Relativistic calculation of the deuteron quadrupole and magnetic moments

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The deuteron quadrupole and magnetic moments are calculated assuming light-front dynamics. The advantage of this approach is that conventional bound-state wave functions and empirical nucleon form factors can be used to construct eigenfunctions of the four-momentum and the spin, together with a representation of the electromagnetic current, in such a way that the current-density operator and the wave functions transform consistently under a unitary representation of the Poincaré group. The sensitivity of our model to the choice of the deuteron bound-state wave functions is tested by using Reid soft core, Argonne v_{14}, Paris, and Nijmegen nucleon-nucleon interactions, as well as three Bonn potentials. The exact results increase both quadrupole and magnetic moments by small amounts compared to the nonrelativistic values. Expansions in powers of the nucleon velocity are found to be unreliable.

I. INTRODUCTION

The quadrupole and magnetic moments of the deuteron have had a long history of study from many points of view. The experimental values of the quadrupole[1] and magnetic moments[2] are 0.2860 ± 0.0015 fm^2 and $0.857\,406 \pm0.000\,001$ μ_N, respectively. These experimental values differ from the results of nonrelativistic calculations by a few percent.

The quadrupole and magnetic moments are characteristic quantities derived from the deuteron current matrix elements in the limit of vanishing momentum transfer. Current conservation and the requirement that the current transform as a four vector impose consistency conditions on the current operators and state vectors. Thus, the central features of a consistent relativistic calculation are the construction of a Poincaré invariant matrix representation of a conserved current and the construction of a model interaction. Going beyond the standard nonrelativistic description, one encounters both the effects of special relativity and of non-nucleonic degrees of freedom. These effects are intertwined in approaches that rely on instant-form Fock-space perturbation expansions of meson-nucleon field theory.[3–7] These calculations involve expansions in inverse powers of the nucleon mass, m, which are justified by the questionable assumption that all relevant momenta and energies are small compared to the nucleon mass. Covariant-wave-function models,[8,9] on the other hand, give exact relativistic results, which feature P-wave components in the deuteron wave function. For these models the validity of p/m expansions of the quadrupole and magnetic moments could be tested numerically. To our knowledge this has not been done.

Deuteron models for which the light-front components of the four-momentum transform kinematically[10,11] yield exact results, which can be compared to expansions in inverse powers of the nucleon mass. These models can describe the available data for the deuteron structure functions $A(Q^2)$ and $B(Q^2)$ within the uncertainty of the empirical nucleon form factors.[10,12]

Because nonrelativistic nucleon-nucleon wave functions are eigenfunctions of the rest energy and spin operators, $\mathbf{j}^2$ and j_z, they can be interpreted as eigenfunctions of a Poincaré invariant mass operator. Eigenfunctions of the total four momentum can always be constructed as eigenfunctions of the mass and three independent components of the momentum. The choice of these independent components determines the "form" of the relativistic dynamics.[13] With light-front dynamics it is possible to construct covariant conserved current operators for which all two-body matrix elements are generated from one-body currents by dynamic Lorentz transformations, and an explicit knowledge of these two-body currents is not needed for the calculation of deuteron form factors, which are then unambiguously determined by the nucleon form factors and the deuteron wave function.[10] Implicit effects of subnucleon degrees of freedom, mesons, and/or quarks, must show up in additional two-body currents, which make contributions to the deuteron form factors that are separately Lorentz invariant.

In this paper we examine the deuteron quadrupole and magnetic moments, obtained within this framework for different nucleon-nucleon interactions, and compare the exact results with approximations obtained by expansion in powers of the nucleon velocity to second order. For the magnetic moments our results are in agreement with earlier results.[14]

In Sec. II, we present a brief overview of the exact relations of the moments to the deuteron wave function. Expansions in inverse powers of the nucleon mass are given in Sec. III. Numerical results and conclusions are presented in Sec. IV.

II. RELATION OF QUADRUPOLE AND MAGNETIC MOMENTS TO THE DEUTERON WAVE FUNCTION

For the consideration of relativistic effects in magnetic and quadrupole moments, it is important to verify the correspondence of the calculated and measured quantities. Experimentally the quadrupole and magnetic moments are determined by measuring energy differences in an external field given by the Hamiltonian[15]

$$H'=\int d^3\mathbf{x}\, I^\nu(\mathbf{x})A_\nu^{\rm ext}(\mathbf{x}) \; , \tag{2.1}$$

where $A_\nu^{\rm ext}(\mathbf{x})$ is the vector potential of the external field and $I^\nu(x)$ is the current operator. The measured quadrupole and magnetic moments are expectation values of components of the quadrupole tensor and magnetic-moment vector,

$$\int d^3\mathbf{x}(3x_ix_k-\delta_{ik}|\mathbf{x}|^2)I^0(\mathbf{x}), \quad \text{and} \quad \tfrac{1}{2}\int d^3\mathbf{x}\,\mathbf{x}\times\mathbf{I}(\mathbf{x}) \; . \tag{2.2}$$

For any function $f(\mathbf{x})$, the Poincaré covariance of the current operators $I^\nu(x)$ completely determines the matrix elements

$$\lim_{\substack{\mathbf{P}\to 0\\ \mathbf{Q}\to 0}} \int d^3\mathbf{x}\, f(\mathbf{x})\langle \lambda_d',\mathbf{P}+\tfrac{1}{2}\mathbf{Q}|I^\nu(\mathbf{x})|\mathbf{P}-\tfrac{1}{2}\mathbf{Q},\lambda_d\rangle \tag{2.3}$$

in terms of the invariant form factors. It follows that the quadrupole and magnetic moments of the deuteron are related to the usual quadrupole and magnetic form factors G_2 and G_1 by

$$Q_d=\lim_{Q^2\to 0} 3\sqrt{2}\frac{G_2(Q^2)}{Q^2}, \quad \mu_d=\frac{m}{M_d}G_1(0) \; . \tag{2.4}$$

According to Ref. 10 it follows that the moments can be obtained from the matrix elements, $\langle \lambda_d'|I^+(0)|\lambda_d\rangle$, of the plus component of the current operator, where the unit vector $\mathbf{n}$ specifying the light front is chosen such that $Q^+=Q^0+\mathbf{n}\cdot\mathbf{Q}=0$. Thus the quadrupole moment and the magnetic moment are given by

$$Q_d=-4\lim_{Q^2\to 0}\frac{\langle 1|I^+(0)|-1\rangle}{Q^2} -\frac{1}{M_d^2}[1-\lim_{Q\to 0}\sqrt{2/\eta}\langle 1|I^+(0)|0\rangle] \; , \tag{2.5}$$

and

$$\mu_d=\frac{m}{M_d}[2-\lim_{Q\to 0}\sqrt{2/\eta}\langle 1|I^+(0)|0\rangle] \; , \tag{2.6}$$

where m and M_d are the nucleon and deuteron masses, respectively, and $\eta\equiv Q^2/4M_d^2$.

The matrix elements, $\langle\lambda_d'|I^+(0)|\lambda_d\rangle$, are unambiguously determined by the nucleon form factors and the deuteron wave functions,

$$\langle\lambda_d'|I^+(0)|\lambda_d\rangle=\sum_{\lambda_1,\lambda_1',\lambda_2}\int d^2\mathbf{k}_T'\int d^2\mathbf{k}_T\int_0^1 d\xi\,\delta[\mathbf{k}_T'-\mathbf{k}_T-(1-\xi)\mathbf{Q}_T]\chi^*_{\lambda_d'}(\mathbf{k}_T',\xi,\lambda_1',\lambda_2) \times[F_{1N}(Q^2)\delta_{\lambda_1,\lambda_1'}-\sqrt{\tau}F_{2N}(Q^2)\langle\lambda_1'|i\sigma_2|\lambda_1\rangle]\chi_{\lambda_d}(\mathbf{k}_T,\xi,\lambda_1,\lambda_2) \; , \tag{2.7}$$

where F_{1N} and F_{2N} are the Dirac and Pauli isoscalar nucleon form factors, and the deuteron wave function $\chi_{\lambda_d}(\mathbf{k}_T,\xi,\lambda_1,\lambda_2)$ is related to the conventional deuteron wave function $\chi^c_{\lambda_d}(\mathbf{k})$ by the variable transformation $\mathbf{k}\to\{\mathbf{k}_T,\xi\}$,

$$\mathbf{k}\cdot\mathbf{n}=M_0(\xi-\tfrac{1}{2}), \quad \mathbf{k}_T\cdot\mathbf{n}=0, \quad M_0^2=\frac{m^2+k_T^2}{\xi(1-\xi)} \; , \tag{2.8}$$

and by Melosh rotations, $\mathcal{R}_M(\xi,\mathbf{k}_T,m)$, of the spins,

$$\mathcal{R}_M(\xi,\mathbf{k}_T,m)=\frac{m+\xi M_0-i\boldsymbol{\sigma}\cdot(\mathbf{n}\times\mathbf{k}_T)}{[(m+\xi M_0)^2+\mathbf{k}_T^2]^{1/2}} \; . \tag{2.9}$$

The wave function $\chi_{\lambda_d}(\mathbf{k}_T,\xi,\lambda_1,\lambda_2)$ is thus specified by

$$\chi_{\lambda_d}(\xi,\mathbf{k}_T,\lambda_1,\lambda_2)=\left[\frac{M_0}{4\xi(1-\xi)}\right]^{1/2}[\mathcal{R}_M^\dagger(\xi,\mathbf{k}_T,m)\chi^c_{\lambda_d}(\mathbf{k})\mathcal{R}_M^*(1-\xi,-\mathbf{k}_T,m)]_{\lambda_1,\lambda_2} \; , \tag{2.10}$$

where $M_0/4\xi(1-\xi)$ is the Jacobian of the variable transformation from $\{\xi,\mathbf{k}_T\}\to\mathbf{k}$.

The conventional wave function $\chi^c_{\lambda_d}(\mathbf{k})$ has the standard form[16]

$$\chi^c_{\lambda_d}(\mathbf{k})=\frac{1}{\sqrt{4\pi}}\left[\boldsymbol{\sigma}\cdot\hat{\mathbf{e}}_{\lambda_d}U_0(k)-\frac{1}{\sqrt{2}}(3\boldsymbol{\sigma}\cdot\mathbf{k}\hat{\mathbf{e}}_{\lambda_d}\cdot\mathbf{k}-\boldsymbol{\sigma}\cdot\hat{\mathbf{e}}_{\lambda_d}k^2)U_2(k)\right]\frac{i\sigma_2}{\sqrt{2}} . \tag{2.11}$$

The polarization vector $\hat{\mathbf{e}}_{\lambda_d}$ has components $\hat{\mathbf{e}}_0=(0,0,1)$ and $\hat{\mathbf{e}}_{\pm}=\mp(1,\pm i,0)/\sqrt{2}$. The functions $U_L(k)$ are related to the conventional deuteron S- and D-state wave functions $u_L(k)$ by

$$U_L(k)=\frac{u_L(k)}{k^{L+1}} , \tag{2.12}$$

and normalized according to

$$\int dk[u_0^2(k)+u_2^2(k)]=1 . \tag{2.13}$$

They are related to the radial wave functions $u_0(r)$ and $u_2(r)$ via

$$u_L(k)=i^L k\left[\frac{2}{\pi}\right]^{1/2}\int dr\, u_L(r)j_L(kr)r . \tag{2.14}$$

Without loss of generality, we choose axes such that $\mathbf{n}=\{0,0,1\}$ and $Q=\{0,|\mathbf{Q}|,0,0\}$. The leading powers of $|\mathbf{Q}|$ in the current matrix elements (2.7) can be obtained by expanding the Fourier transform of the delta function,

$$\delta[\mathbf{k}'_T-\mathbf{k}_T-(1-\xi)\mathbf{Q}]=\frac{1}{(2\pi)^2}\int d^2x_T\exp[i(\mathbf{k}'_T-\mathbf{k}_T)\cdot\mathbf{x}_T][1-i(1-\xi)|\mathbf{Q}|x_1-\tfrac{1}{2}(1-\xi)^2Q^2x_1^2+\cdots] . \tag{2.15}$$

We note that $x_1\equiv -i\partial/\partial k_1$, with the other independent variables kept constant. This expansion yields the following expressions for the two matrix elements.

$$\lim_{Q\to 0}\sqrt{2/\eta}\langle 1|I^+(0)|0\rangle=-\sqrt{8}M_d^2\sum_{\lambda_1,\lambda_2}\int d^2\mathbf{k}_T\int d\xi\left[F_{1N}(0)\chi^*_+(\mathbf{k}_T,\xi,\lambda_1,\lambda_2)(1-\xi)\frac{\partial}{\partial k_1}\chi_0(\mathbf{k}_T,\xi,\lambda_1,\lambda_2)\right.$$
$$\left.+\frac{F_{2N}(0)}{2m}\sum_{\lambda_1'}\chi^*_+(\mathbf{k}_T,\xi,\lambda_1',\lambda_2)\langle\lambda_1'|i\sigma_2|\lambda_1\rangle\chi_0(\mathbf{k}_T,\xi,\lambda_1,\lambda_2)\right] , \tag{2.16}$$

and

$$\lim_{Q^2\to 0}\frac{\langle 1|I^+(0)|-1\rangle}{Q^2}=\sum_{\lambda_1,\lambda_2}\int d^2\mathbf{k}_T\int d\xi\left[F_{1N}(0)\chi^*_+(\mathbf{k}_T,\xi,\lambda_1,\lambda_2)\tfrac{1}{2}(1-\xi)^2\frac{\partial^2}{\partial k_1^2}\chi_-(\mathbf{k}_T,\xi,\lambda_1,\lambda_2)\right.$$
$$\left.+\frac{F_{2N}(0)}{2m}\sum_{\lambda_1'}\chi^*_+(\mathbf{k}_T,\xi,\lambda_1',\lambda_2)(1-\xi)\langle\lambda_1'|i\sigma_2|\lambda_1\rangle\frac{\partial}{\partial k_1}\chi_-(\mathbf{k}_T,\xi,\lambda_1,\lambda_2)\right] . \tag{2.17}$$

All the spin summations can be done algebraically, after rearranging Eqs. (2.16) and (2.17) as follows:

$$\lim_{Q\to 0}\sqrt{2/\eta}\langle 1|I^+(0)|0\rangle=-\sqrt{8}M_d^2\int d^2\mathbf{k}_T\int d\xi\frac{1}{4\xi(1-\xi)}\left[F_{1N}(0)(1-\xi)\frac{\partial}{\partial k_1'}\{\mathrm{Tr}[T_2\chi^{c\dagger}_+(\mathbf{k})T_1\chi^c_0(\mathbf{k}')]\sqrt{M_0(\mathbf{k}')M_0(\mathbf{k})}\}\right.$$
$$\left.+\frac{F_{2N}(0)}{2m}\mathrm{Tr}[T_2\chi^{c\dagger}_+(\mathbf{k})T_3\chi^c_0(\mathbf{k}')]\sqrt{M_0(\mathbf{k}')M_0(\mathbf{k})}\right]_{\mathbf{k}=\mathbf{k}'} , \tag{2.18}$$

and

$$\lim_{Q^2\to 0}\frac{\langle 1|I^+(0)|-1\rangle}{Q^2}$$
$$=-\int d^2\mathbf{k}_T\int d\xi\frac{1}{4\xi(1-\xi)}\left[\frac{F_{1N}(0)}{2}(1-\xi)^2\frac{\partial^2}{\partial k_1\partial k_1'}\{\mathrm{Tr}[T_2\chi^{c\dagger}_+(\mathbf{k})T_1\chi^c_-(\mathbf{k}')]\sqrt{M_0(\mathbf{k}')M_0(\mathbf{k})}\}\right.$$
$$\left.-\frac{F_{2N}(0)}{2m}(1-\xi)\frac{\partial}{\partial k_1'}\{\mathrm{Tr}[T_2\chi^{c\dagger}_+(\mathbf{k})T_3\chi^c_-(\mathbf{k}')]\sqrt{M_0(\mathbf{k}')M_0(\mathbf{k})}\}\right]_{\mathbf{k}=\mathbf{k}'} , \tag{2.19}$$

where

$$T_2 \equiv \mathcal{R}_M^*(1-\xi,-\mathbf{k}'_T,m)\mathcal{R}_M^{*\dagger}(1-\xi,-\mathbf{k}_T,m) \quad (2.20)$$

describes the Melosh rotations of the spectator nucleon, and

$$T_1 \equiv \mathcal{R}_M(\xi,\mathbf{k}_T,m)\mathcal{R}_M^\dagger(\xi,\mathbf{k}'_T,m) , \\ T_3 \equiv \mathcal{R}_M(\xi,\mathbf{k}_T,m)i\sigma_2\mathcal{R}_M^\dagger(\xi,\mathbf{k}'_T,m) \quad (2.21)$$

describe the Melosh rotations of the initial and final struck nucleons. The expressions (2.18) and (2.19) do not involve approximations. The right-hand sides could be evaluated to obtain exact expressions for the quadrupole moment and the magnetic moment. It is easier, however, to obtain the exact result from the matrix elements evaluated for very small nonzero values of Q^2. Equations (2.18) and (2.19) are useful as the starting point for expansions in inverse powers of the nucleon mass.

III. EXPANSION IN POWERS OF THE NUCLEON VELOCITY

We now turn to calculating the quadrupole and magnetic moments by expansion in powers of the nucleon velocity to second order beyond the nonrelativistic result. The expressions for the moments are reduced to one-dimensional integrals in four steps: (i) expansion of the integrand of Eqs. (2.18) and (2.19) in powers of $1/m$; (ii) summation over all spin variables; (iii) differentiation with respect to k_1 and k'_1 at $\mathbf{k}_T=\mathbf{k}'_T$; and (iv) integration over angles after change of integration variables $\{\mathbf{k}_T,\xi\}\to\mathbf{k}$.

The result is a sum of terms characterized by products of the wave functions, $U_L(k)$, and/or their derivatives, $U'_L(k)\equiv\partial U_L(k)/\partial k^2$. The individual terms exhibit the sensitivity of the relativistic corrections to the conventional S- and D-wave functions.

From Eqs. (2.6) and (2.18), it follows that the approximate magnetic moment $\mu_d^{\rm app}$ consists of five terms,

$$\mu_d^{\rm app} \equiv \mu_d^{NR}+\mu_d^c+\mu_{SS}^c+\mu_{SD}^c+\mu_{DD}^c \equiv \mu_d^{NR}+\delta\mu_d^{\rm app} , \quad (3.1)$$

where

$$\mu_d^{NR}=\mu_{DD}^{NR}=\mu_p+\mu_n-\tfrac{3}{2}P_D(\mu_p+\mu_n-\tfrac{1}{2}) ,$$
$$\mu_d^c=\frac{2m-M_d}{2m} ,$$
$$\mu_{SS}^c=\frac{1}{6m^2}(F_{1N}-F_{2N})\int dk\, k^4U_0^2(k) , \quad (3.2)$$
$$\mu_{SD}^c=-\frac{\sqrt{2}}{60m^2}(F_{1N}-4F_{2N})\int dk\, k^6U_0(k)U_2(k) ,$$
$$\mu_{DD}^c=\frac{1}{m^2}(\tfrac{11}{24}+\tfrac{1}{60}F_{2N})\int dk\, k^8U_2^2(k) .$$

The leading term in Eq. (3.1) is the nonrelativistic magnetic moment. The second term is due to the binding energy of the deuteron. The remaining three terms are, respectively, S-wave, SD-interference, and D-wave contributions. These expressions for the magnetic moment are in agreement with those of Ref. 14.

In the same manner Eqs. (2.18), (2.19), and (2.5), yield four terms for the approximate quadrupole moment.

$$Q_d^{\rm app}\equiv Q_{SD}^{NR}+Q_{DD}^{NR}+Q_{SD}^c+Q_{DD}^c\equiv Q_d^{NR}+\delta Q_d^{\rm app} , \quad (3.3)$$

where

$$Q_{SD}^{NR}=\sqrt{2}\int dk\, k^4U'_0(k)U_2(k) + \frac{2\sqrt{2}}{5}\int dk\, k^6U'_0(k)U'_2(k) ,$$
$$Q_{DD}^{NR}=-\tfrac{1}{5}\int dk\, k^8U_2'^2(k) , \quad (3.4)$$
$$Q_{SD}^c=\frac{\sqrt{2}}{m^2}\left[(\tfrac{3}{10}+\tfrac{1}{5}F_{2N})\int dk\, k^6U'_0(k)U_2(k) + \tfrac{6}{35}\int dk\, k^8U'_0(k)U'_2(k)\right] ;$$
$$Q_{DD}^c=\frac{1}{m^2}[(-\tfrac{6}{5}+\tfrac{3}{20}F_{2N})P_D-\tfrac{3}{35}\int dk\, k^{10}U_2'^2(k)] .$$

It is easy to verify by a Fourier transform that the first two terms are the SD-interference and D-wave contributions to the nonrelativistic quadrupole moment.

$$Q_{SD}^{NR}=\frac{1}{\sqrt{50}}\int dr\, r^2u_0(r)u_2(r)$$

and (3.5)

$$Q_{DD}^{NR}=-\tfrac{1}{20}\int dr\, r^2u_2^2(r) .$$

The remaining terms, Q_{SD}^c and Q_{DD}^c, are the relativistic corrections to the SD-interference and D-wave contributions.

IV. NUMERICAL RESULTS AND CONCLUSIONS

We have computed the deuteron quadrupole and magnetic moments of the deuteron wave functions due to the Reid soft core,[17] Argonne v_{14},[18] Paris,[19] Nijmegen,[20] and

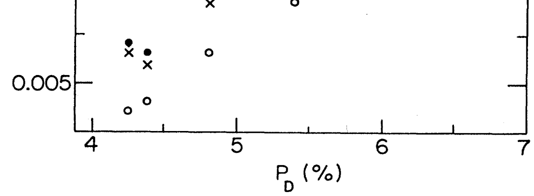

FIG. 1. The relativistic effects in the magnetic moment and the quadrupole moment for different potentials listed in the tables. Besides the exact relativistic corrections $\delta\mu_d$ and δQ_d, we show the approximate corrections $\delta_d^{\rm app}$ obtained by expansion in powers of k/m.

TABLE I. The quadrupole moment Q_d for different wave functions arranged in order of decreasing D-state probability.

		Experimental value $Q_d=0.2860\pm0.0015$				
Potential	P_D (%)	Q_d	Q_d^{NR}	Q_d^{app}	$\delta^{app}Q_d(10^{-3})$	$\delta Q_d(10^{-4})$
Reid soft core	6.47	0.2804	0.2796	0.2762	−3.4	7.2
Argonne v14	6.08	0.2866	0.2859	0.2827	−3.2	6.8
Paris	5.77	0.2795	0.2789	0.2758	−3.0	6.7
Nijmegen	5.39	0.2781	0.2775	0.2747	−2.8	6.6
Bonn R	4.81	0.2742	0.2736	0.2711	−2.5	5.8
Bonn Q	4.38	0.2739	0.2734	0.2711	−2.3	5.2
Bonn E	4.25	0.2812	0.2806	0.2784	−2.2	5.3

TABLE II. The magnetic moment μ_d for different wave functions arranged in order of decreasing D-state probability.

		Experimental value $\mu_d=0.857\,406\pm0.000\,001$				
Potential	P_D (%)	μ_d	μ_d^{NR}	μ_d^{app}	$\delta^{app}\mu_d(10^{-3})$	$\delta\mu_d(10^{-3})$
Reid soft core	6.47	0.8500	0.8429	0.8513	8.4	7.1
Argonne v14	6.08	0.8516	0.8451	0.8526	7.5	6.5
Paris	5.77	0.8531	0.8469	0.8541	7.2	6.2
Nijmegen	5.39	0.8549	0.8491	0.8556	6.5	5.8
Bonn R	4.81	0.8577	0.8524	0.8582	5.9	5.3
Bonn Q	4.38	0.8597	0.8548	0.8601	5.3	4.8
Bonn E	4.25	0.8603	0.8556	0.8610	5.4	4.7

TABLE III. Contributions to the k/m expansion of the quadrupole moment. See Eqs. (3.3) and (3.4).

Potential	P_D (%)	Q_{SD}^{NR}	$Q_{DD}^{NR}(10^{-2})$	$Q_{SD}^{c}(10^{-4})$	$Q_{DD}^{c}(10^{-3})$
Reid soft core	6.47	0.2986	−1.893	9.02	−4.31
Argonne v14	6.08	0.3048	−1.896	8.16	−4.04
Paris	5.77	0.2970	−1.815	8.10	−3.84
Nijmegen	5.39	0.2951	−1.756	7.85	−3.59
Bonn R	4.81	0.2907	−1.703	6.86	−3.21
Bonn Q	4.38	0.2902	−1.682	6.12	−2.93
Bonn E	4.25	0.2974	−1.678	6.26	−2.83

TABLE IV. Contributions to the k/m expansion of the magnetic moment. See Eqs. (3.1) and (3.2). The numerical value of the term μ_d^c is 1.185×10^{-3}.

Potential	P_D (%)	$\mu_{SS}^{c}(10^{-3})$	$\mu_{SD}^{c}(10^{-5})$	$\mu_{DD}^{c}(10^{-3})$
Reid soft core	6.47	2.505	+5.329	4.626
Argonne v14	6.08	2.093	+0.292	4.214
Paris	5.77	2.202	−0.687	3.809
Nijmegen	5.39	1.996	−2.243	3.332
Bonn R	4.81	2.097	−6.849	2.655
Bonn Q	4.38	2.031	−10.84	2.161
Bonn E	4.25	2.117	−6.307	2.178

three Bonn potentials.[21,22] In Tables I and II we list the exact relativistic results, the nonrelativistic values, and the approximate values obtained in Sec. III. The relativistic effects increase the quadrupole moment by 0.19% to 0.26% and the magnetic moment by 0.55% to 0.84%. The relativistic corrections to the quadrupole moments are smaller than the experimental errors and do not significantly improve any disagreement of the nonrelativistic values with the data. The relativistic corrections to the magnetic moments decrease the discrepancy with the experiment, but are not sufficient to achieve agreement with the data. The sensitivity of the corrections to variations in the wave functions is shown in Fig. 1. These results clearly indicate that additional exchange-current effects are needed to achieve agreement of both moments with experiment.

The expansion in powers of k/m yields reasonable approximations for corrections to the magnetic moment. The approximate corrections have the right sign and are between 10% and 20% too large. For the quadrupole moment, however, the expansion yields corrections of the wrong sign and the wrong order of magnitude. The S-wave, D-wave, and SD-interference contributions are shown in Tables III and IV.

The unreliability of the expansion in powers of the nucleon velocity k/m should not be surprising.[23] Not only does the power series diverge, but the expectation values of individual integrals will diverge for sufficiently large powers. When low-order terms give already bad approximations, the addition of higher-order terms is likely to make things worse. In general, relativistic corrections calculated by expansion to order $\langle k^2/m^2 \rangle$ should be considered spurious unless justified by a detailed error analysis of the specific case.

In summary, we found relativistic results that are larger than the nonrelativistic limits by about 0.2% for the quadrupole moment and 0.7% for the magnetic moment. Corrections calculated by expansion in powers of $1/m$ represent unreliable approximations to the relativistic corrections. The remaining discrepancy between the theoretical and experimental results indicates the need to consider the effects of non-nucleonic degrees of freedom, which generate two-body charge-current operators.

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