# Evaluation of $E2$ form factor: $^{24}Mg$

José Ricardo Marinelli
*Departamento de Fisica da Universidade Federal de Santa Catarina, Florianópolis, Santa Catarina, Brazil*

José Roberto Moreira
*Instituto de Física e Instituto de Eletrotécnica e Energia da Universidade de São Paulo, São Paulo, Brazil*
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Longitudinal and transverse electron scattering form factors for the $2^+$ state at 1.37 MeV of the $^{24}Mg$ nucleus were evaluated with rotational model wave functions. Four different approaches were used for the transverse $E2$ form factor: projected Hartree Fock, cranking model, rigid rotor, and irrotational flow. For the nuclear intrinsic wave function, the Nilsson model was assumed in all approaches yielding the calculation of the form factor in the plane-wave and distorted-wave Born approximations. The results are discussed and compared with a recent measurement performed for 180° electrons scattered from this state. The distorted-wave Born approximation calculation, taking into account first-order corrections due to $\langle J_1^2 \rangle^{-1}$, shows that projected Hartree Fock and irrotational flow models give the best agreement with the available data and compete in quality with more complex calculations performed in the "shell model" approach.

## I. INTRODUCTION

Recently, electron scattering cross sections at 180° were measured for several excited levels of the $^{24}Mg$ nucleus.[1] The extracted form factors were described with shell-model calculation so that effects of relativistic dynamics over the structure of bound states were taken into account. In particular, the first level $2^+$ at 1.37 MeV was measured at the effective momentum transfer between 0.87 and 2.07 $fm^{-1}$. Earlier measurements carried out at lower scattering angles yielded accurate knowledge of Coulomb form factors for this level up to 3 $fm^{-1}$.[2,3]

In this paper we have undertaken to analyze these results in a framework different from the one used in Ref. 1. As is well known, $^{24}Mg$ is a quite deformed nucleus and some of its excited states can be described as members of rotational bands. This is the case for the $2^+$ level at 1.37 MeV, which can be understood as the first excited state of a rotational band constructed over the ground-state level. Another argument justifying our approach is the fact that some efforts, in the last few years, had been made to describe transverse form factors in rotational nuclei,[4-6] which means the evaluation of nuclear current densities (in the momentum transfer space).

This kind of analysis provides unique information about how the nucleus rotates. These calculations have shown that rotational form factors are strongly masked in measurements carried with even-odd deformed nuclei, except in some specific situations,[5,7] since the transverse form factor is dominated by the unpaired nucleon. Evaluations of the first excited level $2^+$ of $^{166}Er$ have shown, however, that the $E2$ form factor is quite sensitive to the rotational model used.[6,8] For all these reasons, we believe that the analysis of the transverse form factor from the $2^+$ level of $^{24}Mg$ should also be performed using such an approach. Even knowing that deformed *s-d* shell nuclei in general cannot be treated as "good rotors" as compared with heavy nuclei, rotational models have been used with significant success in this region, yielding good descriptions of some Coulomb form factors.[9]

## II. ROTATIONAL MODELS AND FORM FACTORS

First, let us describe the main results for the form factors obtained by electron scattering, with the use of rotational models from Ref. 5. In that paper, two microscopic models were used to describe nuclear rotations: projection of states of good angular momentum [projected Hartree-Fock (PHF)] and the cranking model. We consider here only even-even nuclei.

Under the PHF approximation, the intrinsic wave function $|\phi_K\rangle$ is tied to the total nuclear wave function by means of (assuming axial symmetry and $K=0$),[5,10]

$$|IOM\rangle = \frac{2I+1}{8\pi^2(N_0^I)^{1/2}} \int d\Omega\, D_{MO}^{*I} R(\Omega)|\phi_0\rangle \ . \qquad (1)$$

The reduced matrix element for a tensor operator $T_\mu^\lambda$ (restricted to intraband transitions) can be written as

$$\langle I_f 0\|T^\lambda\|I_i 0\rangle = \frac{\hat{I}_f \hat{I}_i}{(N_0^{I_f} N_0^{I_i})^{1/2}} \sum_\nu (I_i - \nu\lambda\nu|I_f 0\rangle \int_0^{\pi/2} d\beta \sin\beta\, d_{-\nu,0}^{I_i}(\beta) \langle\phi_0|T_\nu^\lambda e^{-i\beta J_y}|\phi_0\rangle \ . \qquad (2)$$

The integrand in the above expression can be expanded in powers of the Euler angle $\beta$, and the result of the integration can also be written as an expansion in powers of $\langle J_1^2\rangle^{-1}$ where $\langle J_1^2\rangle = 2\langle\phi_0|J_y^2|\phi_0\rangle$, if $I_i=0$ (which is the typical situation for electron scattering measurements). Taking into account only terms up to first order in the expansion we obtain

$$\langle I_f 0 \| T^{C\lambda} \| 00 \rangle = \langle \phi_0 | T_0^{C\lambda} | \phi_0 \rangle \left[ 1 + \frac{\lambda(\lambda+1)}{2\langle J_1^2 \rangle} \right] - \frac{2}{\langle J_1^2 \rangle} \langle \phi_0 | T_0^{C\lambda} J_y^2 | \phi_0 \rangle_L \,, \tag{3a}$$

where

$$\langle \phi_0 | T_0^{c\lambda} J_y^2 | \phi_0 \rangle_L = \langle \phi_0 | T_0^{C\lambda} J_y^2 | \phi_0 \rangle - \langle \phi_0 | T_0^{C\lambda} | \phi_0 \rangle \langle J_y^2 \rangle$$

and

$$\langle I_f 0 \| T^{E\lambda} \| 00 \rangle = \frac{\sqrt{\lambda(\lambda+1)}}{2\langle J_1^2 \rangle} \langle \phi_0 | T_1^{E\lambda} J_- + T_{-1}^{E\lambda} J_+ | \phi_0 \rangle \,, \tag{3b}$$

where $T_\mu^{C\lambda}$ and $T_\mu^{E\lambda}$ are, respectively, Coulomb and transverse electric operators, as defined in Ref. 11. For small $\lambda$ and large nuclear deformation ($\langle J_1^2 \rangle \gg 1$), expressions (3a) and (3b) are good approximations to the exact value of the matrix element; in this situation, even first-order corrections to the Coulomb matrix element should add small contribution to the usual zero-order term.[5]

Within the cranking model, the intrinsic wave function can be approximately written as[5,12]

$$|\phi_w\rangle = |\phi_0\rangle + w \sum_{K' \neq 0} \frac{\langle \phi_{K'} | J_x | \phi_0 \rangle}{\mathcal{E}_{K'} - \mathcal{E}_0} |\phi_{K'}\rangle \,, \tag{4}$$

where $\omega = \sqrt{I(I+1)}/\mathcal{J}_{CR}$ and $\mathcal{J}_{CR}$ is the Inglis moment of inertia.[12]

Applying the Bohr-Mottelson factorization approximation in the above quoted intrinsic wave function, or equivalently, carrying a renormalization of the $T_\mu^\lambda$ operators in order to include first-order corrections in $\omega$, and using the rotor eigenfunctions,[13] it is possible to show that

$$\langle I_f 0 \| T^{C\lambda} \| 00 \rangle \simeq \langle \phi_0 | T_0^{C\lambda} | \phi_0 \rangle \,, \tag{5a}$$

$$\langle I_f 0 \| T^{E\lambda} \| 00 \rangle \simeq \frac{\sqrt{\lambda(\lambda+1)}}{2\mathcal{J}_{CR}} \sum_{K' \neq 0} \left[ \frac{\langle \phi_0 | T_1^{E\lambda} | \phi_{K'} \rangle \langle \phi_{K'} | J_- | \phi_0 \rangle}{\mathcal{E}_{K'} - \mathcal{E}_0} + \frac{\langle \phi_0 | T_{-1}^{E\lambda} | \phi_{K'} \rangle \langle \phi_{K'} | J_+ | \phi_0 \rangle}{\mathcal{E}_{K'} - \mathcal{E}_0} \right] . \tag{5b}$$

It should be noted that under this approximation Siegert's theorem is fully satisfied.

Finally, let us consider the collective rotational model, with an *a priori* assumed type of rotation. In general, Coulomb matrix elements can be written as[11]

$$\langle I_f \| T^{C\lambda} \| 00 \rangle = i^\lambda \int_0^\infty dr\, r^2 \rho_\lambda^p(r) j_\lambda(qr) \,, \tag{6}$$

where $\rho_\lambda^p(r)$ is the multipole charge density of $\lambda$ order, which can be obtained from a phenomenological distribution or from single-particle wave functions for all protons in the nucleus.

The transverse electric matrix element is

$$\langle I_f 0 \| T^{E\lambda} \| 00 \rangle = i^{\lambda+1} \int_0^\infty dr\, r^2 \left[ \left[ \frac{\lambda+1}{2\lambda+1} \right]^{1/2} j_{\lambda-1}(q) J_{\lambda\lambda-1}(r) - \left[ \frac{\lambda}{2\lambda+1} \right]^{1/2} j_{\lambda+1}(qr) J_{\lambda\lambda+1}(r) \right] , \tag{7}$$

where $J_{\lambda\lambda\pm1}$ is the multipolar nuclear current density. Assuming that under nuclear rotation the system is rigid (rigid rotor model), we get[8]

$$J_{\lambda\lambda-1}^{RR}(r) = \frac{i\lambda(\lambda+1)}{2\mathcal{J}_{\rm rig}} [\lambda(2\lambda+1)]^{-1/2} r \rho_\lambda^p(r) \tag{8a}$$

and

$$J_{\lambda\lambda+1}^{RR}(r) = \left[ \frac{\lambda}{\lambda+1} \right]^{1/2} J_{\lambda\lambda-1}^{RR}(r) \,, \tag{8b}$$

where $\mathcal{J}_{\rm rig}$ is the rigid rotor moment of inertia.

Another classical description assumes the nucleus as a permanent deformed incompressible irrotational fluid. Under this framework we can write[7]

$$J_{\lambda\lambda-1}^{IR}(r) = \frac{i}{R_0^{\lambda-2}} \frac{\lambda(\lambda+1)}{2\mathcal{J}_{\rm irr}} \left[ \frac{2\lambda+1}{\lambda} \right]^{1/2} \beta_\lambda r^{\lambda-1} \frac{\rho_0^p(r)}{\sqrt{4\pi}} \,,$$

$$J_{\lambda\lambda+1}^{IR}(r) = 0 \,, \tag{9}$$

where $\beta_\lambda$ is the nuclear deformation of $\lambda$ order and quadratic terms in this parameter are neglected, $R_0$ is the radius of the equivalent sphere, and $\mathcal{J}_{\rm irr}$ is the corresponding moment of inertia.

The results for the four models summarized above can be used for the evaluation of the scattering cross section in plane-wave Born approximation (PWBA), which is

$$\frac{d\sigma}{d\Omega}=\sigma_{\rm Mott}\left\{\sum_\lambda\left[[F^{C\lambda}(q)]^2+\left[\tfrac{1}{2}+tg^2\frac{\theta}{2}\right][F^{T\lambda}(q)]^2\right]\right\}, \tag{10}$$

where

$$F^{C\lambda}(q)=\frac{\sqrt{4\pi}}{Z}\langle I_f 0\|T^{C\lambda}\|00\rangle$$

and

$$F^{T\lambda}(q)=\frac{\sqrt{4\pi}}{Z}\langle I_f 0\|T^{E\lambda}|00\rangle \ .$$

A better result can be obtained with a distorted-wave Born approximation[14] (DWBA) calculation; to perform this treatment it is necessary to write expressions similar to the ones already written for the reduced matrix elements, where $T_\mu^\lambda$ will be replaced by charge and current multipole operators, respectively, in the Coulomb and transverse electric form factors expressions. These matrix elements can then be used straightforwardly for DWBA cross section evaluation.

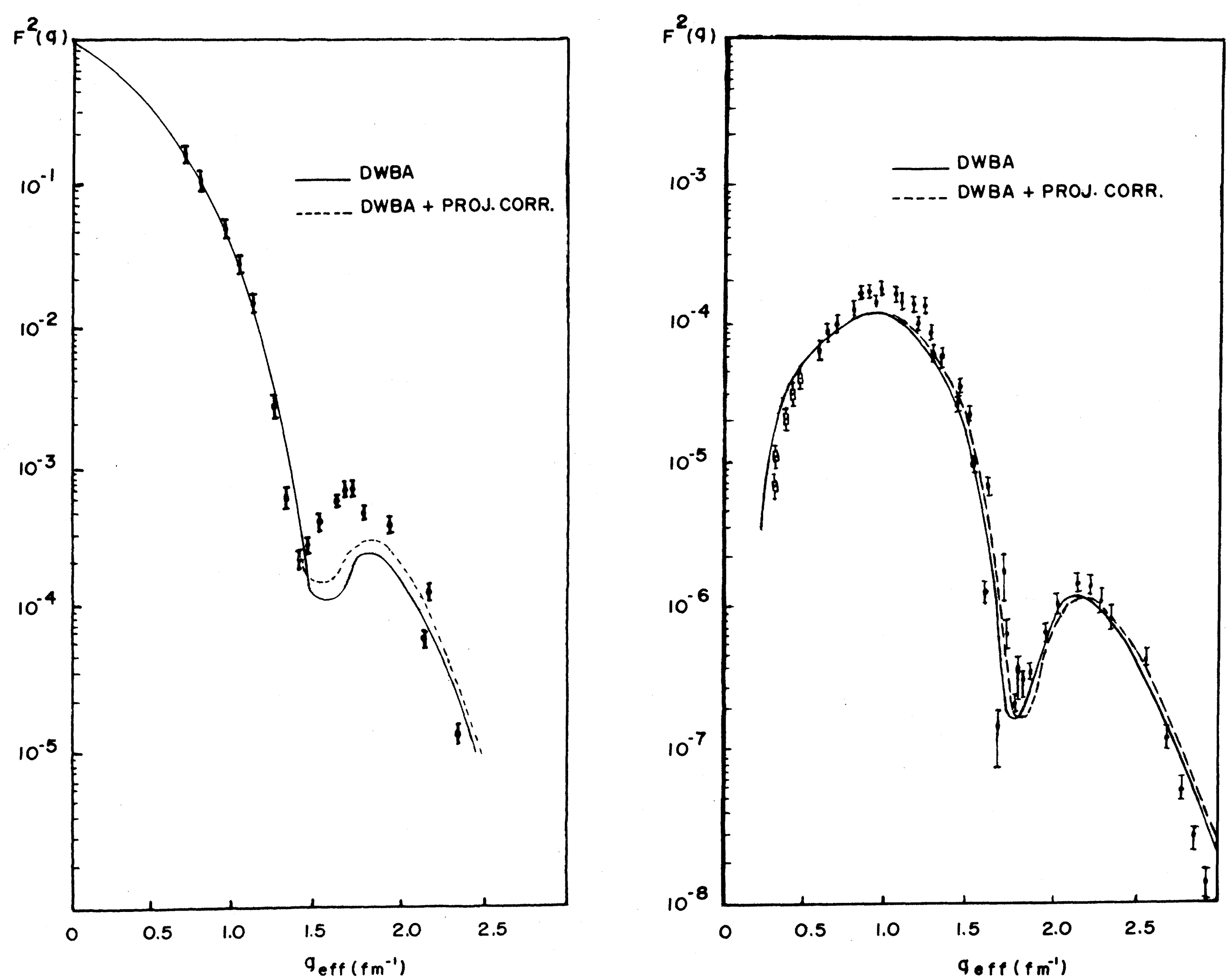


FIG. 1. (a) Elastic Coulomb form factor for $^{24}$Mg evaluated in DWBA, without (——) and with (— — —) first-order correction, in PHF approximation (see text). Data are taken from Ref. 2; (b) The same as (a), for the $2^+$ level of $^{24}$Mg. Data are taken from Refs. 1, 2, and 3.

## III. RESULTS AND CONCLUSIONS

In order to generate the intrinsic nuclear wave function, we used the Nilsson model. The deformation parameter was chosen to reproduce the $BE2$ $(0^+ \rightarrow 2^+)$ transition probability for $^{24}$Mg; the spin orbit and the "$l^2$" parameters were used as prescribed in Ref. 15. It is important to note that $N$ mixing are taken into account in the Nilsson Hamiltonian diagonalization procedure; independent-particle states on the deformed potential are expanded in a spherical basis including $N$ values up to 7. This procedure yields good convergence in the form-factors calculation and includes the contribution of all nucleons, avoiding the necessity of using effective charges. Pairing correlations were neglected, since we are dealing with an $N = Z$ nucleus, in the low-mass region.[16,18] Finally, form factors were evaluated with the usual center of mass correction and the nucleon form-factor correction.[17]

Figures 1(a) and 1(b) show results for the Coulomb elastic and first inelastic ($2^+$ level) form factors for $^{24}$Mg obtained in DWBA. Two theoretical curves are shown: The first one displays the results under the usual factorization approximation for the nuclear wave function; the second one includes first-order corrections due to $\langle J_1^2 \rangle^{-1}$ [see Eq. (3a)]. Differences between both cases are very small as already discussed, except on the diffraction minimum. Obviously, if we neglect these first-order corrections, the four approximations presented in Sec. II would provide identical results for the Coulomb form factors if the same intrinsic charge distribution is used. It is further remarkable that the inelastic Coulomb form factor shows an excellent agreement with the experimental results.

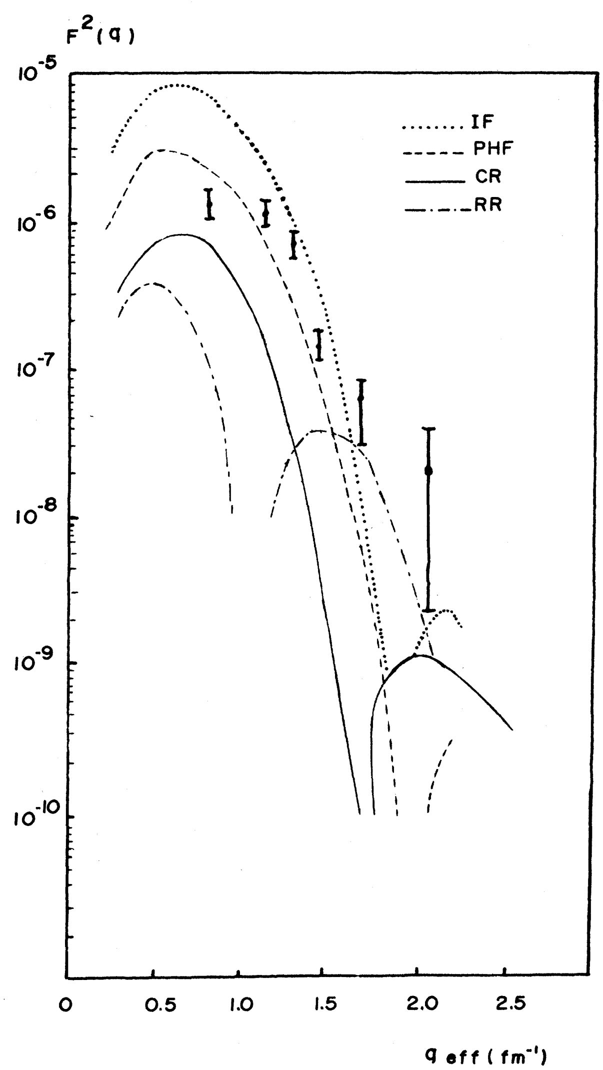


FIG. 2. Transverse $E2$ form factor from the first $2^+$ level of $^{24}$Mg, calculated in PWBA with the following approximations: PHF (— — —), irrotational flow (· · · ·), cranking (——), and rigid rotor (—·—·—·). Data are taken from Ref. 1.

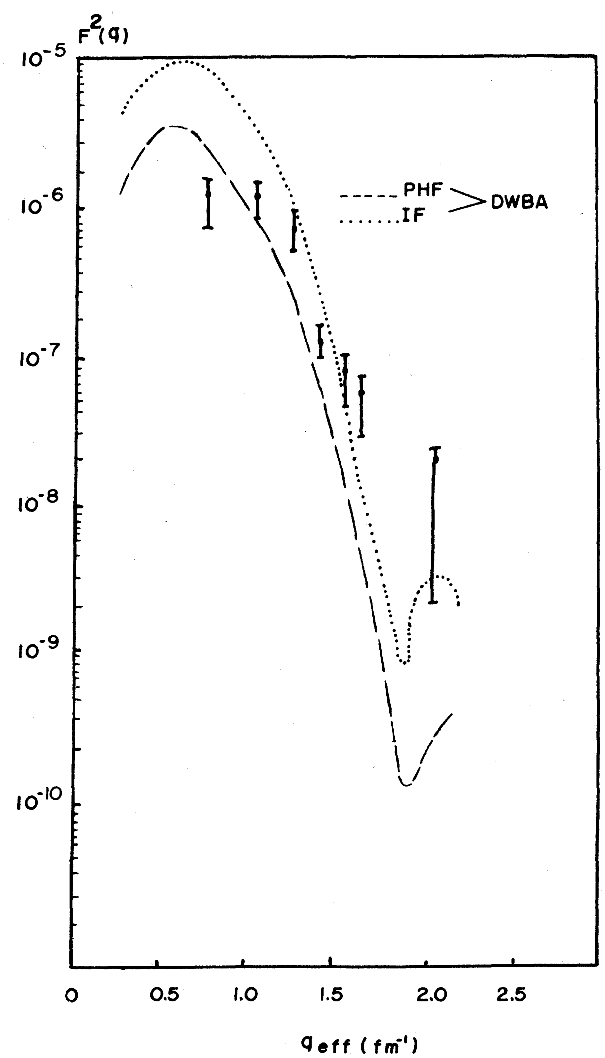


FIG. 3. Transverse $E2$ form factor from the first $2^+$ level of $^{24}$Mg calculated in DWBA, with the following approximations: PHF (— — —) and irrotational flow (· · · ·).

Figure 2 shows results for the $E2$ form factor, evaluated in PWBA, with the four different approximations quoted above, that is, PHF, cranking model, rigid rotor, and irrotational flow. For the two latter models, the transition charge densities $\rho_{\lambda}^{g}(r)$ [see Eqs. (8a), (8b), and (9)] are obtained from the same Nilsson wave function used for PHF and cranking model. For the two latter models, we have to add, to the convective current, the contribution due to spin magnetization. It is noticed from the figure that (1) the calculations in PHF and with irrotational flow give the best agreements with the available data; (2) the rigid rotor model form factor displays a dependence quite different from what is suggested by the data; and (3) the cranking model presents a form factor trend as a function of $q$ very much the same as the PHF and the irrotational flow results, but the overall disagreement with the data is much greater. It is important to notice that the energy denominators have an important participation in the cranking calculation [see Eq. (5b)], since they are very different, mainly for even-even nuclei, when compared with results yielded from Hartree-Fock approximation.[18] For $^{24}$Mg using our Nilsson wave functions, we derived, within the cranking model, a moment of inertia approximately 45% greater than the experimental value—which has direct impact over the total strength of the $E2$ form factor. Since the energy denominators contribute to the value of the moment of inertia and also to the transverse $E\lambda$ matrix element, it is difficult to draw a final conclusion about the behavior of these form factors, when other approximations are used to describe the intrinsic nuclear structure, without explicit calculation.

Finally, in Fig. 3, we repeated PHF and irrotational flow calculations for the form factor, with a more precise technique (DWBA). This is not a real calculation in DWBA, since in this circumstance it would also be required to evaluate Coulomb contribution, even at backward ($\theta \simeq 180°$) scattering angle. We avoided this requirement by plotting the experimental points without the longitudinal contribution, which is already conveniently subtracted.[1] Our main concern with the DWBA calculation was to quantify the Coulomb distortion effect over the $E2$ multipole. So, with respect to Fig. 2, results from Fig. 3. introduce important corrections only in the diffraction minimum of the form factor, as expected.

As a final finding it is clear that measurements of $E\lambda$ form factors for even-even deformed nuclei provide a unique possibility to investigate current densities produced by the nuclear rotation movement. Our results for the first $2^+$ level of $^{24}$Mg have shown that some of the models commonly used to describe nuclear rotation yield a reasonably realistic forecast, and also that it is enough to have modestly accurate measurements, such as the ones presented in Ref. 1, to discriminate between models. For the latter objective, new and strong tools are presently under consideration, such as inelastic electron scattering in coincidence with $\gamma$ rays emitted by the excited nucleus.[19]

Obviously, a more definite answer to the selective determination of the most accurate nuclear model will require the utilization of other intrinsic structure assumptions, or the utilization of other approximations to describe nuclear rotation. Nevertheless, if we compare our calculations with form factors evaluated within the "shell-model" approach, as shown in Ref. 1, it is possible to see that the quality of agreement obtained by us to the data is as good as that obtained in Ref. 1, for the $2^+$ $^{24}$Mg level.

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