Symmetric collision of two slabs in the framework of the Vlasov equation

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A phase-space description of the collision between two slabs of nuclear-matter is presented. Anisotropic distortions of the Fermi surface are included in the distribution function. The present results are compared with the predictions of the time-dependent Hartree-Fock and hydrodynamical approaches.

INTRODUCTION

The Vlasov equation has been extensively used for the description of several processes observed in many particle systems.[1-4] It is an appropriate tool for studying a classical system of fermions moving in a mean field determined by the density. The Vlasov equation can be identified as the leading term of a Wigner-Kirkwood expansion of the time-dependent Hartree-Fock (TDHF) equations in powers of $\hbar$.[5] When the classical limit is taken, only the quantum statistical effects are maintained. Indeed, the Pauli principle is incorporated into the dynamics by allowing the distribution function to take only the values 0 and 1.

The exact solution of the Vlasov equation for finite systems can only be obtained by numerical methods.[6] Approximate solutions for the Vlasov equation have been obtained by means of different parametrizations of the trial distribution function.[7,8] Some of these approaches have given the energies and the fractions of EWSR for the collective excitations of atomic nuclei in good agreement with experimental results. In a very recent paper,[9] the validity of such kind of approximations is discussed by comparing them with the exact solution in an infinite system.

The present note reports an application of the variational description of nuclear fluid dynamics presented before.[10] The system under consideration is now composed of two slabs of nuclear matter, infinite in the transverse directions, and limited in the z coordinate. We are interested in describing the time evolution of the system in the framework of the Vlasov dynamics, starting from given initial conditions.

The motivation for the present calculation is two-fold. In one way, it is instructive to compare the results now obtained with those of the simpler version of this kind of application.[11] One expects to get a better understanding of the main features resulting from a richer parametrization of the distribution function. On the other hand, comparison with the predictions of TDHF (Ref. 12) as well as with nuclear hydrodynamics[13,14] can give a better insight into the advantages and also the limitations of the semiclassical models based on the Vlasov equation.

Although the geometry of the system is quite simplified (in order to limit the complexity of the numerical calculations) one hopes that it exhibits some important aspects present in the real collisions of heavy ions.

NORMAL MODES FOR A SINGLE SLAB

We describe the excited modes of the system by the approximate solutions of the Vlasov equation. In the present approach these solutions are obtained by means of a suitable parametrization of the trial distribution function

$$f=f_E+\{f_E,Q\}+\cdots , \tag{1}$$

where

$$f_E=f_0+\{f_0,P\}+\cdots =\theta\left[\frac{p_F^2}{2m}-h_0-W(\mathbf{r},t)-\sum_{\alpha\beta}\frac{p_\alpha p_\beta}{2m}\chi_{\alpha\beta}(\mathbf{r},t)\right] \tag{2}$$

takes into account the distortions of the Fermi surface (static deformations) and the generator function $Q(\mathbf{r},t)$

$$Q(\mathbf{r},t)=\phi(\mathbf{r},t)+\sum_{\alpha\beta}\frac{p_\alpha p_\beta}{2}\phi_{\alpha\beta}(\mathbf{r},t) \tag{3}$$

introduces the dynamical deformations (currents, . . .).

The Lagrangian of the system in the harmonic approximation reads now as

$$L^{(2)}=\int d^3r\left[-\left[\rho_1-\frac{\rho_0}{2}\chi_{\alpha\alpha}\right]\left[\dot\phi+\frac{p_F^2}{6}\dot\phi_{\alpha\alpha}\right]+\frac{p_F^2\rho_0}{10}\left[\chi_{\alpha\beta}-\frac{\delta_{\alpha\beta}}{3}\chi_{\gamma\gamma}\right]\left[\dot\phi_{\alpha\beta}-\frac{\delta_{\alpha\beta}}{3}\dot\phi_{\gamma\gamma}\right]\right]$$
$$-\int d\mathbf{S}\cdot\delta\mathbf{R}\rho_0\left[\dot\phi+\frac{p_F^2}{10}\dot\phi_{\alpha\alpha}\right]-E[\rho_1,\chi_{\alpha\beta}]-T[\phi,\phi_{\alpha\beta}] , \tag{4}$$

where $E[\rho_1,\chi_{\alpha\beta}]$ and $T[\phi,\phi_{\alpha\beta}]$ are, respectively, the potential and the kinetic-energy functionals. The Landau parameter F_0 is given by

$$F_0=\frac{3m\rho_0}{p_F^2}\frac{\partial^2}{\partial\rho_0^2}v(\rho_0)=\frac{3m}{p_F^2}(a\rho_0+b\rho_0^2)\ . \tag{5}$$

By allowing for the free variation of the fields one obtains the following equations of motion:

$$\dot\rho_1-\frac{\rho_0}{2}\dot\chi_{\alpha\alpha}=\frac{\delta T}{\delta\phi}\ , \tag{6a}$$

$$\delta_{\alpha\beta}\left[\frac{p_F^2\dot\rho_1}{6}-\frac{p_F^2\rho_0}{20}\dot\chi_{\alpha\alpha}\right]-\frac{p_F^2\rho_0}{10}\dot\chi_{\alpha\beta}=\frac{\delta T}{\delta\phi_{\alpha\beta}}\ , \tag{6b}$$

$$\dot\phi+\frac{p_F^2}{6}\dot\phi_{\beta\beta}=-\frac{\delta E}{\delta\rho_1}\ , \tag{6c}$$

$$\delta_{\alpha\beta}\left[\frac{\rho_0}{2}\dot\phi+\frac{p_F^2\rho_0}{20}\dot\phi_{\gamma\gamma}\right]+\frac{p_F^2\rho_0}{10}\dot\phi_{\alpha\beta}=\frac{\delta E}{\delta\chi_{\alpha\beta}}\ , \tag{6d}$$

and the boundary conditions

$$\hat{\mathbf{x}}_\beta\left[\partial_\beta\phi+\frac{p_F^2}{5}(\partial_\alpha\phi_{\alpha\beta}+\tfrac{1}{2}\partial_\beta\phi_{\alpha\alpha})\right]-m(\delta\dot{\mathbf{R}}\cdot\hat{\mathbf{n}})\bigg|_{\mathrm{surf}}=0\ , \tag{7a}$$

$$\delta_{\alpha\beta}\left[\hat{\mathbf{x}}_\eta\left[\partial_\eta\phi+\frac{p_F^2}{7}(\partial_\alpha\phi_{\alpha\eta}+\tfrac{1}{2}\partial_\eta\phi_{\gamma\gamma})-m(\delta\dot{\mathbf{R}}\cdot\hat{\mathbf{n}})\right]\right]+\hat{\mathbf{x}}_\alpha\left[\partial_\beta\phi+\frac{p_F^2}{7}(\partial_\gamma\phi_{\beta\gamma}+\tfrac{1}{2}\partial_\beta\phi_{\gamma\gamma})-\xi_\beta\right]+\alpha\leftrightarrow\beta$$
$$+\hat{\mathbf{x}}_\gamma\frac{p_F^2}{7}(\partial_\gamma\phi_{\alpha\beta}+\partial_\beta\phi_{\alpha\gamma}+\partial_\alpha\phi_{\beta\gamma})\bigg|_{\mathrm{surf}}=0\ , \tag{7b}$$

$$\dot\phi+\frac{p_F^2}{10}\dot\phi_{\alpha\alpha}\bigg|_{\mathrm{surf}}=0\ . \tag{7c}$$

An additional boundary condition is imposed to prevent an infinite current at the surface:[10]

$$\hat{\mathbf{x}}_\alpha\phi_{\alpha\beta}|_{\mathrm{surf}}=0\ . \tag{8}$$

In order to investigate the potentiality of this semiclassical formalism, we begin by considering a slab of nuclear matter, with the nucleons confined to move in the region $-L\le z\le L$ and look for the eigenmode solutions of such a system.

For the longitudinal modes, one has two traceless solutions for the tensor fields $\phi_{\alpha\beta}$:

$$\left[\phi_{\alpha\beta}^{(n)}(z,t)\right]_i=\left[k_{\alpha,i}^{(n)}k_{\beta,i}^{(n)}-\frac{\delta_{\alpha\beta}}{3}k_i^{(n)^2}\right]\times\cos(k_i^{(n)}z)\cos(\omega_n t) \tag{9}$$

with the corresponding coupled scalar fields

$$\phi_i^{(n)}(z,t)=A_i k_i^{(n)^2}\cos(k_i^{(n)}z)\cos(\omega_n t)\ , \tag{10}$$

where

$$A_i=-\frac{11}{42}p_F^2+\frac{m^2c_i^2}{2}\ .$$

Each one of the set of solutions i ($i=1,2$) corresponds to one longitudinal velocity for the propagation of the sound waves, namely,

$$c_{1,2}^2=\frac{p_F^2}{2m^2}\left\{\frac{6}{7}+\frac{F_0}{3}\pm\left[\left[\frac{F_0}{3}\right]^2+\frac{1}{35}\left[\frac{96}{7}+8F_0\right]\right]^{1/2}\right\}\ . \tag{11}$$

This fact, not present in the more simplified version of this formalism,[11] is related with new important features of the dynamics.

The wave vectors $k_1^{(n)}$ and $k_2^{(n)}$ are linked by the equation

$$\omega_n=c_1k_1^{(n)}=c_2k_2^{(n)}\ . \tag{12}$$

We can also add a solution of the type

$$[\phi_{\alpha\beta}^{(n)}(z,t)]_3=\delta_{\alpha\beta}k_3^{(n)^2}\cos(k_3^{(n)}z)\cos(\omega_n t)\ , \tag{13}$$

$$\phi_3^{(n)}(z,t)=-\frac{p_F^2}{2}k_3^{(n)^2}\cos(k_3^{(n)}z)\cos(\omega_n t)\ , \tag{14}$$

which only affects the boundary condition (8).

The complete solutions for the fields ϕ and $\phi_{\alpha\beta}$ are superpositions of the previous particular solutions with convenient coefficients F_{ni}:

$$\phi_{\alpha\beta}=\sum_n c_n \sum_{i=1}^{3} F_{ni}[\phi^{(n)}_{\alpha\beta}]_i \ , \tag{15}$$

$$\phi=\sum_n c_n \sum_{i=1}^{3} F_{ni}\phi^{(n)}_i \ . \tag{16}$$

For the remaining fields, we have

$$\rho_1^{(n)}(z,t)=\rho_1^{(n)}(z)\sin(\omega_n t) \ , \tag{17a}$$

$$\chi^{(n)}_{\alpha\beta}(z,t)=\chi^{(n)}_{\alpha\beta}(z)\sin(\omega_n t) \ , \tag{17b}$$

$$\delta R^{(n)}(z,t)=\delta R^{(n)}(z)\sin(\omega_n t) \ . \tag{17c}$$

The spatial parts $[\rho_1^{(n)}(z),\chi^{(n)}_{\alpha\beta}(z)]$ are related to $\phi^{(n)}(z)$ and $\phi^{(n)}_{\alpha\beta}(z)$ by means of the equations of motion for the normal modes. The surface displacement $\delta R^{(n)}(z)$ ($z=\pm L$) is obtained from the boundary conditions.

SYMMETRIC COLLISION OF TWO SLABS

Before we present the sequence of the results and draw some conclusions, it is important to specify the choice of the dynamical quantities relevant for the evolution of the system as well as the initial conditions.

The nuclear matter density is given by

$$\rho(z,t)=g\int \frac{d^3p}{(2\pi)^3} f_E=\rho_0+\rho_1-\frac{\rho_0}{2}\chi_{\alpha\alpha}+\cdots \ , \tag{18}$$

and the current density along the z-axis is obtained in a similar way:

$$j_z(z,t)=g\int \frac{d^3p}{(2\pi)^3}\frac{p_z}{m}\{f_0,Q\} =\frac{\rho_0}{m}\left[\partial_z\phi+\frac{p_F^2}{5}(\partial_\beta\phi_{\beta z}+\tfrac{1}{2}\partial_z\phi_{\beta\beta})\right] \ . \tag{19}$$

We also define the velocity potential

$$\bar{\phi}=\phi+\frac{p_F^2}{6}\phi_{\gamma\gamma} \tag{20}$$

whose time evolution is determined by the first-order fluctuation of the density [Eq. (6c)]. The current density in the interior of the system is related to the fluctuation of the density by means of the continuity equation [Eq. 6(a)] and at the boundaries it expresses the velocity of the nuclear surfaces as given by Eq. (7a). In Fig. 1 the veloc-

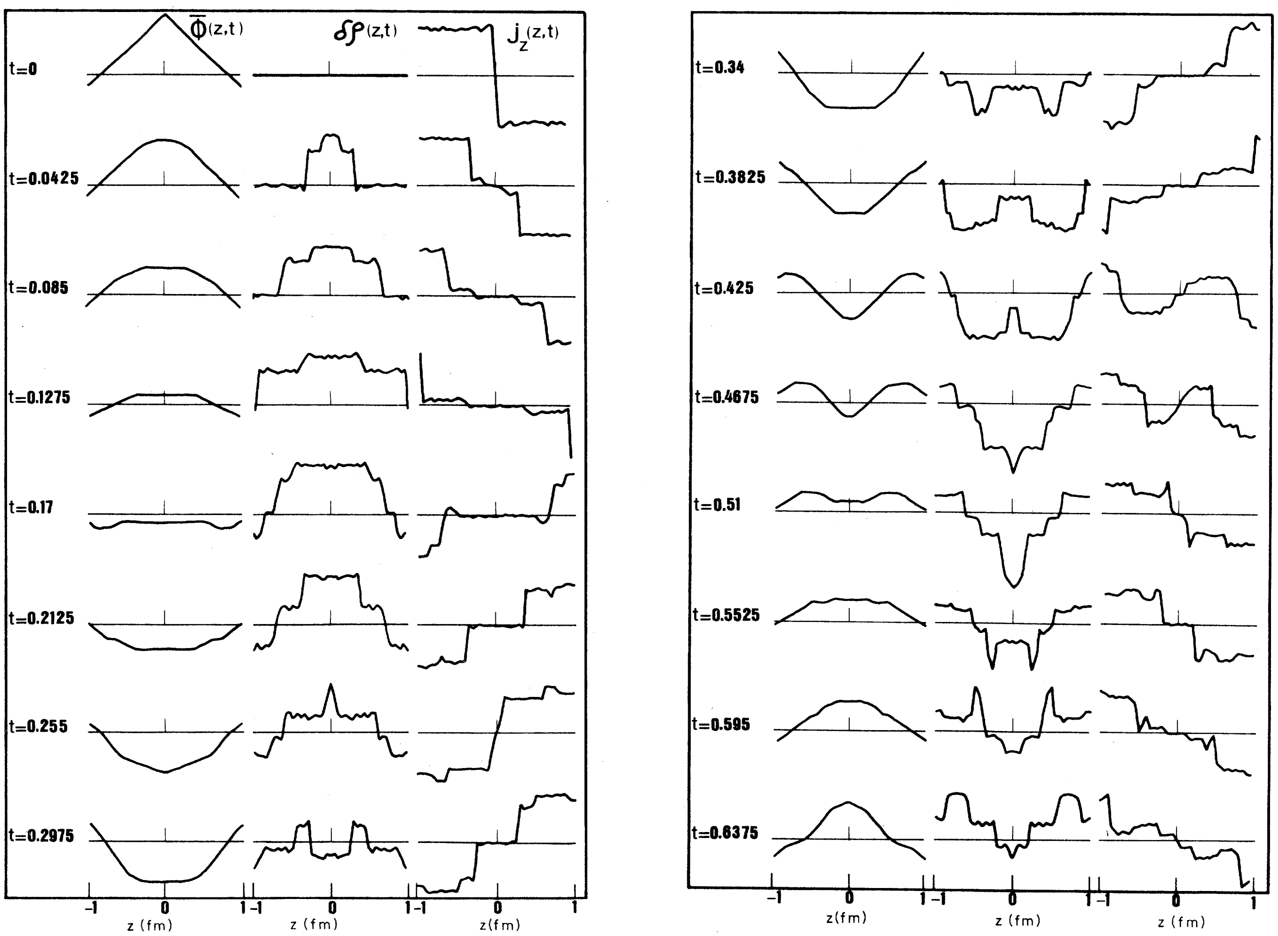

FIG. 1. The potential velocity $\bar{\phi}(z,t)$, the transition density $\delta\rho(z,t)$, and the current density $j_z(z,t)$ are displayed as functions of z, at several instants. (The numbers on the left of the figures give the time in units of 10^{-22} s.)

ity potential $\bar{\phi}$, the first order fluctuation of the density, $\delta\rho=\rho_1-(\rho_0/2)\chi_{\alpha\alpha}$, and the current density j_z are displayed as functions of z, at several instants.

We consider two slabs in motion parallel to the z axis and approaching each other, and choose the instant $t=0$ when the two slabs touch each other. The initial conditions are then

$$\bar{\phi}(z,0)=L+z, \quad z\in[-L,0]$$
$$\bar{\phi}(z,0)=L-z, \quad z\in[0,L] \tag{21}$$

for the velocity potential, all the other fields being zero:

$$\phi_{\alpha\beta}(z,0)=\chi_{\alpha\beta}(z,0)=\rho_1(z,0)=\delta R(z,0)=0 . \tag{22}$$

At the initial time, the density of matter is essentially constant in a wide region in the interior of the slabs. We assume that it is well represented by a square well:

$$\rho(z,0)=\rho_0, \quad -L<z<L ,$$
$$\rho(z,0)=0, \quad z<-L \text{ and } z>L , \tag{23}$$

where ρ_0 is the saturation density of the nuclear matter.

After the collision of the two slabs, a slight increase of the density appears around the contact surface and propagates outwards. The existence of two different velocities for the sound waves produces interferences of the waves generated in the shock region giving rise to the structures shown in the graphs.

At the time $t=0.255\times10^{-22}$ s, the initial situation is more or less inverted. The nuclear current density shows that all the matter is now moving towards the boundaries and away from an imaginary surface dividing the compound slab into two. This effect may be interpreted as an indication of the tendency of the system to separate (fission) into two parts. The effect becomes more and more pronounced at subsequent instants with two distinct lumps emerging at time $t=0.3875\times10^{-22}$ s. However, since a linearized theory cannot explain nonlinear processes, it would be unreasonable to expect a well-defined fragmentation of the compound system.

The shock wave gets reflected on the boundaries and goes on interfering with itself for all times. The initial kinetic energy is transformed into the excitation energy of the collective modes.

The present calculation goes beyond the previous simple Vlasov description[11] by including high-order distortions in the trial distribution function. One thus obtains a more complex pattern for the behavior of the macroscopic fields, in good agreement with TDHF (Ref. 12) and hydrodynamical approaches[13,14] for the same simple systems. One finds now that there is no recurrence time. The loss of memory of the initial conditions, clearly shown, can be interpreted as an indication of the damping of the excitations.

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