^{9}Be and ^{9}B nuclei in a microscopic three-cluster model

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The ^{9}Be and ^{9}B spectra are investigated in a microscopic three-cluster model, using the generator-coordinate method. Both nuclei are described by a ^{8}Be core surrounded by a nucleon. We present the $n+{}^8$Be and $p+{}^8$Be phase shifts, and the electromagnetic transition probabilities in ^{9}Be. A good overall agreement is found with the available experimental data. We suggest spin assignments for different poorly known states. The energy of the $\frac{1}{2}^+$ first excited state in ^{9}B is in excellent agreement with a recent experimental investigation.

I. INTRODUCTION

In spite of numerous experimental (see e.g., Ref. 1) and theoretical (see e.g., Ref. 2) studies, the spectra of ^{9}B and, to a lesser extent, of ^{9}Be remain poorly known.[3] In particular, the analog state of the 1.69 MeV level in ^{9}Be has not been unambiguously determined yet. Moreover, spin assignments remain uncertain in both nuclei. Except for the ^{9}Be ground state, all the ^{9}Be and ^{9}B states present a particle instability. Accordingly, they are broad resonances, whose width prevents an accurate determination of energy and spin.

A microscopic model[4] is very suitable for such a study, since it describes bound, resonant and scattering states in a unified way. This property is especially useful here since ^{9}Be and ^{9}B nuclei present simultaneously bound states and resonances. A further advantage of a microscopic model is that the whole information is obtained from the nucleon-nucleon interaction. When this interaction is chosen, the model does not contain any free parameter, and therefore is able to provide some prediction on the level scheme. In particular, the Coulomb energy shifts between two mirror nuclei have been shown to be accurately given by the microscopic model.[5] The method employed here is the generator coordinate method (GCM) which has been already applied to a large number of spectroscopic studies or nucleus-nucleus collisions.[6]

The ^{9}Be and ^{9}B low-lying states are well known to present a $n+{}^8$Be (or $p+{}^8$Be) structure.[7] Since ^{8}Be is described by an $\alpha+\alpha$ cluster structure, a three-cluster model[8] is required for the present study. A microscopic three-cluster model has been recently applied to a number of systems in which one nucleus presents a two-cluster structure (see Ref. 8). We have shown in Ref. 9 that the ^{8}Be deformation must be taken into account for a valuable description of the spectroscopic properties in ^{12}C, when this nucleus is described by an $\alpha+{}^8$Be structure. With respect to the $\alpha+{}^8$Be system, the ^{9}Be and ^{9}B nuclei involve a cluster with spin $\frac{1}{2}$. This requires the introduction of a spin-orbit force, and additional angular-momentum couplings. Consequently, the calculation times are significantly increased.[8]

In Sec. II, we discuss the microscopic model and present the wave functions. Sections III and IV are devoted to the properties of ^{9}Be and ^{9}B; concluding remarks are presented in Sec. V.

II. THE MICROSCOPIC MODEL

A. Wave functions

The microscopic wave functions are defined in the GCM formalism; the total wave function of the system, with spin J and parity π reads

$$\Psi^{JM\pi}=\sum_{lLI}\mathcal{A}[Y_l(\hat{\rho})\otimes[Y_L(\hat{\rho}')\otimes\phi_n]^I]^{JM}\phi_\alpha\phi_\alpha G_{lLI}^{J\pi}(\rho,\rho') , \tag{1}$$

where $\mathcal{A}$ is an antisymmetrizor operator, and ϕ_α and ϕ_n are the internal wave functions of the alpha particle and of the orbiting nucleon. In this expression, $G_{lLI}^{J\pi}$ is a radial wave function depending on the relative coordinates ρ' between the α particles, and ρ between the nucleon and the center of mass of ^{8}Be. The orbital momentum of ^{8}Be is given by L, while l denotes the orbital momentum of the external nucleon around the ^{8}Be core; I is the channel spin.

In the generator coordinate method, the wave function (1) is written as

$$\Psi^{JM\pi}=\sum_{lLI}\int F_{lLI}^{J\pi}(R_1,R_2)\Phi_{lLI}^{JM\pi}(R_1,R_2)dR_1dR_2 , \tag{2}$$

where R_1 is the generator coordinate associated to the ^{8}Be+ nucleon motion, and R_2 describes the ^{8}Be nucleus; $F_{lLI}^{J\pi}(R_1,R_2)$ is the generator function, and $\Phi_{lLI}^{JM\pi}(R_1,R_2)$ is a projected Slater determinant.[9] Equation (2) is valid when the internal wave functions ϕ_α and ϕ_n are described in the harmonic oscillator model, with a common oscillator parameter. The unknown generator function is determined from the Hamiltonian of the problem.

Equation (1) stresses the three-cluster structure of the system; however, it is not appropriate for the study of the nucleon$+{}^8$Be scattering. To this end, let us consider a ^{8}Be wave function, defined by

$$\phi_{^8\mathrm{Be}}^{Lm\omega}=\mathcal{A}\phi_\alpha\phi_\alpha \tilde{g}^{L\omega}(\rho')Y_L^m(\hat{\rho}') , \tag{3}$$

where ω denotes the level of excitation in the relative motion, and where function $\tilde{g}^{L\omega}(\rho')$ is given by the Hamiltonian of the eight-nucleon system. We have shown in Ref. 9 that an equivalent definition of the wave function (1) is

$$\Psi^{JM\pi}=\sum_{lLI\omega}\mathcal{A}[Y_l(\hat{\rho})\otimes[\phi^{L\omega}_{^8\text{Be}}\otimes\phi_n]^I]^{JM}g^{J\pi}_{lLI\omega}(\rho)\ , \quad (4)$$

where function $g^{J\pi}_{lLI\omega}(\rho)$ is provided by the three-cluster Hamiltonian. The form (4) is suitable for collision studies since it involves the wave functions of the colliding nuclei explicitly. A similar three-cluster model has been employed in Ref. 7 for the description of ^{9}Be but the order of the angular-momentum couplings used here involves the channel spin I, which represents an asymptotic good quantum number. The collision properties obtained here are therefore expected to be somewhat different from those of Ref. 7. The determination of function $g^{J\pi}_{lLI\omega}(\rho)$ [and also of the generator function $F^{J\pi}_{lLI}(R_1,R_2)$] requires the knowledge of matrix elements of the Hamiltonian and of the overlap between two projected Slater determinants $\Phi^{JM\pi}_{lLI}(R_1,R_2)$. We have shown that these matrix elements involve five dimensional integrals. This evaluation requires very large calculation times, with respect to the two-cluster approach. We refer the reader to Ref. 8 for more details on the calculation of GCM matrix elements. In the GCM, the basis wave functions have a Gaussian asymptotic behavior which is not physical for bound states as well as for scattering states. This drawback is corrected by the microscopic R-matrix method (MRM). We refer the reader to Refs. 6 and 10 for details concerning the MRM.

B. Conditions of the calculation

The calculations are performed with the V_2 nucleon-nucleon interaction[11] and a zero-range spin orbit force[12] (with $S_0=50$ MeV fm^5). The Majorana parameter is chosen as $M=0.54$ for positive parity and $M=0.57$ for negative parity. This choice provides a good overall agreement for the ^{9}Be and ^{9}B nuclei. Let us point out that the same interaction is used for both nuclei. This is essential for providing meaningful Coulomb shifts. The oscillator parameter is $b=1.36$ fm, which represents the optimal value for the alpha particle with the V_2 force. The generator coordinates R_1 are taken from 1.2 to 8.4 fm with a step of 1.2 fm. For the ^{8}Be description, we select a single generator coordinate $R_2=3.8$ fm, which represents the minimum of the expectation value of the $\alpha+\alpha$ Hamiltonian. This restriction to a single generator coordinate is necessary to avoid too large calculation times. The $L=0$ and $L=2$ components (1) are included in the model. They represent an approximation of the ^{8}Be ground state and first excited state, respectively. The excitation energy of the ^{8}Be(2^+) state is 3.5 MeV, in nice agreement with experiment.

III. THE ^{9}Be NUCLEUS

The theoretical properties of the ^{9}Be low-lying states are gathered in Table I. The energies are calculated in the c.m. frame with respect to the $n+{}^8$Be threshold. The energies and reduced widths θ^2_L (where L is the spin of ^{8}Be) of bound states and narrow resonances are calculated in the MRM framework,[5] with a channel radius equal to 8.4 fm. For broad resonances, the collision matrices $U^{J\pi}$ are employed to determine the locations and widths. They are parametrized as

$$U_{lI,l'I'}=\eta_{lI,l'I'}\exp(2i\delta_{lI,l'I'})\ . \quad (5)$$

The phase shifts δ and the reflection coefficients η are presented in Figs. 1 and 2 for negative and positive parity partial waves, respectively. For the phase shifts we show the diagonal elements corresponding to the smallest l value. The reflection coefficients are presented with $I=\frac{1}{2}$ and $l'=|J-I'|$.

With the interaction chosen, the ^{9}Be ground state has too small a binding energy but its theoretical quadrupole moment (47.7 $e$mb) and magnetic moment ($-1.52\ \mu_N$) are in nice agreement with experiment[3] (53 ± 3 $e$mb and $-1.18\mu_N$, respectively). This indicates that the

TABLE I. Properties of the ^{9}Be nucleus. The experimental data are from Ref. 3. The bracketed values are tentative assignments. Energies are expressed in MeV and dimensionless reduced widths in %.

J^π	E_x^{exp}	$E_{\text{c.m.}}^{\text{exp}}$	$E_{\text{c.m.}}^{\text{GCM}}$	θ_0^2	θ_2^2	Γ_0	Γ_2	Γ_{exp}
$\frac{3}{2}^-$	0	−1.66	−0.98	5.0	0.7			
$\frac{5}{2}^-$	2.43	0.76	1.80	0.02	5.2	2.0×10^{-4}		7.7×10^{-4}
$\frac{1}{2}^-$	2.78	1.12	0.91	26.4	1.1	0.64		1.08
$\frac{3}{2}^-_2$	(4.70)	(3.04)	3.08	4.0	10.8	0.22		(0.74)
$\frac{7}{2}^-$	6.76	5.10	∼8	∼2	∼40	∼0.2	∼3.0	1.54
$\frac{1}{2}^+$	1.69	0.03	−0.05	17.9	0.5			∼0.15
$\frac{5}{2}^+$	3.05	1.39	1.58	14.4	1.7	0.31		0.282
$\frac{3}{2}^+$	(7.94)	(6.28)	∼6	∼20	∼0	∼2	∼0	(∼1)
$\frac{7}{2}^+$			∼8	∼0	∼40	∼0	∼2	

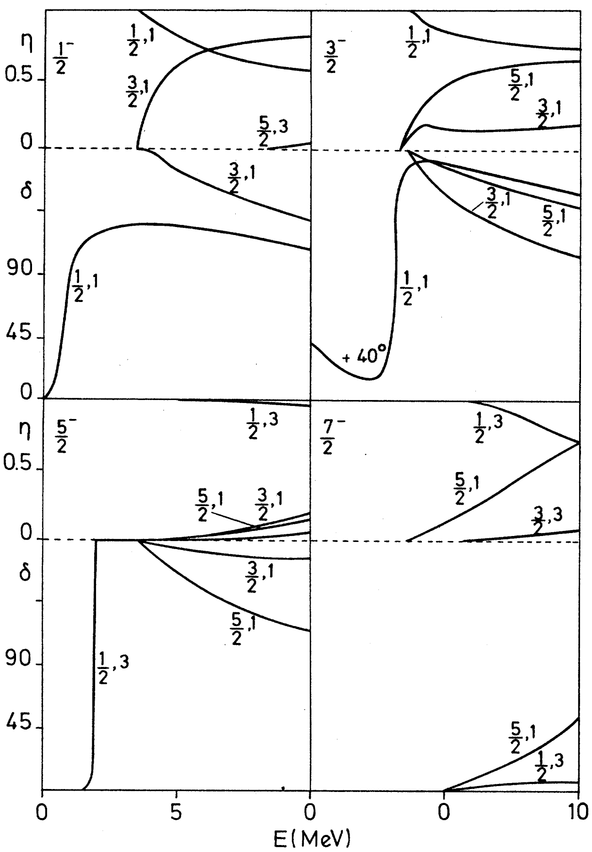

FIG. 1. $n+{}^{8}$Be collision matrices (5) for negative parity. The phase shifts δ correspond to $l=l'$, $I=I'$; l and I are indicated on the figure. For the η coefficients, we choose $I=\frac{1}{2}$, $l'=|J-I'|$; I' and l' are indicated.

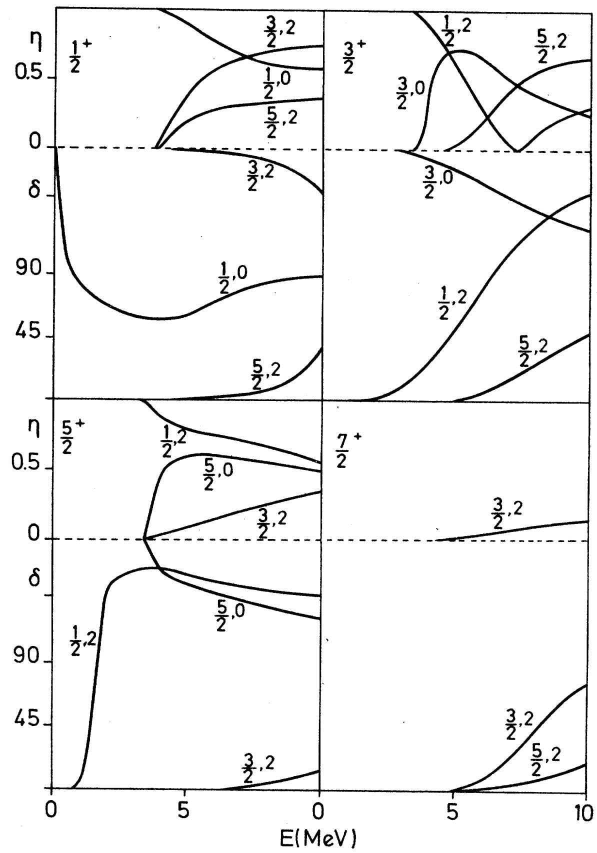

FIG. 2. See Fig. 1 for positive parity.

$n+{}^{8}\mathrm{Be}(0^{+})$ structure of this state, revealed by its large θ_0^2 value is well supported by experiment. The first excited state with negative parity is the $\frac{5}{2}^{-}$ (2.43 MeV) state, whose excitation energy is slightly overestimated in the GCM. The $n+{}^{8}\mathrm{Be}(0^{+})$ configuration is hindered since it corresponds to the angular momentum $l=3$. For this reason, the theoretical $\frac{5}{2}^{-}$ state rather presents a $n+{}^{8}\mathrm{Be}(2^{+})$ structure, which contains an $l=1$ component. In fact, the analysis of Kadija *et al.*[1] favors a ^{5}Li$+\alpha$ structure for the analog ^{9}B state. This indicates that the introduction of the ^{5}He$+\alpha$ configuration should lower the energy and increase the total width of the $\frac{5}{2}^{-}$ state owing to the occurrence of a nonvanishing α width. The η coefficients displayed in Fig. 1 show that the transfer between the $n+{}^{8}\mathrm{Be}(0^{+})$ and $n+{}^{8}\mathrm{Be}(2^{+})$ is very weak, essentially because of the different angular momentum occurring in both channels. At $E_{\mathrm{c.m.}}=3.08$ MeV, the GCM predicts a $\frac{3}{2}^{-}$ state, with a relatively small width of 0.22 MeV. This state has not been observed experimentally but a resonance with parity + and spin tentatively assigned to $\frac{3}{2}$ is known[3] at 3.04 MeV. Since the agreement between both energies is excellent and since the widths are comparable, we think that the resonance observed at 3.04 MeV might be a $\frac{3}{2}^{-}$ excited state, with a marked $n+{}^{8}$Be structure. Let us point out that the calculation of Furutani *et al.*[7] also predicts a $\frac{3}{2}^{-}$ resonance near 2.5 MeV, but these authors assign it to an uncertain 2.1 MeV state. The $\frac{7}{2}^{-}$ phase shift presents a broad resonance in the $l=1$, $I=\frac{5}{2}$ partial wave. Because of its large width, the properties indicated in Table I are indicative only. A broad $\frac{7}{2}^{-}$ resonance is known experimentally at 5.10 MeV, an energy significantly lower than in our model. For such a high-spin state, it is most likely that more symmetric configurations, like ^{5}He$+\alpha$, should be important because of their smaller centrifugal barrier. However, the introduction of such a configuration does not seem to be necessary for low-spin states, and would require a very important increase of calculation times. Let us now discuss the positive-parity states; the corresponding phase shifts are depicted in Fig. 2. The $\frac{1}{2}^{+}$ first excited state is well reproduced by the GCM. The experimental width[3] corresponds to a dimensionless reduced width $\theta_0^2=25\%$, in nice agreement with our calculation. This supports a $n+{}^{8}\mathrm{Be}(0^{+})$ structure for the $\frac{1}{2}^{+}$ resonance. The fact that the GCM succeeds in reproducing the properties of this state is important for the study of the ^{9}B nucleus since the existence of the analog state is

TABLE II. Electromagnetic transition probabilities (in W.U.) in ^{9}Be.

	$B(E2)$		$B(M1)$	
	GCM	exp[a]	GCM	exp[a]
$\frac{5}{2}^- \to \frac{3}{2}^-$	23.5	24.4±1.8	0.10	0.30±0.03
$\frac{1}{2}^- \to \frac{3}{2}^-$	1.9		1.38	
$\frac{5}{2}^-$	11.4			
$\frac{7}{2}^- \to \frac{3}{2}^-$	~7	6.3±2.7		
$\frac{3}{2}^+ \to \frac{1}{2}^+$	4.5		5.0×10^{-3}	
$\frac{5}{2}^+ \to \frac{1}{2}^+$	7.8			

	$B(E1)$	
	GCM	exp[a]
$\frac{1}{2}^+ \to \frac{3}{2}^-$	0.68	0.22±0.09
$\frac{5}{2}^+ \to \frac{3}{2}^-$	0.32	$(3.6\pm3.0)\times10^{-2}$
$\frac{5}{2}^- \to \frac{5}{2}^+$	7.7×10^{-3}	

[a]Reference 3.

open to discussion. We shall come back to this point in the next section. The microscopic model gives rise to a $\frac{5}{2}^+$ state, whose location and width are in excellent agreement with experiment. This resonance has its main component in the $n+{}^{8}$Be(0^+) channel, with a small θ_2^2 value. The $\frac{3}{2}^+$ phase shift suggests the existence of a broad resonance near 6 MeV with a width of about 2 MeV. This leads to a large θ_0^2 value of 20%. Hence, the positive-parity states can be interpreted as a ^{8}Be core in its ground state, surrounded by a neutron. As discussed above, a possible $\frac{3}{2}^+$ state has been observed at 3.04 MeV but, owing to its theoretical energy, we rather suggest the 6.28 MeV state whose width is about 1 MeV and whose spin is uncertain as candidate to the $\frac{3}{2}^+$ resonance. This assignment is necessary if one considers the attribution of the 3.04 MeV state to a $\frac{3}{2}^-$ spin (see the discussion above). A broad $\frac{7}{2}^+$ resonance is predicted near 8 MeV, mainly in the $l=2$, $I=\frac{3}{2}$ channel, but owing to its large width, an experimental candidate has not been observed yet.

The electromagnetic transition probabilities (expressed in Weisskopf units) in ^{9}Be are displayed in Table II. The $E2$ transitions towards the ground state are very well reproduced. Note however that, because of the large width of the $\frac{7}{2}^-$ state, the bound-state approximation which is used here is indicative only. For the $M1$ transition probabilities, the GCM underestimates the $B(M1,\frac{5}{2}^- \to \frac{3}{2}^-)$; this confirms that the $\frac{5}{2}^-$ state contains other configurations than the $n+{}^{8}$Be configuration introduced here. Note the small $B(M1)$ for the $\frac{3}{2}^+ \to \frac{1}{2}^+$ transition. If one neglects the $n+{}^{8}$Be(2^+) channel (a reasonable assumption according to the reduced widths presented in Table I), this transition occurs by $l=2$ to $l=0$ transition, which is forbidden if the antisymmetrization is removed. Consequently, the nonvanishing value of the $B(M1,\frac{3}{2}^+ \to \frac{1}{2}^+)$ is due to the occurrence of the antisymmetrization or to a small component in the $n+{}^{8}$Be(2^+) channel. The $B(E1)$ from the first excited state has been extensively studied by Barker,[13] who suggests $B(E1)=0.38^{+0.07}_{-0.06}$ W.U. as experimental value from a fit of the ^{9}Be(γ,n)^{8}Be data. By using the R-matrix formalism and taking the unbound character of the $\frac{1}{2}^+$ state into account, Barker finds $B(E1,\frac{1}{2}^+ \to \frac{3}{2}^-)=0.400$ W.U. The value presented in Table I is obtained within the bound-state approximation, which is valid if the transition involves a bound state and a narrow resonance (note in addition that in our calculation the binding energy of

TABLE III. Properties of the ^{9}B nucleus. The experimental data are from Ref. 3. The bracketed values are tentative assignments. Energies are expressed in MeV and dimensionless reduced widths in %.

J^π	E_x^{exp}	$E_{\text{c.m.}}^{\text{exp}}$	$E_{\text{c.m.}}^{\text{GCM}}$	θ_0^2	θ_2^2	Γ_0	Γ_2	Γ_{exp}
$\frac{3}{2}^-$	0	0.19	0.46	9.8	0.8	0.031		$(5.4\pm2.1)\times10^{-4}$
$\frac{5}{2}^-$	2.36	2.55	3.24	0.03	8.0	5.3×10^{-4}		$(8.1\pm0.5)\times10^{-2}$
$\frac{1}{2}^-$			2.34	28.8	1.4	1.10		
$\frac{3}{2}^-$	(4.8)	(5.0)	4.3	3.7	19.0	0.2	0.2	(1.0±0.2)
$\frac{1}{2}^+$	1.16[a]	1.35[a]	1.34	48.3	0.6	1.3		1.3±0.05[a]
$\frac{5}{2}^+$	2.79	2.98	3.11	17.8	2.6	0.63		0.55±0.09
$\frac{3}{2}^+$			~6	~23	~0	~1.5	~0	

[a]Reference 1.

the $\frac{1}{2}^+$ state is found slightly negative). The $B(E1)$ obtained here is quite consistent with the experimental values. However, the $B(E1,\frac{5}{2}^+\rightarrow\frac{3}{2}^-)$ transition probability is overestimated by the model. This is surprising in view of the above discussion which states that the $\frac{3}{2}^-$ and $\frac{5}{2}^+$ states are both well described by the $n+{}^{8}$Be configuration. This discrepancy deserves further investigation. The $E1$ transition probability from the $\frac{5}{2}^-$ state to the $\frac{5}{2}^+$ state is found very small in the GCM. This can be explained by the different structures of these levels (see Table I). The $\frac{5}{2}^-$ resonance presents a dominant $n+{}^{8}$Be(2^+) structure, while the $n+{}^{8}$Be(0^+) configuration is more important in the $\frac{5}{2}^+$ state.

IV. THE ^{9}B NUCLEUS

The properties of the ^{9}B nucleus are presented in Table III, and the $p+{}^{8}$Be phase shifts in Figs. 3 and 4. Let us recall that, in order to provide meaningful Coulomb shifts between the ^{9}Be and ^{9}B nuclei, we have used the same nucleon-nucleon interaction as in Sec. III. Let us first discuss the negative-parity states. The ground state is predicted at somewhat too high an energy. However, the experimental reduced width ($\theta_0^2=7.5\pm2.9\,\%$) is in remarkable agreement with our result. We find for the ground state a quadrupole moment $Q=9.0$ $e$mb, and a magnetic moment $\mu=3.38\ \mu_N$. The $\frac{5}{2}^-$ resonance is located too high in the spectrum, and presents too small a width. Both effects are most likely due to missing channels like ^{5}Li$+\alpha$ which are expected to play a significant role in this state.[1] Anyway, our GCM investigation confirms that the $\frac{5}{2}^-$ resonance does not present a $p+{}^{8}$Be structure. Our calculation predicts a $\frac{1}{2}^-$ resonance near $E_x=2.5$ MeV, which is the analog of the well known 2.78 MeV state in ^{9}Be. This state is expected to decay mainly in the $p+{}^{8}$Be(0^+) channel, with a width of about 1.1 MeV. Although there is a strong theoretical evidence for the existence of this $\frac{1}{2}^-$ state, it has not been observed experimentally. The model suggests the presence of a broad $\frac{3}{2}{}_2^-$ resonance for which the 4.8 MeV level, whose spin is unknown, seems to be a valuable candidate.

Let us now turn to the positive-parity states. The existence of the analog of the $\frac{1}{2}^+$ (1.69 MeV) level in ^{9}Be is a long-standing problem. It has recently received much attention experimentally as well as theoretically. Sherr and Bertsch[14] have suggested an excitation energy of 0.93 MeV corresponding to $E_{\mathrm{c.m.}}\approx1.12$ MeV. However, this work is open to criticism because the definition of the en-

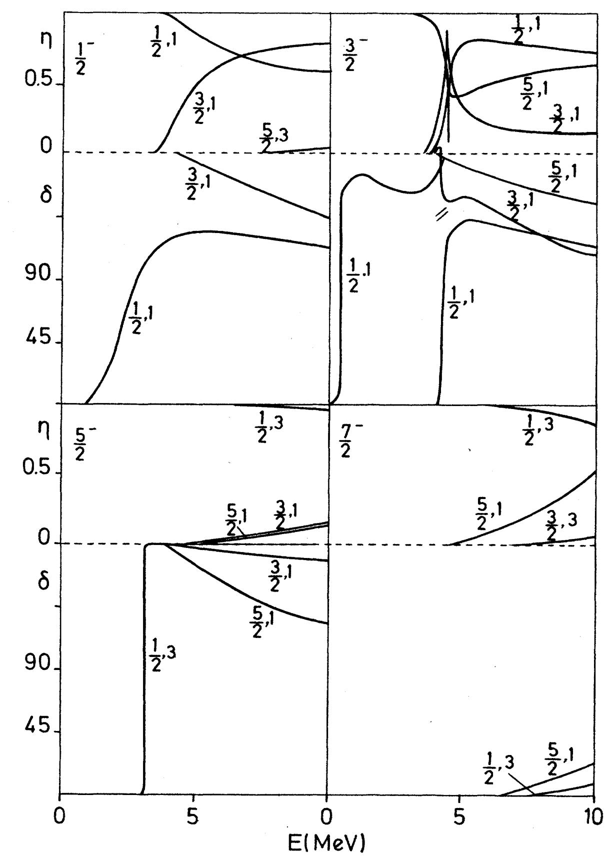

FIG. 3. $p+{}^{8}$Be collision matrices for negative parity. See caption to Fig. 1.

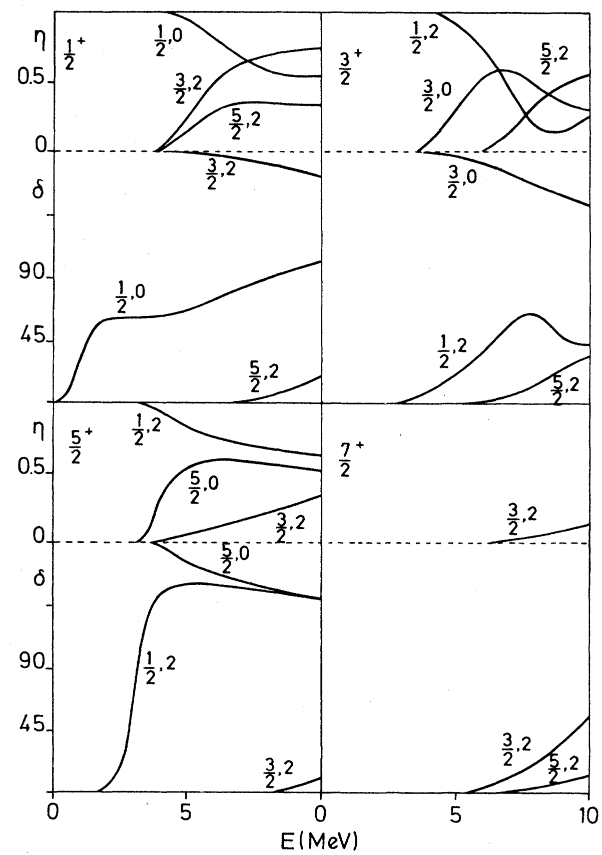

FIG. 4. See Fig. 3 for positive parity.

ergy for unbound levels is inaccurate (see discussion in Ref. 2). In an R-matrix analysis, Barker[2] rather finds a c.m. energy of 2.0 MeV, with a width comprised between 1 and 2 MeV. Since the analog state in ^{9}Be is well described by the $n+{}^8$Be configurations, and since the shift between the mirror nuclei is given by the Coulomb interaction which is treated exactly, we think that our result is reliable. It is very nicely supported by the recent measurement of Kadija *et al.*[1] who suggest $E_{\text{c.m.}}=1.35$ MeV with $\Gamma=1.3\pm0.05$ MeV. The GCM gives rise to a $\frac{5}{2}^+$ state ($E_{\text{c.m.}}=3.11$ MeV), with a large reduced width in the $p+{}^8\text{Be}(0^+)$ channel. Consequently, the experimental 2.79 MeV state ($E_{\text{c.m.}}=2.98$ MeV) whose spin is $\frac{3}{2}$ or $\frac{5}{2}$ and whose parity is positive can be assigned to the predicted $\frac{5}{2}^+$ resonance. This suggestion is strengthened by the good agreement between the theoretical and experimental widths. The $\frac{7}{2}^+$ phase shift starts rising from 6 MeV in the $p+{}^8\text{Be}(2^+)$ channel, but no definite conclusion can be drawn concerning the existence of a $\frac{7}{2}^+$ level.

V. CONCLUSION

The microscopic three-cluster model has been shown to reproduce very well many properties of the ^{9}Be and ^{9}B nuclei. The model takes the deformation of ^{8}Be into account and treats bound, resonant and scattering states in a unified way. This eliminates the problem of the definition of resonance energies. The low-lying states of ^{9}Be have been shown to present a marked $n+{}^8$Be structure. The model suggests that the experimental 4.70 and 7.94 MeV states might be $\frac{3}{2}^-$ and $\frac{3}{2}^+$, respectively. When the spin increases, the more symmetric $^5\text{He}+\alpha$ configuration is expected to play an important role because of a larger reduced mass. Anyway, the present study shows that the well known $\frac{5}{2}^-$ resonance does not present a $n+{}^8\text{Be}(0^+)$ structure. The electromagnetic transition probabilities confirm that the present description of ^{9}Be is quite good. We have investigated the ^{9}B level scheme with the same interaction as for the ^{9}Be nucleus. The ^{9}B ground state presents a strong $p+{}^8\text{Be}(0^+)$ clustering; this is supported by the experimental reduced width which is in excellent agreement with our value. The model suggests a $\frac{1}{2}^-$ state, analog to the $\frac{1}{2}^-$ (2.78 MeV) state observed in ^{9}Be which should be located near 2 MeV. A $\frac{1}{2}^+$ resonance, to which several theoretical and experimental works have been devoted, is predicted here at $E_{\text{c.m.}}=1.34$ MeV, with a width of 1.3 MeV, in excellent agreement with the recent experimental data of Kadija *et al.*[1]

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