Nonmesonic photonuclear reactions in the three-nucleon system

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We derive sets of equations which describe $\gamma+{}^3\mathrm{H}_e \rightarrow {}^3\mathrm{H}_e$, $p+d$, and $2p+n$, where the photon is either real or virtual, by requiring four-body unitarity. The initial and final wave functions and the final-state interactions (in the case of $p+d$ and $2p+n$) are the solutions of the three-body (Faddeev) equation in the bound and continuum states, respectively. The driving terms in the sets of equations for the reactions consist of two parts: the impulse one-baryon current [the nucleon, $\Delta(1232)$, and $N^*(1440)$,...] and an amplitude which satisfies a four-body equation and which includes the meson-exchange currents in lowest order.

I. INTRODUCTION

There has been increasing interest in the study of electromagnetic interactions with nuclei, among which the trinucleon (i.e., ${}^3\mathrm{H}_e$ or ${}^3\mathrm{H}$, which we will not distinguish between in this paper) has had special attention.[1] This is because it is the lightest nucleus whose nucleon density is high enough to be compared with those of heavier nuclei, and because it is the heaviest nucleus from which one can derive the wave function both in the bound and continuum states exactly, at least from the present computational point of view.

The charge form factor of the trinucleon for $Q^2 \lesssim 10\,\mathrm{fm}^{-2}$ is known to be well reproduced in the impulse approximation, using the realistic wave function for the trinucleon. However, for $Q^2 \gtrsim 10\ \mathrm{fm}^{-2}$, the impulse approximation fails to reproduce the first minimum and second maximum.[1] Several different reasons have been given to explain this discrepancy: the meson exchange current (MEC),[2] the $\Delta(1232)$ component in the trinucleon wave function,[3] multiquark components (especially, six-quark states),[4] and quark exchanges.[5] The common feature among these effects is that they all change the high-momentum component of the trinucleon wave function. For the magnetic form factor, the discrepancy between theory and the theoretical prediction in the impulse approximation is more severe and can be observed even for $Q^2 \lesssim 10\ \mathrm{fm}^{-2}$. It is well known that the MEC contribution to the isovector magnetic current is in the same order as the contribution from the impulse magnetic current, which almost explains this discrepancy. However, the study of the other three effects has not yet been exhausted.

It should be noted here that the electromagnetic form factors of the trinucleon have so far been calculated independently of the related two reactions, i.e., $\gamma+{}^3\mathrm{H}_e \rightarrow p+d, 2p+n$. This is mainly because the discrepancy of the electromagnetic form factors between theory and experiment has been one of the major topics in the field of electromagnetic interactions in few-nucleon systems.[1] However, in order to see the details of the interactions between the baryons, either at the meson-baryon level or the quark level, we need to gather together more relevant information, for example, the polarization quantities of the reactions where the continuum states of the trinucleon are involved.

Two-body photodisintegration of the ${}^3\mathrm{H}_e, \gamma+{}^3\mathrm{H}_e \rightarrow p+d$, has previously been studied at low energies ($T_\gamma \lesssim 100$ MeV).[1,6–9] Two groups[6–9] have obtained the cross section by solving the Faddeev equation both in the bound and continuum states, respectively, for the ${}^3\mathrm{He}$ wave function and the final state interaction. Recently, the inverse reaction, $p+d \rightarrow \gamma+{}^3\mathrm{H}_e$, was measured[10] at $T_p \lesssim 500$ MeV (which is equivalent to $T_\gamma \lesssim 300$ MeV) and compared with the theoretical prediction of Laget,[11] who included a certain number of diagrams in addition to the impulse diagram. Agreement between these is relatively good at lower energies ($T_p < 350$ MeV); however, some disagreement becomes evident at higher energies. More surprisingly, however, is that a similar calculation[12] for $p+d \rightarrow \pi^0+{}^3\mathrm{H}_e$ at the same incident energies, shows a significant disagreement with the experimental results. More recently, however, Ueda[13] calculated the cross sections of the reaction for $p+d \rightarrow \pi^0+{}^3\mathrm{H}_e$ by taking into account the $NN\Delta$ and $N\Delta\Delta$ reactions, and found them to be remarkably in good agreement with the data. He also claimed in this same reaction, a possible signature of the $NN\Delta$ resonance, based on his microscopic calculation,[14] where the dynamics of the coupled NNN-$NN\Delta$-πdN system is treated with a Faddeev-type-equation. The success of these calculations implies that, for the reactions where three-nucleon continuum states are involved, one must (a) take into account the Δ and (b) include all orders of the rescattering diagrams, at higher energies. This important message should be kept in mind when calculating $\gamma+{}^3\mathrm{H}_e \rightarrow p+d$ at higher energies.

At present, a number of intermediate energy and high duty-factor electron facilities are planned, typically at Continuous Electron Beam Accelerator Facility (CEBAF), which is believed will provide an electron beam up to 4 GeV. Therefore, it is reasonable to require that at such energies, the theories dealing with the reactions under consideration should take into account (1) Lorentz invariance, (2) the effects of one- and multiple-pion production as well as isobars (at least the Δ), (3) exotic channels such as the six- and nine-quark states, and (4) gauge invariance.

In this paper, we will present a unified theory of $\gamma+{}^3\mathrm{H}_e$ (or ${}^3\mathrm{H}$) $\rightarrow {}^3\mathrm{H}_e$ (or ${}^3\mathrm{H}$), $p+d$ (or $n+d$), and $n+2p$ (or $p+2n$), where the photon can be real or virtual. (Recent reviews of the electromagnetic form factors and electrodisintegration of the trinucleon can be found in Refs. 1 and 15, respectively.) For these reactions, we require four-body unitarity and derive sets of equations for the reasons which will be explained in Sec. II. The sets of equations for the reactions with the final states $p+d$ and $n+2p$, are similar to those in Refs. 8 and 16 except that (1) we have, apart from the impulse nucleon current term, a term which obeys a (Mitra-Yakubovski[17] type) four-body equation which includes the usual one-pion exchange current term in lowest order, (2) we will allow inclusion of isobars as well as the explicit quark degrees of freedom, into our Hilbert space, in addition to the nucleon, and (3) our equation is basically relativistic.

We will now outline the remaining sections of this paper. In Sec. II, we will give derivations of the sets of equations. In Sec. III, we will try to answer the following question: How should we modify the conventional Faddeev computer codes available for low-energy calculations[8,16,25] so that we can adapt them to the need at higher energies above one-pion threshold? Our conclusion is given in Sec. IV.

II. THEORY

In this section we derive sets of equations for the reactions under consideration. For this purpose, we will use a similar technique applied to the π-N-N (Refs. 18 and 19), π-π-N (Ref. 20), γ-π-N (Refs. 21 and 22), and γ-N-N (Ref. 23) systems, all of which are based on the classification method first theorized by Taylor.[24] In order to demonstrate his method, let us take an example of the $\gamma N \rightarrow \pi N$ amplitude.[21] The diagrams which contribute to the full (or physical) amplitude for this reaction, are classified as zero-particle irreducible, and can be decomposed (as shown in Fig. 1) into two parts: (i) those which are one-particle irreducible and which we denote by $\tilde{t}^{(1)}$ and (ii) those which are not included in (i) and which turn out to be the s-channel pole diagrams. Therefore, we can now write the $\gamma N \rightarrow \pi N$ amplitude as

$$\tilde{t}^{(0)}=\tilde{t}^{(1)}+f^{(1)}d_B\tilde{f}^{(1)} , \tag{1}$$

where the superscript in the parentheses refers to the irreducibility of the diagrams that contribute to the corresponding amplitude. The important point here is that the s-channel pole terms are only constructed with the renormalized masses and vertices, which are one-particle irreducible. This guarantees that the full amplitude has a pole at its physical mass and also shows its correct threshold behavior. Similarly, from $\tilde{t}^{(1)}$ in Eq. (1), the term which has a two-particle unitarity cut can be isolated. This is diagrammatically shown in Fig. 2, and can be written in the following two forms:

$$\tilde{t}^{(1)}=\tilde{t}^{2}+t^{(2)}g^{(1)}\tilde{t}^{(1)} \tag{2a}$$

$$=\tilde{t}^{(2)}+t^{(1)}g^{(1)}\tilde{t}^{(2)} , \tag{2b}$$

where t is the t matrix for the π-B scattering (where B is a baryon) and $g^{(1)}$ is the dressed propagator for the π-B system. Equation (2) is the result of both the last-cut lemma and the "complete unitarity" in the theory by Taylor.[24] The last-cut lemma states that one can uniquely expose an n-particle unitarity cut in the n-particle irreducible amplitude, closest either to the final state [corresponding to Eq. (2a)] or to the initial state [corresponding to Eq. (2b)]. The "complete unitarity" means that in the above procedure of exposing an n-particle unitarity cut, the two amplitudes which have the same irreducibility and the same number of external lines in the initial and final states, are identical. This leads to, for example, Eq. (2a), an integral equation. We refer the reader to Refs. 18–24 for further details of Taylor's method.

In Refs. 21–23, the photonuclear and electronuclear reactions in one- and two-nucleon systems are discussed. Their derivations, in principle, do not depend on the details of the interaction. However, they have used a chiral bag model Lagrangian in order to justify the omission of the highly irreducible amplitudes. They attribute this omission to the truncation of the pion-quark coupling at a low order as is usually exercised in the model. In this paper, we do not specify any particular Lagrangian or Hamiltonian in order to allow a comparison between our present theory and the conventional theories[2–8,16] in which realistic (or phenomenological) N-N interactions are used. This can be achieved when constructing the N-N interactions, either by including all types of mesons as well as the pion, or by using the potentials which are parametrized so as to fit to the experimental data. In a practical sense, however, this generalization does not alter any of the results obtained in Refs. 21–23 since one can interpret the pion, in the intermediate states, as being one of the other mesons as well. However, the following should be kept in mind: (i) If we want to introduce the Δ or the explicit quark degrees of freedom such as the six- and nine-quark states and the q-$\bar{q}$ pair contributions[22,27] without ambiguity (as will be discussed in Sec. III), then

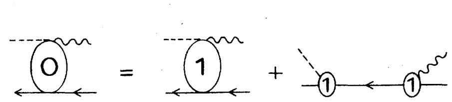

FIG. 1. Diagrammatic representation of Eq. (1), where the number in the circle denotes irreducibility. Time goes from right to left as the arrows indicate. Solid lines represent baryons, broken lines pions, and wiggly lines photons.

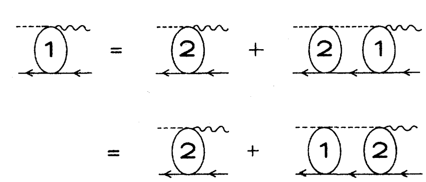

FIG. 2. Diagrammatic representation of Eqs. (2a) and (2b).

one should employ a specific interaction Hamiltonian in order to construct the input amplitudes. (ii) The basic Lagrangian has to be renormalizable when we are to deal with the Feynman diagrams which include all types of the meson-baryon interactions. It should be noted here that at the present time there do not exist any chiral bag models which contain types of mesons other than pions. However, we can easily imagine such a chiral bag model that involves any type of meson. It is expected that we can achieve renormalization in such a model. This is because we have in our theory the vertex functions (which may be related to the bag size) and also because the higher-order pion-baryon coupling terms would be suppressed if other mesons were incorporated. This is conjectured from our experience in the semiclassical calculation of the static properties of nucleons in the chiral potential model.[28] We will relegate further comments on this topic to Sec. III C.

The notation of the amplitudes that we deal with in this paper are summarized in Fig. 3. It should be noted here that the figures throughout this paper (as in Refs. 21–23) are drawn as time goes from right to left, as the arrows indicate.

In order to derive sets of equations to describe the reactions under consideration, we require four-body unitarity for the following two reasons: (1) We need to expose the triton pole of the amplitude in the initial state. This is because we rely on the generalized Lehmann-Symanzik-Zimmermann (LSZ) reduction technique for composite particles, which is described in Ref. 29 for the three-particle systems, in order to extract the physical amplitudes from the corresponding off-shell Green's functions. (2) Taylor's method[19–24] will provide us with equations with specified potentials whose forms are given consistently with renormalization and four-body unitarity. In other words, the form of the potentials can be specified by application of his method, and each vertex function satisfies a certain equation (to guarantee unitarity). Consequently, our final equations are valid even above the four-body breakup threshold.

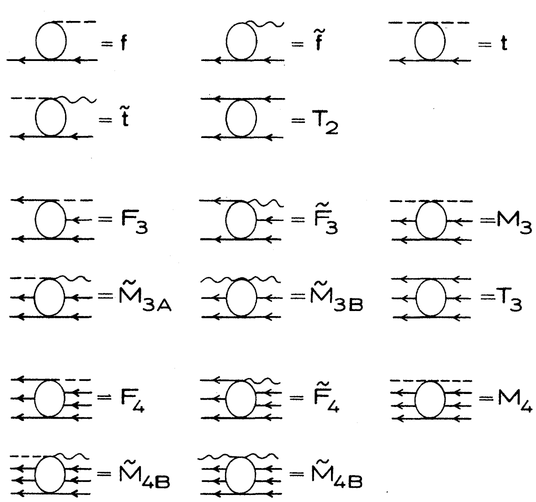

FIG. 3. Diagrammatic representation of all of the amplitudes which are dealt with in this paper. Their corresponding symbols are also shown.

We will now turn to the exposure of unitarity in the amplitude $\tilde{F}^{(2)}_{4;C}$ for $BBB \leftarrow \gamma BBB$ [where $B = N, \Delta(1232)$, $[N^*(1440), \ldots]$, we can expose the three-particle unitarity] cut by separating the amplitude into two parts in the following ways:

$$\tilde{F}^{(2)}_{4;c} = \tilde{F}^{(3)}_{4;c} + (T_3^{(3)} G^{(2)} \tilde{F}_4^{(2)})_c \tag{3a}$$

$$= \tilde{F}^{(3)}_{4;c} + (T_3^{(2)} G^{(2)} \tilde{F}_4^{(3)})_c \; . \tag{3b}$$

Here, the subscript c means that we collect only the connected parts of the Feynman diagrams that contribute to the amplitude. $G^{(2)}$ is the three-baryon propagator and can be written as

$$G^{(2)} = d_B(1) \otimes d_B(2) \otimes d_B(3) \; ,$$

in which $d_B(i)$, $i = 1,2,3$, are the baryon propagators, and where $\otimes$ denotes the direct product as explained in Ref. 18 and which is important only when antisymmetry of the amplitudes are discussed. We will not discuss the antisymmetry of the amplitudes under consideration in this paper. Therefore, we will use a simpler notation, $G^{(2)} = d_B d_B d_B$, in the following discussions. From Eq. (3b), we derive

$$\tilde{F}^{(2)}_{4;c} = (1 + T_2^{(1)} G^{(1)} + T^{(2)}_{3;c} G^{(2)}) \tilde{F}^{(3)}_{4;c} \; , \tag{4}$$

where $G^{(1)} = d_B d_B$, and where we have dropped the disconnected part of $\tilde{F}_4^{(3)}$ because it does not contain the trinucleon pole in the initial state.

We shall now examine $\tilde{F}^{(3)}_{4;c}$. We can write this three-particle irreducible amplitude for $BBB \leftarrow \gamma BBB$, according to the last-cut lemma, as

$$\tilde{F}^{(3)}_{4;c} = \tilde{F}^{(4)}_{4;c} + (F_4^{(4)} G^{(3)} \tilde{M}^{(3)}_{4A})_c + (\tilde{F}_4^{(4)} \tilde{G}^{(3)} \tilde{M}^{(3)}_{4B})_c \tag{5a}$$

$$= \tilde{F}^{(4)}_{4;c} + (F_4^{(3)} G^{(3)} \tilde{M}^{(4)}_{4A})_c + (\tilde{F}_4^{(3)} \tilde{G}^{(3)} \tilde{M}^{(4)}_{4B})_c \; , \tag{5b}$$

where $G^{(3)} = d_B d_B d_B d_\pi$ and $\tilde{G}^{(3)} = d_B d_B d_B d_\gamma$, and where d_π and d_γ are the propagators of the pion and the photon, respectively. To derive Eq. (5), we have chosen $\tilde{F}^{(4)}_{4;c} = 0$ as it involves at least five-particle intermediate states, and we expect the inelastic channel [i.e., $\pi\pi NNN$] to be dominated by the $\Delta\Delta N$, $\pi\Delta NN$, or N^*NN states, which contribute to three- or four-body unitarity. In Figs. 4(a)–(c), we show some examples of the diagrams that contribute to $\tilde{F}^{(3)}_{3;c}$, which is contained in the third term in Eq. (5a). In the paper by Araki and Afnan (Ref. 23) for the study of photonuclear reactions in two-nucleon systems, $\tilde{F}^{(3)}_{3;c}$ was set to be zero. This is because the diagrams that contribute to $\tilde{F}^{(3)}_{3;c}$ involve the interaction $\langle B | H | B\pi\pi \rangle$. In a practical sense, the consistent inclusion of this interaction term makes the computation of the final equations difficult. In this paper we include the diagrams both by modifying the πBB coupling constant in the corresponding diagrams, Figs. 4(a′) and (b′), and by parametrizing the two-pion contribution with the σ and ρ exchanges [Fig. 4(c′)]. Note that the diagrams in Figs. 4 (a′)–(c′) belong to $\tilde{F}^{(2)}_{3;c}$. Therefore, we can put $\tilde{F}^{(3)}_{3;c} = 0$.

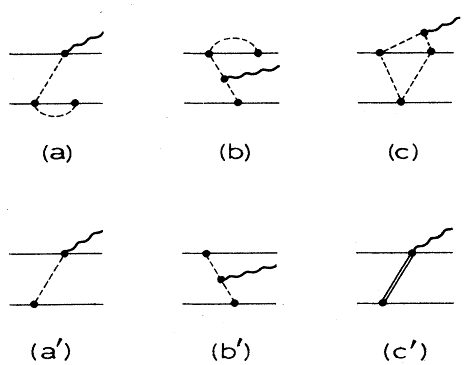

FIG. 4. Some examples of the diagrams that contribute to $\widetilde{F}^{(3)}_{3;C}$ are shown in (a)–(c). Furthermore, their approximate diagrams, due to the modifications which are discussed in the text, are also shown in (a′)–(c′). The two thin solid lines in parallel represent the ρ or σ meson.

A similar argument holds for $F^{(3)}_{3;c}$; thus, we can put $F^{(3)}_{3;c}=0$ as well. Equation (5a) now reads

$$\widetilde{F}^{(3)}_{4;c}=(F^{(4)}_{4;d}G^{(3)}\widetilde{M}^{(3)}_{4A})_c+(\widetilde{F}^{(4)}_{4;d}\widetilde{G}^{(3)}\widetilde{M}^{(3)}_{4B})_c \ , \tag{6}$$

where the subscript d indicates that only the disconnected part of the Feynman diagrams is taken into account. Since $F^{(3)}_{3;c}=\widetilde{F}^{(3)}_{3;c}=0$, we can write

$$F^{(n+2)}_{4;d}=\sum_{i\neq j\neq k\neq i} d_B^{-1}(i)\otimes d_B^{-1}(j)f^{(n)}(k) \tag{7}$$

and

$$\widetilde{F}^{(n+2)}_{4;d}=\sum_{i\neq j\neq k\neq i} d_B^{-1}(i)\otimes d_B^{-1}(j)\widetilde{f}^{(n)}(k) \ , \tag{8}$$

where $f^{(n)}$ and $\widetilde{f}^{(n)}$ are the n-particle irreducible amplitudes (or vertex functions) for $B\leftarrow\pi B$ and $B\leftarrow\gamma B$, respectively. In Refs. 20–23, these vertex functions correspond to the bare couplings which were calculated directly from the chiral bag model Lagrangian because the $\langle B|H|B\pi\pi\rangle$ interaction was neglected. In this paper we include $\langle B|H|B\pi\pi\rangle$ by adjusting the bare coupling constants, as is done for $F^{(3)}_{3;c}$ and $\widetilde{F}^{(3)}_{3;c}$. This adjustment is reasonable since we neglect the five-particle unitarity cuts and thus $f^{(2)}$ has no energy dependence. Since we include the electromagnetic coupling to first order only, we can write $\widetilde{M}^{(3)}_{4B;d}$ in Eq. (6), as

$$\widetilde{M}^{(3)}_{4B;d}=d^{(2)}_\gamma+d_\gamma^{-1}d^{(1)}_B T^{(1)}_{2;c} \ . \tag{9}$$

By taking into account all of these considerations, Eq. (6) now becomes

$$\widetilde{F}^{(3)}_{4;c}=f^{(2)}d_\pi\widetilde{M}^{(2)}_{3A;c}+f^{(2)}d_\pi d_B\widetilde{M}^{(3)}_{4A;c}+\widetilde{f}^{(2)}d_B T^{(2)}_{3;c} \ . \tag{10}$$

In order to derive physical amplitudes for the reactions under consideration, we need to take the pole residue of the amplitude in Eq. (10) at the trinucleon-bound-state energy in the initial state. In Eq. (10), it is obvious that the first term on the right-hand side (rhs) does not contribute to this residue, while at first glance the contribution of the second term on the rhs is not obvious. However, the second term does indeed contribute to the residue as will be shown later. Before turning to this point, however, let us consider $T^{(2)}_{3;c}$ in the last term. By applying the last-cut lemma to it, we can write

$$T^{(2)}_{3;c}=T^{(3)}_{3;c}+(T^{(3)}_3G^{(2)}T^{(2)})_c \ . \tag{11}$$

If the total energy of the three-nucleon system is less than the one-pion threshold (i.e., $\sqrt{s}<3m_N+m_\pi$), we can ignore the diagrams that contain the four-body cuts, which are included in $T^{(3)}_{3;c}$. Furthermore, if we ignore both the three-nucleon forces and the nine-quark interactions, which are also included in $T^{(3)}_{3;c}$, we can write $T^{(3)}_{3;c}=0$. Above the one-pion threshold, we have to include $T^{(3)}_{3;c}$. This means that we are required to solve a four-body problem for the πNNN system, provided that we restrict ourselves to include only the nucleon in the Hilbert space. If we now decide to include the isobars (at least the Δ) in the Hilbert space (and we should note here that such calculations have been done for the trinucleon bound states[3] as well as the $NN\Delta$ continuum state,[14] then we can incorporate the inelastic effect. Therefore it seems reasonable to choose $T^{(3)}_{3;c}=0$.

This idea is also supported by the recent studies[26] of N-N scattering above threshold, using the Bethe-Salpeter equation. Kloet[30] has pointed out that the Bethe-Salpeter approach for the N-Δ system and the three-body approach for the πNN system have many similarities. There are differences, too, though: the Δ in the Faddeev approach for the πNN system has a width, while the Δ in the Bethe-Salpeter approach does not; the two thresholds for the N-Δ and πNN systems are different, although it is usually claimed that the effect is small.

If we, therefore, *assume* that the inelastic channel through the (zero-width) Δ is dominant, and incorporate the Δ into our Hilbert space, then we can choose $T^{(3)}_{3;c}=0$. In this case, we can write Eq. (11) as

$$T^{(2)}_{3;c}=\sum_{i\neq j,k}[T^{(2)}_2(i)d_B(j)\bar{\delta}_{ik}T^{(1)}_2(k)+T^{(2)}_2(i)d_B(j)d_B(k)T^{(2)}_{3;c}] \ , \tag{12}$$

where $T^{(n)}_2(i)$ denotes the B-B interaction between the pair (j,k), and where $\bar{\delta}_{ik}=1-\delta_{ik}$. By writing the amplitude $T^{(2)}_{3;c}$ as the sum,

$$T^{(2)}_{3;c}=\sum_{i=1}^{3}T_i \ , \tag{13}$$

then T_i satisfies the equation,

$$T_i=v_iG^{(2)}\sum_{j=1}^{3}(\bar{\delta}_{ij}t_j+T_j) \ , \tag{14}$$

where

$$v_i=T^{(2)}_2(i)d_B^{-1}(k) \quad\text{and}\quad t_j=T^{(1)}_2(j)d_B^{-1}(k) \ .$$

By using the familiar technique of Faddeev,[31] we can solve Eq. (14) and obtain the result

$$T_i = t_i G^{(2)} \sum_j \bar{\delta}_{ij}(t_j + T_j) , \quad (15)$$

where use has been made of the fact that[18,19,23]

$$t_i = v_i + v_i d_B(j) d_B(k) t_i . \quad (16)$$

If we introduce the amplitude U_{ij} as

$$T_j = \sum_i T_2^{(1)}(i) G^{(1)}(i) U_{ij} G^{(1)}(j) T_2^{(1)}(j) , \quad (17)$$

where

$$G^{(1)}(i) = d_B(j) d_B(k), \quad i \neq j \neq k \neq i , \quad (18)$$

and insert Eq. (17) into Eq. (15), we find that U_{ij} obeys

$$U_{ij} = \bar{\delta}_{ij} G^{(2)-1} + \sum_k U_{ik} G^{(2)} T_{3;d}^{(2)}(k) \bar{\delta}_{kj} , \quad (19)$$

which is found to be the Alt-Grassberger-Sandhas (AGS) equation.[31] In Eq. (19),

$$T_{3;d}^{(2)}(i) = T_2^{(1)}(i) d_B^{-1}(i) . \quad (20)$$

It is obvious now that $T_{3;c}^{(2)}$ in Eqs. (11) and (12) has a trinucleon pole and thus can be expanded around the pole, using the eigen functions, or wave functions, as

$$G^{(2)} T_{3;c}^{(2)} G^{(2)} = \frac{|\Phi(^3\mathrm{He})\rangle\langle\Phi(^3\mathrm{He})|}{E + |\varepsilon_{^3\mathrm{He}}|} + \cdots . \quad (21)$$

The trinucleon wave function $|\Phi(^3\mathrm{He})\rangle$ is related to the AGS amplitude[31] [without the inhomogeneous term in Eq. (19)] in the bound state.[32]

In order to see that the second term on the rhs of Eq. (10) has a trinucleon pole and to derive its pole residue, we will now examine $\tilde{M}_{4A;c}^{(3)}$. Due to the last-cut lemma, we can write the three-particle irreducible amplitude in two ways:

$$\begin{aligned} \tilde{M}_{4A;c}^{(3)} &= \tilde{M}_{4A;c}^{(4)} + (M_4^{(4)} G^{(3)} \tilde{M}_{4A}^{(3)})_c \\ &\quad + (\tilde{M}_{4A}^{(4)} \tilde{G}^{(3)} \tilde{M}_{4B}^{(3)})_c & (22a) \\ &= \tilde{M}_{4A;c}^{(4)} + (M_4^{(3)} G^{(3)} \tilde{M}_{4A}^{(4)})_c \\ &\quad + (\tilde{M}_{4A}^{(3)} \tilde{G}^{(3)} \tilde{M}_{4B}^{(4)})_c . & (22b) \end{aligned}$$

We neglect five-body unitarity in the πBBB and γBBB systems and four-body unitarity in the πBB and γBB systems, which means that we have

$$\tilde{M}_{4A;c}^{(4)} = M_{4;c}^{(4)} = M_{3;c}^{(3)} = \tilde{M}_{3A;c}^{(3)} = T_{3;c}^{(3)} = 0 .$$

If we were to include the three-body force type interactions in the πBB system as well as the nine-quark states or three-body force type interactions in the BBB system, we would have $M_{3;c}^{(3)} \neq 0$ and $T_{3;c}^{(3)} \neq 0$. These effects can be included in perturbation theory within the present scheme. Therefore, we can write Eq. (22a) as

$$\begin{aligned} \tilde{M}_{4A;c} &= \tilde{t}^{(2)} d_B T_{3;c}^{(2)} \\ &\quad + [(t^{(2)} d_\pi + T_2^{(2)} d_B)(\tilde{M}_{3A;c}^{(2)} + d_B \tilde{M}_{4A;c}^{(3)})]_c . \end{aligned} \quad (23)$$

We now find that $\tilde{M}_{4A;c}^{(3)}$ has a trinucleon pole, due to the first term on the rhs of Eq. (23). To express this explicitly, we shall write $\tilde{M}_{4A;c}^{(3)}$ as

$$\tilde{M}_{4A;c} = \tilde{T}_{\pi\gamma}^{(3)} G^{(2)} T_{3;c}^{(2)} , \quad (24)$$

where

$$\begin{aligned} \tilde{T}_{\pi\gamma}^{(3)} = \sum_i \{ & \tilde{t}^{(2)}(i) G^{(1)-1}(i) \\ & + [t^{(2)}(i) d_\pi G^{(1)-1}(i) \\ & + T_2^{(2)}(i) d_B^{-1}(i)] G^{(2)} \tilde{T}_{\pi\gamma}^{(3)} \} . \end{aligned} \quad (25)$$

To derive Eq. (25), we have dropped the term involving $\tilde{M}_{3A;c}^{(2)}$ in Eq. (23) as this term does not acquire the trinucleon pole in the initial state.

Equation (25) is a four-body equation, which contains a disconnected part in the kernel. We shall now rewrite Eq. (25) in order to have a connected kernel equation and, at the same time, to dress the input amplitudes, which are two-particle irreducible in Eq. (25). First, we shall label the three baryons with the numbers 1, 2, and 3, respectively, and the pion with number 4. We will then label the π-B pairs with numbers 1, 2, and 3, which correspond to the pairs (4,1), (4,2), and (4,3). We will also then label the B-B pairs with the numbers 4, 5, and 6, which correspond to the pairs (1,2), (2,3), and (3,1). We can thus write $\tilde{T}_{\pi\gamma}^{(3)}$ as a sum of the six terms,

$$\tilde{T}_{\pi\gamma}^{(3)} = \sum_{i=1}^{6} Y_i , \quad (26)$$

where Y_i obeys the following quasi-two-body equation,

$$Y_i = b^{(2)}(i) + c^{(2)}(i) G^{(2)} \sum_{j=1}^{6} Y_j , \quad (27)$$

where

$$b^{(2)}(i) = \begin{cases} \tilde{t}^{(2)}(i) G^{(1)-1}(i) & \text{for } i = 1,2,3, \\ 0 \quad \text{for } i = 4,5,6, \end{cases} \quad (28)$$

and

$$c^{(2)}(i) = \begin{cases} t^{(2)}(i) d_\pi G^{(1)-1}(i) & \text{for } i = 1,2,3 \\ T_2^{(2)}(i) d_B^{-1}(i) & \text{for } i = 4,5,6 . \end{cases} \quad (29)$$

We first move the term that involves Y_i on the rhs of Eq. (27), to the left-hand side, and then use Eq. (16) and the following equations:[21]

$$t^{(1)}(i) = t^{(2)}(i) + t^{(1)}(i) d_\pi d_B(i) t^{(2)}(i) , \quad (30)$$

$$\tilde{t}^{(1)}(i) = \tilde{t}^{(2)}(i) + t^{(1)}(i) d_\pi d_B(i) \tilde{t}^{(2)}(i) , \quad (31)$$

and

$$T_2^{(1)}(i) = T_2^{(2)}(i) + T_2^{(2)}(i) G^{(1)}(i) T_2^{(1)}(i) . \quad (32)$$

Equation (32) is equivalent to Eq. (14) if $T_2^{(2)} = v_i$. We then finally arrive at the desired equation,

$$Y_i = b^{(1)}(i) + c^{(1)}(i) G^{(2)} \sum_{j=1}^{6} \bar{\delta}_{ij} Y_j . \quad (33)$$

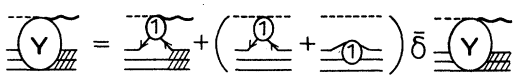

FIG. 5. Diagrammatic representation of Eq. (33).

This is a Mitra-Yakubovsky[17] type four-body equation whose kernel is connected, and the input amplitudes are renormalized, i.e., one-particle irreducible. Equation (30) is diagrammatically shown in Fig. 5. At this stage, we shall not go any further into the details of solving Eq. (33).[33]

We will now return to Eq. (10). By inserting Eqs. (24), (25), (18), and (33) into Eq. (10), we obtain

$$\tilde{F}^{(3)}_{4;c} = \left[\tilde{F}^{(4)}_{4;d} + F^{(4)}_{4;d} G^{(3)} \sum_{i=1}^{6} Y_i \right] G^{(2)} T^{(2)}_{3;c} \ , \tag{34}$$

where we have dropped the first term on the rhs of Eq. (10) because of the lack of the trinucleon pole in the initial state. Above, $\tilde{T}^{(3)}_{\pi\gamma}$ (or Y_i) is a solution of the integral equation [i.e., Eq. (25) or Eq. (33)], whose input is dressed (or one-particle irreducible, while Eq. (34) contains "bare" couplings, i.e., $f^{(2)}$ and $\tilde{f}^{(2)}$.[34] Since our aim is to compare the final equations with the conventional calculations,[6–8] in which all of the input into the two-body amplitudes is phenomenological (or dressed), it is ideal to rewrite Eq. (34) so as to have the input amplitudes which are one-particle irreducible. To accomplish this, we insert Eq. (33) into Eq. (34) and then use the equality[21] for $B \leftarrow \gamma B$, i.e.,

$$\tilde{f}^{(1)}(i) = \tilde{f}^{(2)}(i) + f^{(2)}(i) d_B(i) d_\pi \tilde{t}^{(1)}(i) \ , \tag{35}$$

in order to dress the first term on the rhs of Eq. (34), which then yields

$$\tilde{F}^{(3)}_{4;c} = \sum_{i=1}^{3} \left[\tilde{f}^{(1)}(i) d_B(i) + f^{(2)}(i) d_B(i) d_\pi \sum_{j=1}^{6} \left[\bar{\delta}_{ij} b^{(1)}(j) + c^{(1)}(j) G^{(2)} \sum_{k=1}^{6} \bar{\delta}_{jk} Y_k \right] G^{(2)} \right] T^{(2)}_{3;c} \ . \tag{36}$$

We then pick up the $i \neq j$ component from the last term and combine it with the second term on the rhs of Eq. (36). By using Eq. (33) and then the equalities[20,21] for $B \leftarrow \pi B$, i.e.,

$$f^{(1)}(i) = f^{(2)}(i) + f^{(2)}(i) d_B(i) d_\pi t^{(1)}(i) \ , \tag{37}$$

we finally arrive at the following simple expression for $\tilde{F}^{(3)}_{4;c}$:

$$\tilde{F}^{(3)}_{4;c} = \tilde{R}^{(3)} G^{(2)} T^{(2)}_{3;c} \ , \tag{38}$$

where

$$\tilde{R}^{(3)} = \sum_{i=1}^{3} \left[\tilde{F}^{(3)}_{4;d}(i) + F^{(3)}_{4;d}(i) G^{(3)} \sum_{j=1}^{6} \bar{\delta}_{ij} Y_j \right] . \tag{39}$$

Equation (39) and its decomposition are diagrammatically shown in Fig. 6.

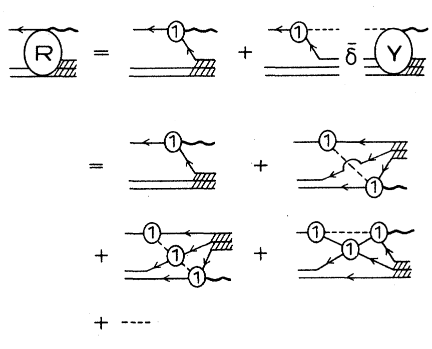

FIG. 6. Decomposition of $\tilde{R}^{(3)}$ as given in Eq. (39).

We are now in a position to write the physical amplitudes for the reaction processes under consideration. To derive the physical amplitude, we must firstly take the pole residue of $\tilde{F}^{(2)}_{4;c}$ [Eq. (4)] at the trinucleon-bound-state energy in the initial state. We then take the pole residue at the trinucleon pole or deuteron pole in the final state which corresponds to the ^{3}He or $p+d$ final states, respectively. The two-body t matrix can be expanded around the deuteron-bound-state energy as[18,23]

$$G^{(1)} T^{(1)}_2 G^{(1)} = \frac{|\Psi(d)\rangle\langle\Psi(d)|}{E + |\varepsilon_d|} + \cdots \ , \tag{40}$$

where $|\Psi(d)\rangle$ is the deuteron wave function. Assuming that the two-body interaction can be approximated in a separable form, then the t matrix can be written as

$$T^{(1)}_2 = \sum_{\mu} |\phi_\mu\rangle \tau_\mu \langle \phi_\mu| \ , \tag{41}$$

where $|\phi_\mu\rangle$ and τ_μ are the form factor and propagators for the quasiparticle, respectively.[18,22] We could use either Eq. (40) or Eq. (41) to derive the amplitude for $p+d \leftarrow \gamma + {}^3$He. However, in the following derivation, we will only use Eq. (41) in order to take into account the effects of the rescattering process before reaching the final state, $p+d$ or $2p+n$. The rescattering effect (usually referred to as the final state interaction) is taken into account through $T^{(2)}_{3;c}$ in Eq. (4), which can be written, by inserting Eq. (41) into Eq. (17), as

$$T^{(2)}_{3;c} = \sum_{\substack{ij \\ \mu\nu}} |\phi_\mu(i)\rangle \tau_\mu(i) X_{\mu i;\nu j} \tau_\nu(j) \langle \phi_\nu(j)| \ , \tag{42}$$

where

$$X_{\mu i;\nu j} = \langle \phi_\mu(i)| G^{(1)} U_{ij} G^{(1)} |\phi_\nu(j)\rangle \tag{43}$$

and

$$X_{\mu i;\nu j}=Z_{\mu i;\nu j}+\sum_{\rho k} Z_{\mu i;\rho k}\tau_\rho(k)X_{\rho k;\nu j} . \tag{44}$$

We use Eq. (21) exclusively in order to obtain the pole residue at the trinucleon-bound-state energy. The resulting physical amplitudes are as follows:

$$X({}^3\text{He}\leftarrow\gamma\,{}^3\text{He})=\langle\Psi({}^3\text{He})|T({}^3\text{He}\leftarrow\gamma\,{}^3\text{He})|\Psi({}^3\text{He})\rangle , \tag{45a}$$

$$X(p+d\leftarrow\gamma\,{}^3\text{He})=\langle\phi_d,\chi_p|T(p+d\leftarrow\gamma\,{}^3\text{He})|\Psi({}^3\text{He})\rangle , \tag{45b}$$

and

$$X(2p+n\leftarrow\gamma\,{}^3\text{He})=\langle\chi_p,\chi_p,\chi_n|T(2p+n\leftarrow\gamma\,{}^3\text{He})|\Psi({}^3\text{He})\rangle , \tag{45c}$$

where

$$T({}^3\text{He}\leftarrow\gamma\,{}^3\text{He})=\tilde{R}^{(3)} , \tag{46a}$$

$$T(p+d\leftarrow\gamma\,{}^3\text{He})=\left[1+\sum_{ij\nu}X_{di;\nu j}\tau_\nu(j)\langle\phi_\nu(j)|\right]G^{(1)}\tilde{R}^{(3)} , \tag{46b}$$

and

$$T(2p+n\leftarrow\gamma\,{}^3\text{He})=\sum_{\substack{ij\\ \mu\nu}}|\phi_\mu(i)\rangle\tau_\mu(i)[\langle\phi_\mu(i)|+X_{\mu i;\nu j}\tau_\nu(j)\langle\phi_\nu(j)|]G^{(1)}\tilde{R}^{(3)} . \tag{46c}$$

In Eq. (45), χ_p and χ_n are the asymptotic wave functions for the proton and the neutron, respectively. Equations (45a)–(45c) are diagrammatically shown in Fig. 7. The amplitudes in Eqs. (45b) and (45c) with the help of Eqs. (46b) and (46c) are the same as those of Refs. 9 and 15, except that (1) our equations include the four-body correlation term, which is given in the second term on the rhs of Eq. (39), as a result of the requirement of four-body unitarity, (2) our equations are Lorentz invariant, (3) the N and other higher-mass isobars can be included in the intermediate states, and (4) we can include both of the three-body type forces in the πBB and BBB systems as well as the exotic channels, in the context of the six- and nine-quark states. A more detailed discussion on points (2)–(4) is the subject of the next section.

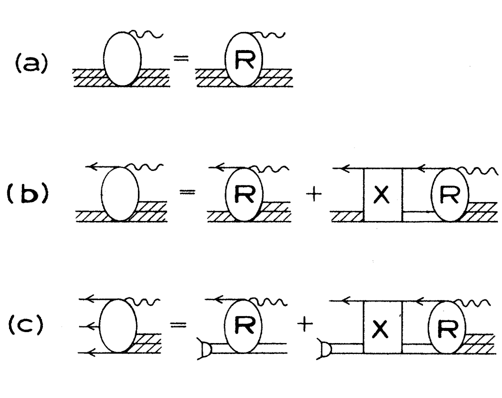

FIG. 7. Diagrammatic representation of the amplitudes for $\gamma+{}^3\text{He}\rightarrow{}^3\text{He}$, $p+d$, and $2p+n$, which are given in Eqs. (45) and (46).

III. DISCUSSION

As mentioned in Sec. I, the equations that describe the reactions under consideration at energies above pion threshold, are desirable to satisy (1) Lorentz invariance, (2) multiparticle unitarity and isobars [at least the Δ, possibly $N^*(1440)$, and some others] (3) gauge invariance, and (4) the exotic channels, or explicit quark degrees of freedom, such as the six- and nine-quark states. We will now describe how these requirements can be fulfilled in our present approach and implemented into the conventional Faddeev codes.[6,8,16,25]

A. Lorentz invariance

Our derivation of the equations in Sec. II is based on a (nonspecified but) relativistic equation. At present, a number of Lorentz invariant two- and three-body equations are known: the Bethe-Salpeter, one-particle on-shell,[34], and light-front equations[35] as well as the Blankenbeclar-Sugar[36] (BbS) and Aaron-Amado-Young[37] (AAY) equations as approximations for the Bethe-Salpeter equation in the two- and three-body systems. [For example, see Ref. 38 for a current review.] For three-body systems, the only method which is ready for numerical computation is the AAY equation, which is a generalization of the BbS technique (for two-body systems) to three-body systems. The BbS and AAY equations require the removal of the antibaryon states because of the minimal implementation of the unitarity constraint. This removal breaks gauge invariance.[39] One can relate the BbS and AAY equations to the two-body Lippman-Schwinger and three-body AGS (Ref. 31) equations by keeping Lorentz invariance; this requires a factor which arises from Lorentz invariance of the two-body t

matrices, in the translation from the two-body center-of-mass (c.m.) frame to the three-body c.m. frame.[40] This factor becomes unity in the nonrelativistic limit, meaning that the AAY equation is reduced to the nonrelativistic Faddeev (or the AGS) equation in this limit. In our theory, no particular relativistic equation is meant to be used. However, the AAY equation is found to be the most convenient for computation at the present time. The use of this equation requires minimal modification of the presently available nonrelativistic Faddeev codes for the three-nucleon system,[25] with respect to the implementation of Lorentz invariance.

B. Multiparticle unitarity and isobars

Because of the energy available in electron machines of the future, we need to take into account the production of the multipions and isobars. In this paper we only expose the four-body unitarity cuts, and neglect the five- (or more) particle unitarity cuts. This is because we expect that the inelastic channels are dominated by the πNNN state and the states which contain the isobars, i.e., $\pi\Delta NN$, $\Delta\Delta N$, $\Delta\Delta\Delta$, and $N^*(1440)\Delta N$, etc., all of which contribute to three- or four-body unitarity.

Below one-pion threshold, our equation has made one correction to the method of Refs. 8 and 16, which is the four-body correlation term [the second term on the rhs of Eq. (39)]. Above threshold, one has to consider the coupling of the Δ state. There are such Faddeev computer codes available for bound states,[3] and for continuum states.[14] In most conventional Faddeev computer codes,[6–8,16,25] the two-body (N-N) input amplitudes are parametrized in separable forms. The simple inclusion of the N-Δ and Δ-Δ states, by parametrizing them in a similar way, causes a double counting problem, as the N-Δ (in the input) may include the π-N-N continuum states as well. Therefore, unless one knows the exact method to separate the N-Δ state from the π-N-N state, then one should not apply the method blindly.

Not the easiest, but the least unambiguous solution for this double counting problem, is to construct the input amplitudes (in the two-body systems) from a Lagrangian. One could start from the chiral bag model Lagrangian as we did for the photonuclear reactions in the one- and two-baryon systems.[21–23] However, problems with this are that there have been no chiral bag models which contain mesons other than the pion, and also that the chiral soliton models[41] and the Skyrme model[42] (as alternative to the chiral bag model) are not developed or ready for our method to be applied to them, with respect to quantization.

It is conceivable then that one could use an alternative method, as an extension of the conventional Faddeev codes, by sacrificing uniqueness to a certain extent. For example, one could construct the N-Δ and Δ-Δ potentials in the one-boson-exchange (OBE) model.[26] (It should be noted here that the Δ has no width.) The zero width Δ is considered to be a bare particle (e.g., the Δ in the bag model,[43] in the context of three quarks, without dressing due to the chiral pion). We also know that the mass shift in the real axis, due to the pion self energy, is the order of $\sim 30\%$ for the bag radius, $R \sim 1$ fm.[44] Therefore, if one solves a coupled two-channel problem for the N-Δ systems, using real and symmetric separable potentials, and drops the π-B-B intermediate states that appear in the four-body equation [Eq. (33)], then we can avoid the double counting problem to the extent that we ignore the mass shift. It should be noted here that the π-B-B intermediate states are not included in the AGS equation,[31] Eq. (19), for the BBB system.

C. Gauge invariance

In the photonuclear reactions for one-,[21] two-,[11,23,45] and three-baryon[8,16,45] systems, the electromagnetic $U(1)$ gauge invariance is violated for several different reasons. For example, certain diagrams are omitted for the sake of computational simplicity,[8,16,21,23] and a double counting problem occurs due to the misidentification of $\tilde{t}^{(1)}$ with $\tilde{t}^{(0)}$ [see Eq. (1)] in some of the theories.[11,45] It is conceivable that, as in Ref. 46, the application of the Ward-Takahashi identity[47] to the present theory guarantees gauge invariance. However, this requires that we lump together $\tilde{t}^{(1)}$ and the pole term in Eq. (1) in order to make $\tilde{t}^{(0)}$. This is not attempted in our present theory. Recently, we proposed a theory in order to maintain gauge invariance in a coupled channel (πB-γB) approach.[48] This will give us a gauge invariant $\tilde{t}^{(0)}$ amplitude, both on and off the energy shell. Therefore, if $\tilde{t}^{(0)}$ is the only ingredient in a theory for photonuclear reactions, then we can guarantee gauge invariance. However, in our present formulations, both the $\tilde{t}^{(1)}$ and $\tilde{f}^{(1)}$ are separate and neither of them is gauge invariant by itself, except for the isobars. In the case of the isobars, the presence of the derivative couplings make it possible to write $\tilde{f}^{(1)}$ in terms of the gauge invariant tensor, $F_{\mu\nu}$; thus, $\tilde{t}^{(1)}$ can also be made gauge invariant because its only ingredients are $f^{(1)}$ and $\tilde{f}^{(1)}$. As a result, gauge invariance is not guaranteed in our present approach.

However, it may be worth noting here that it is conceivable that by taking advantage of Eq. (1), our final equations can be split into two: a manifestly gauge invariant piece and the rest. We expect that the gauge noninvariant part is suppressed considerably at all of the energy ranges because (i) the gauge noninvariant part is zero in the soft photon limit, and this factor is expected to carry over to the low-energy regime, (ii) near the resonance energies [$\Delta(1232)$,$N^*(1470)$,. . .], the isobar pole contributions (which are gauge invariant) dominate, and (iii) the MEC contribution (or the four-body correlation term in our present theory) and the higher multipole amplitudes are expected to be more important at higher energies. Both of these are automatically included in our present theory, therefore contributing to the compensation for the loss of gauge invariance.

The quark-antiquark (q-$\bar{q}$) pair contribution may also be incorporated, either within the chiral bag model,[21,34] or in a more phenomenological way by parametrizing the input amplitudes, in order to compensate for the loss of gauge invariance caused by the neglect of the B-$\bar{B}$ pair terms, as mentioned in point (1).

Before closing this subsection, we will mention the fact that there is a minimal way to guarantee gauge invariance for the amplitudes which are derived within a unitary theory. For this purpose, we will introduce an operator and define a new electromagnetic current ($\tilde{J}$) from its old current (J) which is gauge noninvariant:

$$\tilde{J}_\mu = H_{\mu\nu} J^\nu ,$$

where

$$H_{\mu\nu} = g_{\mu\nu} - \frac{p_\mu k_\nu}{(p \cdot k)} .$$

It is obvious that the new current $\tilde{J}^\mu$ satisfies both the unitarity and gauge condition. A drawback of this method, however, is its nonuniqueness. This method, therefore, should be compared to the K-matrix method, which is commonly used to recover unitarity, but which is not unique in the choice of the K matrix as an input in order to compute the t matrix.

D. Quark degrees of freedom

It is believed that there exists duality between quarks and mesons.[42] This means that one might be able to describe the strong interactions between baryons, using either quarks or mesons. In one case the quark description may be simpler, and in another, the meson description may be more so.[41,42] In our formulation, however, both the six-quark states and the four- (or more) meson intermediate states can be incorporated through $T_2^{(3)}$, which is contained in $T_2^{(2)}$ of Eq. (16). Furthermore, both the nine-quark states and three-body forces are incorporated through $T_{3;c}^{(3)}$ which is found in Eq. (11) and which was dropped in order to derive Eq. (19).

IV. CONCLUSION

In this paper, we have presented a unified theory of $\gamma + {}^3\text{He} \rightarrow {}^3\text{He}$, $p+d$, and $2p+n$. We have required four-body unitarity and derived sets of equations to describe these three reactions. The trinucleon wave functions in the initial or final states satisfy a three-body (Faddeev) equation. The final state interaction was taken into account by solving the three-body equation in the continuum state. The interaction term consists of two parts: the impulse one-baryon current and a few-body correlation term which satisfies a (Mitra-Yakubovski–type[17]) four-body equation. Our equations are convenient for implementation, into the conventional nonrelativistic Faddeev codes (Refs. 3, 6–8, 16, and 25), of the following: (1) relativistic invariance, (2) isobar degrees of freedom, and (3) the exotic channels due to the manifestation of the explicit quark degrees of freedom. In fact, our equations for the $p+d$ and $2p+n$ final states are identical to those of Gibson and Lehman,[8,9,16] apart from the above three points and the four-body correlation term.

In this paper, we have not attempted to reduce the four-body equation to a computationally viable form. This may be done, however, by introducing the quasiparticle states or by approximating the four-body amplitudes with effective two-body amplitudes. Our present approach can be extended to unify the electromagnetic and nonelectromagnetic interactions in the three-nucleon system involving nonmesonic and mesonic final states, e.g., $p+d \rightarrow p+d$, $\pi^0 + {}^3\text{He}$, $\gamma + {}^3\text{He} \rightarrow p+d$, and $\pi^0 + {}^3\text{He}$. This will be discussed elsewhere.

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