Law of corresponding states. Comparison of the equation of state for a van der Waals gas (shown by dashed curves) and for a system interacting through the Brink-Boeker force (shown by solid curves).

III. CALCULATIONS FOR FINITE SYSTEM

As discussed earlier, the finite-size effect is expected to influence very significantly the critical properties of the nuclear system. In this section, we calculate the critical temperature and critical density of the nuclear system with Brink-Boeker force for a few hundred nucleons, which will resemble a realistic situation occurring in a heavy-ion reaction. We will also attempt to take the Coulomb interaction into account. We will follow the procedure of Jaqaman *et al.*[15] It has been argued by them that for the finite system, instead of calculating the isotherms P vs ρ, it is advantageous to calculate μ vs ρ. Thus the calculation of the chemical potential for an interacting Fermi system has to be performed. We have done such a calculation, the result of which is presented in Table I. Here, we have listed the critical temperature and critical density obtained for systems with varying neutron number N and proton number Z, keeping $N=Z$. Table I shows, how the finite size of the system affects the critical point. It is seen that, as we go from nucleon number ∞ to nucleon number 200, T_c and ρ_c decrease from 14 MeV to 10 MeV and $0.35\rho_0$ to $0.223\rho_0$, respectively. We have taken $\rho_0=0.17\ \mathrm{fm}^{-3}$. Thus the effect of the finite size of the system is to reduce the critical temperature and critical density. Compared to the infinite nuclear matter, T_c has been reduced by 4 MeV. In Fig. 5, we have plotted several isotherms calculated for the system with $N=150$ and $Z=150$. The isotherms show that the critical temperature is 11 MeV and the critical density is 0.235 ρ_0. Thus the present calculation with finite-range force shows a similar trend, as has been seen in the calculation with the zero-range Skyrme interaction.

We have tried to calculate the effect of Coulomb interaction on critical properties. This has been done in an approximate way by maintaining the neutron-proton symmetry in the $N=Z$ system, as in Ref. 15. Thus the chemical potentials for the neutron and proton are assumed to be same. The inclusion of this effect reduces the critical temperature. For the system with $A=500$, we find $T_c=8$ MeV and $\rho_c=0.223\rho_0$. Thus a further reduction of 3 MeV in the value of T_c is found. A similar

TABLE I. Effect of finite size on the critical point of nuclear matter for the $N=Z$ systems. T_c, ρ_c, and ρ_0 are the critical temperature, critical density, and normal nuclear matter density, respectively.

No. of nucleons	T_c (MeV)	ρ_c/ρ_0
∞	14	0.35
10^6	14	0.35
10^3	12	0.294
500	11	0.235
300	11	0.235
200	10	0.223

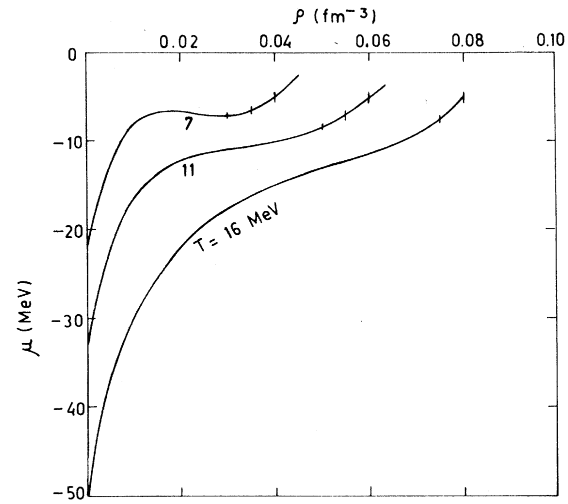

FIG. 5. Isotherms for a system of 150 protons and 150 neutrons interacting via the finite-range Brink-Boeker force. Error bars indicate computational errors involved in summing the required series in the calculation of chemical potential μ. ρ refers to the density.

reduction has been found in Ref. 15. This is very much in conformance with our expectation, since the repulsive Coulomb force will effectively reduce the binding of nucleons and, consequently, with less temperature, the system can go to a gas phase.

IV. DISCUSSION AND CONCLUSION

We have investigated the occurrence of a liquid-gas phase transition in a nuclear system, in the temperature-dependent HF formalism, using a finite-range Brink-Boeker force. Such a force favors a less compact dilute configuration for the system compared to the zero-range force. For the infinite system, we find the critical temperature to be 14 MeV, and the critical density is $0.35\rho_0$. This value of critical temperature is, in general, a few MeV less than those obtained with the zero-range Skyrme interactions. Thus a higher range of the force leads to a lowering of the critical temperature. We have calculated the corresponding state form of the equation of state, and compared it with the same one obtained for the van der Waals gas. A close resemblance to nuclear system is seen for reduced temperature $t \geq 1$. For $t < 1$, the van der Waals system has a much deeper minimum. When the finite-size effect is taken into account, the value of T_c and ρ_c steadily decreases. For mass number $A=200$, we get $T_c=10$ MeV and $\rho_c=0.223\rho_0$. The effect of the Coulomb force on T_c and ρ_c has been estimated approximately. Its effect is to further reduce the critical temperature by a couple of MeV. The Coulomb force reduces the overall binding of the system and hence transition can take place at lower values of T_c. For the system having $A=500$, the value of T_c was found to be 8 MeV. Thus, for a typical heavy ion reaction, the critical temperature will be expected to be around this value. The average binding energy per nucleon in a heavy nucleus is about 8 MeV. Thus the critical temperature obtained here agrees with the binding energy per particle, which approximately may be the case.

Generally, temperature is determined from the measurement of the kinetic energy of the emitted particles. Until recently such measurement usually yielded a value of 15 MeV.[4] This was called the slope temperature. However, recently it has been found that, if one measures the kinetic energy of the particles emitted in backward angle, one obtains a slope temperature of 5–6 MeV.[18,19] These particles truly correspond to the thermally equilibrated compound nuclear system. Such a result is also obtained by measuring the relative populations of the excited states. Analyzing the available experimental data, the limiting temperature of about 6 MeV is found[19] to be reached in the formation of thermally equilibrated fusion nuclei. This shows that finite nuclei can be considered to remain in the liquid state up to around $T=6$ MeV, a value very much in the region of the present result.

The present calculation implies that nuclei will undergo a transition to a gas phase consisting of nucleons at the critical temperature around 8 MeV. Below that, they will remain in the liquid state, which is corroborated from the limiting temperature obtained from the study of hot fusion nuclei.

One must be careful in relating the multifragmentation process to the phenomenon of the liquid-gas phase transition, because, in the former, one will find several fragments of different size which are not free nucleons. While in the liquid-gas phase transition, the gas phase is composed of only free nucleons, which other microscopic calculations and the present study suppose. The mechanism of the formation of various fragments is not contained in these microscopic studies.

The statistical fragmentation model of Bonderf *et al.*[6] deals with the formation of fragments. In this model, the nucleus remains as a compound system below the crack temperature which is about 6 MeV. Thus it will be reasonable to say that this temperature is the limiting temperature below which a nucleus remains in a liquid state. Bondorf *et al.*[6] find another temperature of 11 MeV, above which small clusters with mass number $A \leq 4$ are predominantly formed. In the Gross model,[12] also, at similar temperature, the small clusters are formed. From what has been said above, we are reluctant to attribute this as the transition signaling the phenomenon of the liquid-gas phase transition. We would like to further point out that, in these models, the interfragment interactions have not been taken into account, which may change the picture considerably.[20]

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