Δ component in the nuclear ground state

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The Δ component in the nuclear ground state is evaluated in connection with the Δ dynamics. The various microscopic processes which could originate such components are separately studied. The role of short-range correlations is emphasized, as well as that of random-phase-approximation corrections. It is shown moreover that a rather large number of Δ's in a nucleus does not lead to contradictions with the standard shell model.

I. SETTING THE PROBLEM

It is a well grounded achievement of modern nuclear physics that not only nucleons are relevant in the study of nuclear dynamics, but that mesons and barionic resonances, like Δ's, play an important role too. Surprisingly enough, however, only a small number of theoretical works have been devoted, in the recent years, to the determination of number of Δ's in large[1] as well as in small[2] nuclei, the mainstream of the research being devoted to the study of dynamical properties of Δ's and pions in nuclei in higher-energy regions. Only recently Lipkin and Lee[3] renewed interest in the presence of Δ's in nuclei, suggesting a possible signature of Δ components in the ground state of ^{3}He.

The question, nevertheless, is by no means trivial. In fact, to our knowledge, the only detailed calculation of the ratio N_Δ/N in nuclear matters is that of Ref. 1, which provided the surprisingly high value of about 7.5% (at least). Of course a critical discussion of such an outcome is needed, and is deferred to the end of this section, but we may anticipate (also in view of our results in Sec. III) that the procedure and the results are essentially convincing.

If one takes the naive attitude of generalizing the shell model considering an assembly of nucleons and Δ as a system of independent particles moving in two different potential wells, one for nucleons and one for Δ's, then (since the Pauli principle does not work for different particles) 7.5% of Δ's is clearly too much, as it would completely destroy the shell structure. But in this respect even extremely small numbers should be too much, since, for instance, 0.8% will render 127 a magic number instead of 126.

One is consequently led to examine more refined schemes. For instance, there has been recently proposed[4] a Hartree-Fock-like method for a system of nucleons and Δ's, where the Δ is taken as a stable particle. One should remark that on the one hand this assumption could be questionable (and we shall examine this point in what follows, but our conclusion is that the free Δ width is not a relevant parameter, so enforcing the conclusions of Ref. 4); on the other hand, in Ref. 4 the consequent shell structure is not examined in detail, so that no answer is given to the previous objection.

As a matter of fact a set of works of Bleuler *et al.*[5] regards instead the high number of Δ in nuclear matter as a dramatic drawback of the standard shell model and as a justification to overcome it, building up a shell model composed of quarks instead of nucleons.[6]

However, this is not the only possible way out: we could instead describe a heavy nucleus in terms of a configuration mixing, as on the other hand is already required by the occupation number of the higher shells in heavy nuclei, as is the case of ^{206}Pb, which experimentally turns out to be of about 0.7.[7] Then it may happen that a large Δ component appears in the added configurations without altering the occupation number of the closed shells, but only subtracting strength from the shells above the Fermi level.

There are many questions arising from this brief discussion.

(1) As a first point we need to understand in detail which is the microscopical origin of the result of Ref. 1. Let us remark first of all that their conclusions are strictly linked to two quite satisfactory achievements. The first point is that the *N-N* interaction used in the calculations comes out from a very delicate analysis[8] of the *N-N* scattering in terms of the mesonic theory. Once the interaction has been obtained, then *N-N* phase shifts are fairly described, and a detailed knowledge of the underlying dynamics is available. Particular care has been devoted to the 2-π exchange which is responsible for the most delicate feature of the interaction. In this context a peculiar role is played by the so-called *box diagrams* (see Fig. 1) in which one or two Δ's appear in the intermediate state. The second relevant feature is that within this starting point a good description of the nuclear matter is obtained in lowest-order Brückner theory.[9] In particular it is emphasized in Ref. 9 that a large contribution to the binding comes from the previously quoted box diagrams. Then the number of Δ's easily follow by deriving the binding energy with respect to the chemical potential of the Δ which in our case is in practice just the mass difference $M_\Delta - M$.

Looking to the disadvantages of the approach, we may remark that the parameters (coupling constants and cutoffs of the form factors) are essentially determined

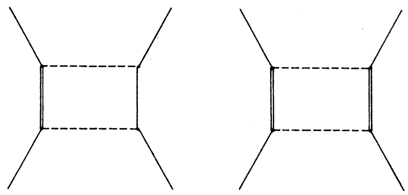

FIG. 1. Box diagrams: (a) with one intermediate Δ, (b) with two intermediate Δ's. Dashed lines denote both π and ρ.

from a best fit on the phase shifts on elastic N-N scattering, while more detailed information could be obtained from other reactions involving mesons. For example, in Refs. 1 and 9 the coupling constant for the $\pi N\Delta$ vertex is given by $f^2_{\pi N\Delta}/4\pi=0.27$ and the $\rho N\Delta$ vertex follows in the strong-coupling scheme ($C_\rho=2.3$) according to Höler and Pietarinen.[10] The cutoffs are of about 1200 MeV/c for both vertices. All these parameters preserve a good description of N-N phase shifts. Now, the $\rho N\Delta$ vertex is of course badly known, but for the first one, instead, better information should come from π-N scattering. Here a recent fit[11] which takes into account in a rather refined way the nucleon Born term and one-loop corrections leads just to a cutoff of about 700 MeV/c but with a larger coupling constant (0.42, to be compared with the previous 0.27 and the Chew-Low value 0.32, which is used throughout this work). Previous works of the Bonn group[12] used indeed a smaller cutoff, without destroying the agreement with experimental phase shifts. We cannot give any insight for what concerns the relevance of the increase of the coupling constant.

Another drawback of this scheme is intrinsic to the complications of the calculations, since of course much information is lost about the relevance of the elementary ingredients involved. In this sense it is worthwhile to consider a complementary point of view, in which one avoids the heavy numerical calculations involved in the solution of the Bethe-Goldstone equation, simulating them by means of parameters of straight physical meaning, but keeping continuously under control the elementary ingredients and their connection with the nuclear dynamics.

(2) As we have remarked above, one could enquire about the role of the Δ width in this context. There are two different aspects of the question: in fact the Δ has by itself a noteworthy width in the vacuum, and on the other hand many-body effects may strongly affect the width in the medium.[13,28] As far as we are concerned with the ground state, we shall show that the free width is ineffective (then justifying the approach of Ref. 4). We shall instead examine in detail the medium-induced effects, which are directly connected with the ratio N_Δ/N.

(3) The last point concerns the prevision we could obtain from the analysis of the momentum distribution of the Δ's and nucleons inside the nuclear matter (which is the counterpart of the occupation number in finite nuclei). For instance our previous naive model should correspond to a momentum distribution for the Δ peaked around small momenta, while nucleon momentum distributions should remain similar to a θ function with reduced Fermi momentum. In this case we should inevitably give up the standard shell model and look for new schemes based, say, on QCD. On the contrary, should we find a distribution slowly decreasing up to k_F and with a reduced tail above k_F, then no decisive breakdown of the usual mesonic theory of nuclear forces should occur, the difficulties of this approach (which remain nonetheless noteworthy) being instead on the level of sophistication required in handling the nuclear many-body problem and in the too-high number of parameters (coupling constants, cutoffs) involved, which are often neither well determined nor of clear physical meaning.

In Sec. II we shall briefly describe the formalism used throughout the paper; in Sec. III we shall describe some microscopical models to evaluate the ratio N_Δ/N in strict connection with the Δ dynamics inside the nuclear medium. In Sec. IV the momentum distribution is examined.

II. GENERAL FORMALISM

Our approach starts from the elementary consideration that the average of the number operator $\int d^3x\, \psi^\dagger_\Delta(x)\psi_\Delta(x)$ is connected to the Δ Green's function by

$$\frac{N_\Delta}{V}=\langle\Psi_0|\psi^\dagger_\Delta(x)\psi_\Delta(x)|\Psi_0\rangle = -i\lim_{\substack{t'\to t^+\\ \mathbf{r}'\to\mathbf{r}}}\mathrm{Tr}G_\Delta(\mathbf{r}',\mathbf{r},t'-t) = -i\,\mathrm{Tr}\int\frac{d^4p}{(2\pi)^4}G_\Delta(p)e^{i\eta p_0} \tag{1}$$

(the traces refer to spin-isospin sums). The Δ Green's function is furthermore conveniently expressed by means of a Dyson's equation

$$G_\Delta(p)=G^0_\Delta(p)+G^0_\Delta(p)\Sigma_\Delta(p)G_\Delta(p) \tag{2}$$

with $G^0_\Delta(p)$ defined as

$$G^0_\Delta(p)=\frac{1}{p^0-\dfrac{p^2}{2M_\Delta}-\delta M+i\eta} \tag{3}$$

(where of course $\delta M=M_\Delta-M$). The connection between N_Δ and the Δ dynamics is evident by itself. The connection with the nuclear binding energy follows by observing that the diagrammatic meaning of Eq. (1) is that of a Δ Green's function closed on itself. Topologically this may by visualized as the class of all closed diagrams with a dot on a Δ line. Then each diagram is easily put in a 1-1 correspondence with the class of diagrams containing at least one Δ line of the conventional Brückner theory for binding energy, by simply squaring one energy denominator containing a Δ. Since the presence of a Δ in a denominator is always accompanied by a factor δM we simply conclude that

$$N_\Delta = -\frac{\partial}{\partial(\delta M)}\text{B.E.} \tag{4}$$

Equation (4) translates the relations (2.17) and (2.18) of Ref. 1.

The same relation may also be obtained in a more compact way as follows. Consider the partition function in the canonical ensemble:

$$Z = \text{Tr}e^{-\beta H} . \tag{5}$$

The binding energy for the ground state is easily recognized to be

$$\text{B.E.} = -\lim_{\beta\to\infty}\frac{\ln Z}{\beta} . \tag{6}$$

On the other hand Z admits the functional-integral representation

$$Z = \int \mathcal{D}\psi^* \mathcal{D}\psi \exp\left[-\int_0^\beta d\tau\, H(\tau)\right] . \tag{7}$$

The structure of H is of course the following: it will contain (in the nonrelativistic limit) a kinetic part, an interaction term (whatever it will be) and a term

$$\int d^3x\, \delta M \psi_\Delta^\dagger(\mathbf{x})\psi_\Delta(\mathbf{x})$$

which accounts for the mass difference between nucleon and Δ. From this it follows that

$$\frac{\partial}{\partial(\delta M)}\ln Z = \int_0^\beta d\tau \int d^3x \langle n_\Delta(\mathbf{x},\tau)\rangle , \tag{8}$$

where of course $\langle n_\Delta(\mathbf{x},\tau)\rangle$ is the average value of the Δ density in a thermodinamical sense. In the limit $\beta\to\infty$, on the one hand, $n_\Delta(\mathbf{x},\tau)$ becomes constant in time, and on the other hand only the ground state contributes to the average: this means

$$\lim_{\beta\to\infty}\frac{1}{\beta}\frac{\partial}{\partial(\delta M)}\ln Z = \int d^3x\, n_\Delta(\mathbf{x}) . \tag{9}$$

Comparing Eqs. (6) and (9) the result (4) follows at once.

Until now, our results are exact. Now we need some approximations, and we have the choice of approximating either the binding energy or the Δ self-energy. As far as binding energy is considered, we shall be concerned in practice with lowest-order perturbation theory only (at least formally), as we shall see in detail in the next section (provided that some suitable approximations have been introduced).

We shall give here some more comments on the Δ self-energy in order to clarify how some results of the next section have been obtained. In particular we shall be concerned with self-energy diagrams like those of Fig. 2. The potential employed in the next section will be either a one-pion or one-ρ exchange plus a Landau-Migdal parameter. In the case of pion we shall consider

$$V_{\text{eff}}(q,q_0) = \frac{f_{\pi NN}^2}{m_\pi^2}\left[g' + \frac{q^2}{q_0^2 - q^2 - m_\pi^2}\right] v^2(q^2) \tag{10}$$

(for the ρ exchange only trivial changes are required, which we shall avoid here for sake of brevity). Then the diagram of Fig. 2(a) will read

$$\Sigma(p) = i\frac{f_{\pi n\Delta}^2}{f_{\pi NN}^2}\int \frac{d^4q}{(2\pi)^4}\mathbf{T}\cdot\mathbf{T}^\dagger(\hat{\mathbf{q}}\cdot\mathbf{S})(\hat{\mathbf{q}}\cdot\mathbf{S}^\dagger) \times G^0(p-q)V_{\text{eff}}^2(q,q_0)\Pi(q,q_0) , \tag{11}$$

where Π (hatched bubble) denotes both a particle-hole and a Δ-hole polarization propagator. Exploiting the traces inside Π we find

$$\Pi(q,q_0) = \left[4\Pi_0(q,q_0) + \frac{16}{9}\frac{f_{\pi N\Delta}^2}{f_{\pi NN}^2}\Pi_\Delta(q,q_0)\right] . \tag{12}$$

We shall use throughout the Chew-Low value $f_{\pi N\Delta} = 2f_{\pi NN}$. The self-energy of Fig. 2(b) is simply obtained by the replacement

$$\Pi(q,q_0) \to \frac{\Pi(q,q_0)}{1 - V_{\text{eff}}(q,q_0)\Pi(q,q_0)} = \Pi_{\text{RPA}}(q,q_0) \tag{13}$$

in Eq. (11).

Once an approximation for the self-energy has been given, we put it into the expression for G_Δ,

$$G_\Delta(p) \approx [G_\Delta^0(p)]^2\Sigma_\Delta(p) \tag{14}$$

(the reason why we do not like to use the exact Dyson's equation for G_Δ will be clarified later).

A nontrivial point (in principle) is the evaluation of the matrix products: we have[14]

$$\langle \tfrac{3}{2}, T|\mathbf{T}\cdot\mathbf{T}^\dagger|\tfrac{3}{2}, T'\rangle = \delta_{T,T'} \tag{15}$$

and

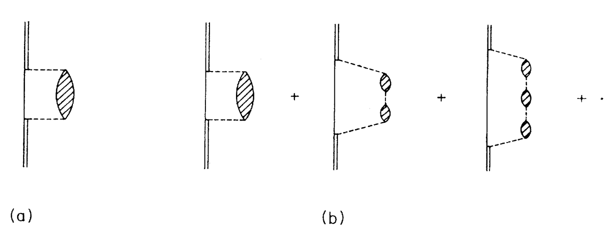

FIG. 2. Self-energy diagrams for the Δ: (a) to the second order, (b) in RPA. Dashed lines may denote both π and ρ.

$$\langle \tfrac{3}{2},M|(\mathbf{q}\cdot\mathbf{S})(\mathbf{q}\cdot\mathbf{S}^{\dagger})|\tfrac{3}{2},M'\rangle = q^2\delta_{M,M'}\sum_{\lambda=0,2}(-1)^{M-\frac{1}{2}}4(2\lambda+1)\begin{bmatrix}1 & 1 & \lambda\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\frac{3}{2} & \frac{3}{2} & \lambda\\ -M & M & 0\end{bmatrix}\begin{Bmatrix}\frac{3}{2} & \frac{3}{2} & \lambda\\ 1 & 1 & \frac{1}{2}\end{Bmatrix}P_{\lambda}(\cos\widehat{\mathbf{pq}}) . \tag{16}$$

Here the term with $\lambda=0$ turns out to be simply $q^2/3$, while the tensor term has the structure

$$\text{cost}\cdot\begin{bmatrix}\frac{3}{2} & \frac{3}{2} & 2\\ M & -M & 0\end{bmatrix} = \text{cost}\cdot\frac{(-1)^{3/2-M}}{2} .$$

However, Eq. (1) requires a trace over M at the end of the calculation, then the tensor term is clearly averaged to 0. This lucky circumstance enables us to express the self-energy, in this approximation, as

$$\Sigma(p,p_0) = \frac{i}{3}\frac{f^2_{\pi N\Delta}}{f^2_{\pi NN}}\int\frac{d^4q}{(2\pi)^4}\left[\frac{\theta(|\mathbf{p}-\mathbf{q}|-k_F)}{p_0-q_0-\epsilon(\mathbf{p}-\mathbf{q})+i\eta}+\frac{\theta(k_F-|\mathbf{p}-\mathbf{q}|)}{p_0-q_0-\epsilon(\mathbf{p}-\mathbf{q})-i\eta}\right]V^2_{\text{eff}}(q,q_0)\Pi(q,q_0) \tag{17}$$

with $\epsilon(p)=p^2/2m$.

This quantity has to be inserted into Eq. (14) and then in (1), the integration over p_0 being trivial. To render the evaluation of the ratio N_Δ/N more reliable from a numerical point of view it is convenient to make the substitution $\mathbf{p}\rightarrow\mathbf{p}+\mathbf{q}$ and to perform a Wick rotation on the integration variable q_0. We end up with the expression

$$\frac{N_\Delta}{N} = \frac{-2}{\pi^3k_F^3}\frac{f^2_{\pi N\Delta}}{f^2_{\pi NN}}\int_0^{k_F}p^2dp\int_0^{\infty}q^2dq\int_{-\infty}^{\infty}dq_0\frac{V^2_{\text{eff}}(q,iq_0)\Pi(q,iq_0)}{\left[iq_0-\frac{p^2+q^2}{2M\Delta}-M_\Delta+M+\frac{p^2}{2M}\right]^2-\frac{4p^2q^2}{M^2_\Delta}} \tag{18}$$

with the nontrivial gain, from a computational point of view, that the functions $V_{\text{eff}}(q,iq_0)$ and $\Pi(q,iq_0)$ are real, smooth, and with definite sign.

Of course this procedure applies equally well to the diagram of Fig. 2(b). This approximation is also theoretically appealing, since it has been proven[15] that a consistent theory may be built up with an effective Lagrangian, containing only bosons dressed with the random-phase-approximation (RPA) series interacting via some "elementary" vertices with all the nucleon dynamics, and in such a theory our RPA self-energy amounts to the one-loop approximation.

Two points remain to be explained. First of all we want to justify the convenience of the approximation (14). As a matter of fact, had we chosen to set

$$G_\Delta(p) = \frac{G^0_\Delta(p)}{1-G^0_\Delta(p)\Sigma(p)} , \tag{19}$$

then a Wick rotation should no longer be possible. As a consequence we should have to do with an integral (over p_0) of a highly singular function. This task is not impossible, but the numerical errors so obtained come out to be comparable with the (small) numbers we are looking for.

The last remark is that in any case we always started with a second-order (at least) term. It remains to be explained why the first-order contribution has to be discarded. As a matter of fact, if we make use of Eq. (4) we may consider such a term as the derivative of the diagram of Fig. 3, which in turn may be described either as an approximation to $[G^0_\Delta(p)]^{-1}G_\Delta(p)$ integrated over all p or to $[G^0(p)]^{-1}G(P)$ integrated over the Fermi sea. In this second point of view the corresponding approximation for $G(p)$ is, of course,

$$G(p)=G_0(p)\Sigma^{(1)}(p)G_0(p) ,$$

with $\Sigma^{(1)}(p)$ given in Fig. 4. It is clear that such a self-energy contribution is nothing but a renormalization of the nucleon propagator, and if we choose to neglect, as it is reasonable, all the effects which are not brought about by the nuclear medium we may renormalize the theory by subtracting to the self-energy its value taken at $k_F=0$.[16] In this way the finite part of the diagram vanishes and of course its derivative too.

There remains to be proven two formulas needed in Sec. IV concerning the occupation number of nucleons and Δ's. As far as Δ's are concerned, the occupation number is nothing but $\langle\Psi_0|\tilde{\psi}^\dagger_\Delta(p)\tilde{\psi}_\Delta(p)|\Phi_0\rangle$ [$\tilde{\psi}_\Delta(p)$ being the Fourier transform of $\psi_\Delta(x)$], normalized in such a way to coincide with the residuum of the propagator. In practice we simply drop in Eq. (18) the integration over d^3p and change the normalization factor, getting (in RPA approximation)

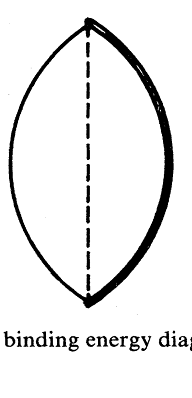

FIG. 3. First-order binding energy diagram with a Δ line.

FIG. 4. First-order self-energy diagram for the nucleon, corresponding to the B.E. diagram of Fig. 3.

$$n_\Delta(p)=\frac{1}{3}\frac{1}{(2\pi)^3}\int_0^{k_F} q^2dq\int_{-1}^{1} d\cos\widehat{\mathbf{pq}}\int_{-\infty}^{\infty} dq_0\frac{f^2_{\pi N\Delta}}{f^2_{\pi NN}}\frac{V^2_{\rm eff}(|\mathbf{p}+\mathbf{q}|,iq_0)\Pi_{\rm RPA}(|\mathbf{p}+\mathbf{q}|,iq_0)}{\left[iq_0-\delta M-\frac{(\mathbf{p}+\mathbf{q})^2}{2M_\Delta}+\frac{q^2}{2M}\right]^2} . \tag{20}$$

An analogous formula shall be used in Sec. IV for determining the occupation number of nucleons. This is a much more difficult task, which will be examined in more detail in a subsequent paper. Here we shall limit ourselves to give $n(p)$ in a similar approach assuming that nucleon self-energy has the same structure as the Δ's one, i.e., receives its dominant contribution from the RPA-dressed pion. So we may simply change some trivial spin-isospin factors in (20) and recall the different analytical structure of the nucleon propagator which may also have a hole part. Following the same path as before an easy calculation provides

$$n(p)=\theta(k_F-p)+\frac{1}{4}\int\frac{d^4q}{(2\pi)^4}\frac{\theta(k_F-|\mathbf{p}-\mathbf{q}|)-\theta(k_F-p)}{(iq_0-\epsilon_p+\epsilon_{q-p})^2}V^2(q,iq_0)\Pi_{\rm RPA}(q,iq_0) . \tag{21}$$

Of course other mechanisms are relevant in the determination of $n(p)$. However, our aim here is to understand how the momentum distribution is altered by the presence of Δ's in the ground state (if the nucleons are subtracted from a small region around k_F or if the high-momentum tail is depleted). For this reason we shall only concentrate our attention on that particular dynamics which is also responsible for the presence of Δ's, i.e., those self-energy diagrams having the same intermediate states as the Δ self-energy previously considered.

III. NUMBER OF Δ's IN NUCLEAR MATTER

We come in this section to the evaluation of N_Δ. To start with, we need an approximation for the Δ self-energy, at some level of complexity.

The first job we shall deal with is to examine critically the results of Ref. 1, in order to achieve a better understanding of its dynamical content. The approximation used there is the so-called Brückner-Hartree-Fock theory. In our scheme this amounts to considering the two diagrams of Fig. 5, where the hatched blocks both represent a G matrix built up from the Bonn potential, including box diagrams. Of course the presence of two G matrices does not amount to a double counting (even if the diagram is regarded as a binding energy diagram, i.e., closing the Δ line on itself), but instead selects from all the summed graphs those containing at least one box diagram.

Here, in order to reduce the calculation to its elementary ingredients, we shall consider first of all a crude approximation, by simply replacing the G matrix with a π or ρ exchange. Clearly the box diagrams appear here as the central objects of our problem. Let us, for instance, define

$$V_\pi^{\Delta\Delta}=-\frac{f^2_{\pi N\Delta}}{m_\pi^2}\frac{(\mathbf{S}_1\cdot\mathbf{q})(\mathbf{S}_2\cdot\mathbf{q})}{q^2+m_\pi^2}\mathbf{T}_1\cdot\mathbf{T}_2 , \tag{22}$$

the $2N$-2Δ transition potential originated by a one-pion exchange (the ρ exchange potential as well as the cases of NN-$N\Delta$ transitions are obvious generalizations). The potentials are presently thought of as static. Then the two-pion exchange box diagram with two intermediate Δ's may be written as

$$\langle p,k|V_{\Delta\Delta}^{\rm box}|p',k'\rangle=V_{\Delta\Delta}^{\rm box}(q;p,k)=\int\frac{d^3t}{(2\pi)^3}\frac{[V_\pi^{\Delta\Delta}(t)]^\dagger V_\pi^{\Delta\Delta}(t+q)}{\frac{p^2}{2M}+\frac{k^2}{2M}-2\delta M-\frac{(p'+t)^2}{2M_\Delta}-\frac{(k-q-t)^2}{2M_\Delta}} , \tag{23}$$

where $q=p'-p=k-k'$ denotes the transferred 4-momentum. Of course we avoid for sake of simplicity to give the corresponding potential $V_{N\Delta}^{\rm box}$ for a $N\Delta$ intermediate state, as well as for the 2-ρ exchange, but the generalization is quite obvious.

The number of Δ's turns out to be

$$n_\Delta=-\frac{\partial}{\partial(\delta M)}{\rm Tr}\int\frac{d^3p}{(2\pi)^3}\theta(k_F-p)\int\frac{d^3k}{(2\pi)^3}\theta(k_F-k)[V_{\Delta\Delta}^{\rm box}(0;p,k)+V_{N\Delta}^{\rm box}(0;p,k)] \tag{24}$$

(the trace being taken over spin and isospin variables).

The next step will be to simplify the so obtained potential in such a way to render them local. There are two simple possibilities. The first one is to assume the barions as static, so that previous expressions drastically simplify to

$$n_\Delta=\frac{k_F^6}{36\pi^4}{\rm Tr}\left[\frac{1}{2\delta M^2}\int\frac{d^3t}{(2\pi)^3}[V_{\pi,\rho}^{\Delta\Delta}(t)]^\dagger V_{\pi,\rho}^{\Delta\Delta}(t)+\frac{1}{\delta M^2}\int\frac{d^3t}{(2\pi)^3}[V_{\pi,\rho}^{N\Delta}(t)]^\dagger V_{\pi,\rho}^{N\Delta}(t)\right] . \tag{25}$$

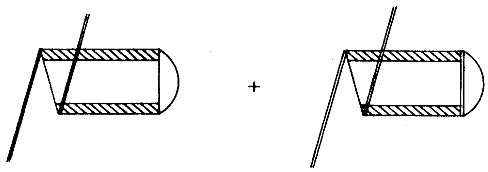

FIG. 5. Self-energy diagrams contributing to N_Δ. Hatched blocks correspond to a G matrix. These diagrams have to be regarded as Goldstone diagrams.

It is obviously expected to be a failure of this scheme, and it happens indeed, since if we use standard expressions for π and ρ exchange, then after traces are performed they behave like

$$\frac{t^2}{t^2+m^2_{\pi,\rho}} ,$$

then implying a bad divergence of the integral. The reason for which we have nevertheless considered this approximation is that we believe that much physics may be understood from such a failure. There are of course three possible reasons for this breakdown. (1) We have neglected here any form factor at the elementary vertices. (2) We have neglected the kinetic energy of the baryons. (3) We have ignored short-range correlations.

The first point is obviously inadequate to solve our problem: it is clear that a form factor will ensure convergence, but the idea that only the internal structure of the nucleon will prevent the number of Δ's from being infinite seems to be really unlikely.

The second point is more subtle, but we should consider, at least for pion exchange, that the pion mass, which plays the role of the range of the interaction, should act in practice as a cutoff, and for momenta of this order of magnitude the kinetic energy of the barions are indeed negligible as compared with δM. The question is of course more delicate for the the ρ exchange because momenta are cut to a much higher value, but nevertheless it is irksome that convergence has to be ensured by barion kinematics. The crucial point is of course the third, since clearly the bad behavior of the potential for $q \to \infty$ reflects the fact that in configuration space the one-pion exchange potential (OPEP) has the well-known pathology of a $\delta(\mathbf{r})$-like contribution, which is rendered physically ineffective by short-range correlations.

Let us first of all examine the points (1) and (2). We may of course improve the convergence by taking into account, even in an approximate way, the barion kinematics. If we imagine that in the model we are considering the external momenta remain small (and in practice we need only momenta cut by k_F) then we could neglect them in the energy denominators, thus again rendering local the potential V^{box}:

$$V^{\text{box}}_{\Delta\Delta}(q) \equiv -\int \frac{d^3t}{(2\pi)^3}[V^{\Delta\Delta}_{\pi,\rho}(t)]^\dagger \frac{1}{2\delta M+\dfrac{t^2}{M_\Delta}} V^{\Delta\Delta}_{\pi,\rho}(t+q) , \tag{26}$$

TABLE I. N_Δ/N in nuclear medium without correlations and different kinematics.

Static approximation	$\Lambda=\infty$	$\Lambda=1300$ MeV/c	$\Lambda=800$ MeV/c
$P^{\Delta\Delta}(\pi)$	∞	9.9%	3.9%
$P^{N\Delta}(\pi)$	∞	20.8%	7.6%
$P^{\Delta\Delta}(\rho)$	∞	47.1%	10.0%
$P^{N\Delta}(\rho)$	∞	> 100%	22.0%
Simplified kinematic			
$P^{\Delta\Delta}(\pi)$	10.9%	4.5%	2.3%
$P^{N\Delta}(\pi)$	10.5%	5.1%	2.6%
$P^{\Delta\Delta}(\rho)$	65.2%	11.7%	3.8%
$P^{N\Delta}(\rho)$	47.7%	11.5%	4.2%
Full kinematics (static mesons)			
$P^{\Delta\Delta}(\pi)$	7.0%	4.8%	2.5%
$P^{N\Delta}(\pi)$	7.5%	5.3%	2.8%
$P^{\Delta\Delta}(\rho)$	70.0%	12.6%	4.1%
$P^{N\Delta}(\rho)$	51.9%	12.5%	4.5%
Full kinematics (dynamical mesons)			
$P^{\Delta\Delta}(\pi)$	11.2%	4.5%	2.3%
$P^{N\Delta}(\pi)$	10.7%	5.1%	2.7%
$P^{\Delta\Delta}(\rho)$	67.2%	12.8%	4.2%
$P^{N\Delta}(\rho)$	49.8%	12.3%	4.4%

$$V_{N\Delta}^{\text{box}}(q) \equiv -\int \frac{d^3t}{(2\pi)^3}[V_{\pi,\rho}^{N\Delta}(t)]^\dagger \frac{1}{\delta M + \frac{t^2}{2M} + \frac{t^2}{2M_\Delta}} \times V_{\pi,\rho}^{N\Delta}(t+q) \ . \quad (27)$$

These potentials are again divergent but less than before. However, inserting them into (24), the derivation with respect to δM and the nonrelativistic kinematics ensure the finiteness of n_Δ.

In Table I the number of Δ's are given in different conditions, beginning from the simplest cases, namely the explicit evaluation of uncorrelated box diagrams within different approximation schemes. We have in fact used both the approximations (25) and (26) and (27) and we have compared them with calculations containing the complete kinematics [along the lines of Eq. (18)] both with static and nonstatic mesons.

Of course a proviso is needed: It must be reminded indeed that the various perturbative schemes employed here in general are not able to preserve the constraint $0 < N_\Delta / N < 1$. Correlations and RPA schemes could, for instance, allow negative values of N_Δ / N and it is not always warranted for this ratio to be smaller than 1. If such a result happens, we must regard it simply as a bad failure of the approximation scheme.

We see that this happens for the ρ exchange in the static case, as could be expected. It is to be remarked that while the case of infinite cutoff leads either to divergences or to completely unrealistic numbers, a sufficiently small cutoff is able to make them at least likely. This in our opinion is somewhat suspicious. We think in fact that the relevant physical effect should not be cutoff dependent, and that the form factor should only be responsible for small adjustments. In this sense the values of Table I for $\Lambda = \infty$ means that the uncorrelated potential is quite unrealistic even if one could artificially strongly reduce the ratio N_Δ / N by operating on the form factor.

A further relevant point which comes out from this table is that the approximation (26) and (27) seems to work quite well. Moreover, contrarily to the previsions of Ref. 17, the effect of the energy dependence in the meson propagator is essentially negligible.

Next we come to the problem of short-range correlations. The reason for which correlations must be included is that of course when the nucleons exchange one or more particles, or when box diagrams occur, the nucleons

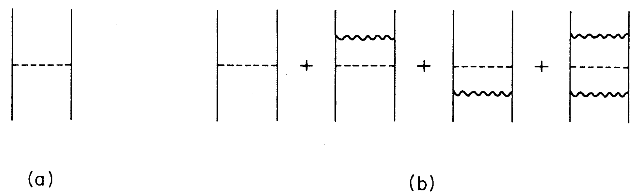

FIG. 6. Diagrams corresponding to short-range correlations: (a) Uncorrelated diagram; dashed line may describe π or ρ exchange or a box diagram. (b) Correlation contributions; wiggly line corresponds to a G matrix built up from ω-meson-exchange potential.

can never be too near one another because the exchange of many ω mesons provides to keep them far apart. In other words the diagrams of Fig. 6(a) must be replaced by the sum of diagrams described in Fig. 6(b). It may be shown that this sum corresponds to evaluate the potential (23) not between plane waves, but between the corresponding states which are solutions of the Bethe-Goldstone equation for the ω-exchange potential.[18] In practice, however, the simplest attitude in the current literature is customarily to multiply the potential (which is required to be local) by the pair correlation function, either deduced from a G-matrix calculation or parametrized in some way.[19] A very appealing parametrization of the pair correlation function[19] is

$$g(r) = 1 - j_0(q_c r) \Longrightarrow \tilde{g}(q) = \delta^{(3)}(\mathbf{q}) - \frac{2\pi^2}{q_c^2}\delta^{(1)}(q - q_c) \quad (28)$$

with q_c roughly of the order of the ω mass. In practice including correlations amounts, more or less, to substituting a potential $V(q)$ with

$$\tilde{V}(q) = \int \frac{d^3k}{(2\pi)^3}\tilde{g}(\mathbf{q}+\mathbf{k})V(k) \ . \quad (29)$$

If we follow the same attitude adopted in Refs. 8 and 9 then we should consider $V^{\text{box}}(q)$ as a piece of the potential, which needs to be correlated by means of the Bethe-Goldstone equation in the case of microscopic calculations or by means of Eq. (29) if we want a simple parametrization.

Our case is particularly simple since $V^{\text{box}}(q)$ is needed at $q=0$ only: one gets, for instance, in absence of barion kinematics,

$$V_{\Delta\Delta}^{\text{box}}(0) = -\frac{1}{2\delta M}\int \frac{d^3t}{(2\pi)^3}[V_{\pi,\rho}^{\Delta\Delta}(t)]^\dagger \left[V_{\pi,\rho}^{\Delta\Delta}(t) - \frac{1}{4\pi q_c^2}\int d^3k \, V_{\pi,\rho}^{\Delta\Delta}(\mathbf{t}+\mathbf{k})\delta(k - q_c) \right] . \quad (30)$$

It is clearly seen that the convergence is improved since both terms have the same asymptotic behavior. Since each one $\rightarrow 1$ for $t \rightarrow \infty$, then the difference goes like t^{-2}. But the original divergence was like $\int t^2 dt$ and we are left with $\int dt$ only. In this approximate scheme then both binding energy and the number of Δ are unsatisfactorily determined. Again we may invoke nuclear kinematics in order to make them convergent. The formula will be quite similar to Eq. (30) with $1/\delta M$ replaced by the energy denominator:

$$V_{\Delta\Delta}^{\text{box}}(0) = -\int \frac{d^3t}{(2\pi)^3} \frac{V_{\pi,\rho}^{\Delta\Delta}(t)}{2\delta M + \frac{t^2}{M_\Delta}} \left[V_{\pi,\rho}^{\Delta\Delta}(t) - \frac{1}{4\pi q_c^2}\int d^3k \, V_{\pi,\rho}^{\Delta\Delta}(\mathbf{k}-\mathbf{t})\delta(k - q_c) \right] . \quad (31)$$

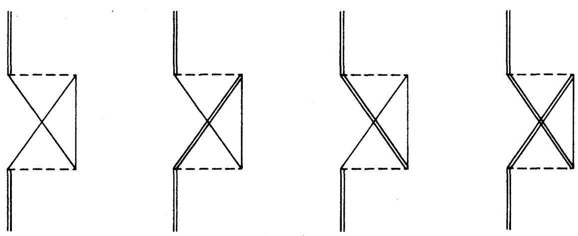

FIG. 7. Some second-order self-energy diagrams neglected in the present approach.

The next step should obviously be to evaluate the number of Δ's in the approximation (31). This time, however, at least in the limit of $\Lambda\to\infty$, we have to do with negative values of V^{box} which in turn entail $N_\Delta/N<0$. Even for finite Λ may results remain unreasonable.

There are many possible origins for this breakdown: one may question, for instance, the oversimplified way of introducing correlations, moreover we cannot forget the already quoted difference between a two-body correlation function and a correlated wave function; furthermore some less relevant diagrams, like those of Fig. 7 have been neglected; and finally we cannot give any insight of what happens when the correlated box diagrams are iterated in a full G-matrix calculation. All these considerations, however, cannot hide an important drawback of this scheme, which is, however, intrinsic to the approach of Refs. 8 and 9, namely the fact that the Δ dynamics is all confined inside the box potentials and has been limited to the 2π (or eventually 2ρ) exchange.

As a matter of fact, what is neglected is that not only the barions before and after the occurrence of the interaction (meson exchange or box diagram) must be kept distant by the action of short-range correlations, but a strong repulsive interaction is also expected between the Δ's inside the box diagram itself. In our simplified model this new feature is introduced by correlating the meson exchange instead of the box potential on the whole.

The analytical expression turns out to be

$$V^{\text{box}}_{\Delta\Delta}(0)=-\int\frac{d^3t}{(2\pi)^3}\frac{1}{2\delta M+\dfrac{t^2}{M_\Delta}}\left[V^{\Delta\Delta}_{\pi,\rho}(t)-\frac{1}{4\pi q_c^2}\int d^3k V^{\Delta\Delta}_{\pi,\rho}(\mathbf{k}-\mathbf{t})\delta(k-q_c)\right]^2 \tag{32}$$

with the obvious gain that this time V^{box} is negative-definite. But another nontrivial achievement is that also the convergence of the integral is improved. As a consequence we expect a strongly reduced effect both for what concerns N_Δ/N and the binding energy. The results obtained from Eq. (32) are given in Table II for two different values of q_c.

It is worthwhile to note at this point that our considerations seems to go beyond the purposes of our paper and suggest that the relevance of the box diagrams in evaluating the nuclear binding energy is overestimated in the previously quoted infinite nuclear matter calculation using the Bonn potential, and that a more coherent calculation would require the explicit introduction of the Δ degree of freedom inside the Bethe-Goldstone equation, together with a repulsive (ω-meson exchange) Δ-Δ and N-Δ interaction.

Coming back to Table II we see first of all that for the most reasonable choice of the parameters ($q_c=800$ MeV/c and $\Lambda=1300$ MeV/c) we find a result analogous to that of Ref. 1. The difference is, however, that in the present calculation the most relevant contribution comes from ρ-meson exchange, for which we expect some reduction once a G-matrix calculation is performed. In this sense the results of Table II may be considered as an overestimate of the true results.

These results are, in our opinion, interesting in that they show the relevance of including the short-range

TABLE II. N_Δ/N in nuclear medium with correlated meson exchanges.

$q_c=500$ MeV/c	$\Lambda=\infty$	$\Lambda=1300$ MeV/c	$\Lambda=800$ MeV/c
$P^{\Delta\Delta}(\pi)$	0.26%	0.17%	0.28%
$P^{N\Delta}(\pi)$	0.18%	0.15%	0.3%
$P^{\Delta\Delta}(\rho)$	6.7%	1.8%	0.47%
$P^{N\Delta}(\rho)$	7.6%	1.9%	0.44%
P^{tot}	14.8%	4.04%	1.48%
$q_c=800$ MeV/c			
$P^{\Delta\Delta}(\pi)$	0.31%	0.62%	0.93%
$P^{N\Delta}(\pi)$	0.26%	0.68%	1.1%
$P^{\Delta\Delta}(\rho)$	2.8%	1.8%	0.62%
$P^{N\Delta}(\rho)$	26.6%	2.9%	0.59%
P^{tot}	50.7%	7.0%	3.2%

TABLE III. N_Δ/N in nuclear medium with RPA correlations.

g'	$P_\pi^{(2)}$	$P_\rho^{(2)}$	$P_{\rm tot}^{(2)}$	$P_\pi^{\rm RPA}$	$P_\rho^{\rm RPA}$	$P_{\rm tot}^{\rm RPA}$
0.5	1.61%	7.65%	9.26%	3.34%	12.55%	15.89%
0.6	0.83%	7.58%	8.41%	1.26%	8.16%	9.42%
0.7	0.33%	8.65%	8.98%	0.4%	6.26%	6.66%
0.8	0.11%	10.86%	10.97%	0.05%	5.61%	5.66%

correlations which modify the asymptotic (high q) behavior of the one-meson exchange potential and that they bring into play the range of the correlations (q_c^{-1}) as the relevant parameter of the model. They are instead not yet satisfactory in that other relevant dynamics has been neglected. To make a simple example, we remind that a correlated pion may propagate in the spin-transverse channel as well and that the correlated ρ meson may propagate in the spin-longitudinal one.[20] That is, however, not the whole story, since other diagrams, like those of Refs. 21 and 22 are also relevant, together with other more complicated and mostly unexplored processes.

The customary attitude of nuclear physicists, besides the quoted cases[21,22,20] in which some attempts have been done of evaluating microscopical processes, is to separate the interaction as a pure one-meson (π,ρ) exchange and an effective interaction including each other diagram but the single-meson exchange and to parametrize the latter by means of Landau-Migdal parameters, along the lines of Eq. (10). Of course the Landau parameters should be different, in principle, for the spin-longitudinal (pion) channel and for the spin-transverse (ρ) one. In practice, for instance, the longitudinal interaction should account for the correlations to the pion exchange coming from the correlations, as discussed above, but it should also include that part of the correlated ρ exchange which propagates in the longitudinal channel, and moreover it should also embody all those diagrams we have neglected so far. The same of course holds for the transverse interaction. It is well known, however, that both the longitudinal (g_L') and transverse (g_T') Landau parameter must have the same limit (g') when the transferred momentum is vanishing. If we assume for the sake of simplicity that the momentum dependence of the Landau parameters is slow, they may be safely assumed as coincident. Moreover it has been outlined many times[23,24] that different values of g' should be chosen for the different channels N-N, N-Δ, and Δ-Δ.

Here we prefer to consider the Landau-Migdal interaction as free parameters and to make use of the so-called universality by setting $g_{NN}'=g_{N\Delta}'=g_{\Delta\Delta}'$ (always for the sake of simplicity). We may now repeat the previous calculation according to Eq. (18) and using the effective interaction of Eq. (10). The results are given in Table III for different values of g'. Here with "π" (or "ρ") we mean the whole effective interaction propagating in the spin-longitudinal (or -transverse) channel.

The difference with respect to Table II is that the effective interaction used here accounts for a much richer dynamics. Nevertheless the results remain of the same order of magnitude. It is to be remarked that while the pion-exchange effect is strongly depressed, the ρ-meson exchange remains quite relevant.

The formalism of Eq. (18) is immediately translated to the RPA scheme by means of Eq. (13). The results of the RPA scheme (with the labels π and ρ denoting, as specified above, the spin-longitudinal and spin-transverse channel) are also reported in Table III. They show a significant decrease of N_Δ for high values of g'.

We want to remind the reader furthermore that the number of Δ's in the nuclear ground state is strictly connected with an observable quantity, namely the Δ width in the medium, as is clearly seen from Eqs. (1) and (2). The first term of (2) is of course irrelevant and since N_Δ/V is obviously real, then only the imaginary part of $\Sigma(p)$ comes into play. We have already remarked that the Δ self-energy in the vacuum is also ineffective, as it may be seen as a renormalization contribution, and we are left with only the contribution of the nuclear medium.

The existence and relevance of such a contribution was already discovered many years ago in studying the pion optical potential: it was found indeed that a spreading potential (i.e., a contribution to the imaginary part of the optical potential coming presumably from two-body absorption) was required in order to make the various Δ-hole models realistic.[25–27]

The microscopical origin of this spreading potential has been examined in great detail in Ref. 28. The self-energy diagrams considered here correspond exactly to what is called "two-body absorption" in the quoted paper. Higher-order diagrams are here neglected, while in Ref. 28 are found to be relevant. It was shown, however,

TABLE IV. N_Δ/N in nuclear medium with RPA correlations. Parameters like in Ref. 8.

g'	$P_\pi^{(2)}$	$P_\rho^{(2)}$	$P_{\rm tot}^{(2)}$	$P_\pi^{\rm RPA}$	$P_\rho^{\rm RPA}$	$P_{\rm tot}^{\rm RPA}$
0.5	1.27%	5.78%	7.05%	2.43%	8.70%	11.12%
0.6	0.66%	5.73%	6.39%	0.96%	5.92%	6.89%
0.7	0.27%	6.56%	6.82%	0.32%	4.7%	5.02%
0.8	0.09%	8.26%	8.35%	0.05%	4.34%	4.4%

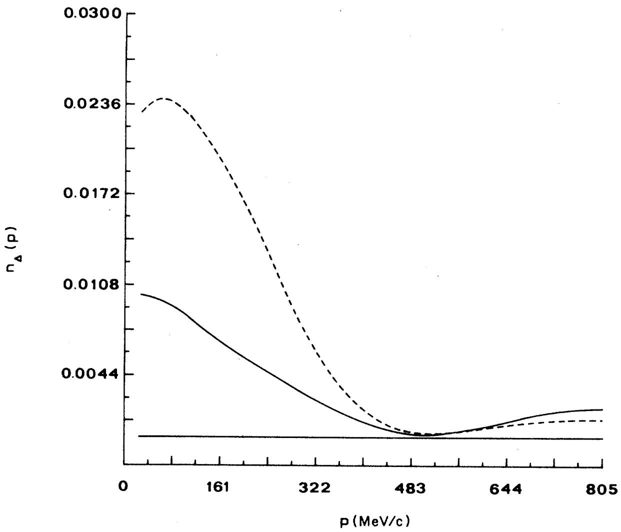

FIG. 8. Δ momentum distribution: solid line refers to RPA calculations, dashed line to second-order approximation.

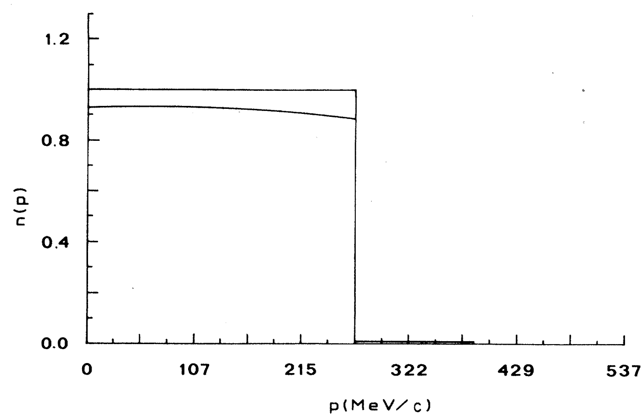

FIG. 9. Nucleon momentum distribution in RPA.

in Ref. 13 that the whole series of 2-,3-,4-,. . . body absorption may be summed up by means of a set of integral equations. In that paper the two-body absorption was obtained as the first iteration of the integral equations, starting from free pion and Δ. There was also shown that the whole series is not convergent, and that the first iteration overestimates the true solution of the integral equations. A direct comparison with the present work and with Ref. 28 is unfortunately not possible because in Ref. 13 the effect of the ρ meson, as well as other off-shell effects, was parametrized and not explicitly evaluated, so that a microscopical description of what was included and what was left out is lacking. Furthermore Ref. 13 provided a good description of the Δ self-energy near the resonance (and in fact Δ photoexcitation[29] and inclusive electron scattering in the Δ region[30] were fairly described), but there was much less confidence about off-shell behavior, and in the present calculation the off-shell effects may give relevant contributions.

A final remark concerns the parameters employed in the calculation. In Table IV the same results as in Table III are reported, but this time with the same coupling constants and cut off of Ref. 1. This clearly shows how much the results depend upon the parameter $f_{\pi N\Delta}$, which turns out to be the crucial factor of the calculation.

IV. THE MOMENTUM DISTRIBUTION

To better understand what is the effect of such a high number of Δ's in the medium it is of relevance to understand how the Δ's are distributed in terms of their momentum and which nucleon levels are depleted. As described briefly in the introduction, if the quoted depletion is distributed smoothly in a wide momentum range, we simply infer that the occupation number of the external shells (mainly the valence shell) is somewhat reduced, in good agreement with the experimental situation as shown firstly in ^{206}Pb,[7] and then examined in a variety of heavy nuclei.[31] The physical effect in this case should not be the destruction of the whole shell model but simply the depletion of the tail of the momentum distribution for $p > k_F$. Since in fact a value of about 0.7 for the occupation number of the valence shell in heavy nuclei is a well established experimental outcome, also reproduced by detailed microscopic calculations[32] we reasonably expect that the same calculation, if the Δ's are not explicitly included, will overemphasize the tail.

Here we have evaluated the occupation number for both Δ and nucleons along the lines of Sec. II [Eqs. (20) and (21)]. The momentum distribution of the Δ is given in Fig. 8, evaluated in the frame of the RPA approximation, with $g'=0.7$. A pronounced peak is apparent at small momenta. On the contrary, the nucleon momentum distribution presents a very smooth behavior up to k_F (see Fig. 9) but a strong depletion of the tail.

We are now in position to draw some conclusions.

(1) We have seen the relevant role played by the so-called box diagrams in the Δ self-energy. This implies that as far as the ground state is concerned the natural Δ width is irrelevant (since we are working well below the threshold) and only those self-energy diagrams which have a truly many-body origin come into play. This justifies the assumption of Ref. 4.

(2) The order of magnitude of the result of Ref. 1 is essentially confirmed. Our analysis, even in absence of complicated microscopical calculations, seems to put in evidence two drawbacks of the previously quoted result: on the one hand, the lack of a ΔΔ or $N\Delta$ short-range repulsion leads to an overestimate of N_Δ; such an overestimate is, however, compensated for by the choice of the parameters (in particular $f_{\pi N\Delta}$ and consequently $f_{\rho N\Delta}=C_\rho f_{\pi N\Delta}$, which are fixed in such a way to reproduce the NN scattering but fail with the πN scattering). As a conclusion the value of N_Δ seems to be of the order of 7%.

(3) This outcome is nevertheless not contradictory with the shell-model structure: it would imply, on the contrary, a reduced occupation number of nucleons above the Fermi level.

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