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Inversion of elastic scattering phase shifts calculated with algebraic scattering theory

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The elastic phase shifts for the $^{16}O+^{28}Si$ scattering obtained by the algebraic approach to scattering theory are used in an inversion procedure at fixed energy. The resulting local optical potential shows oscillations and a dependence on energy. These effects point to an underlying nonlocal interaction.

INTRODUCTION

Recently, an algebraic approach to scattering (AAS) has been developed[1,2] which is based on the special features of noncompact Lie groups. In particular, it was shown by Alhassid and co-workers[1,2] that scattering by a modified Coulomb interaction, e.g., heavy-ion scattering, can be treated in three dimensions with a dynamical symmetry $SO(3,2)\supset SO(3)\otimes SO(2)$, where SO(3) and SO(2) describe angular momentum and interaction strength, respectively. In this case the S matrix for elastic scattering at energy $E_{\text{c.m.}}=\hbar^2k^2/2\mu$ and angular momentum quantum number l can be written as

$$S_l(k)=\frac{\Gamma[(l+w+2+i\eta)/2]\Gamma[(l-w+1+i\eta)/2]}{\Gamma[(l+w+2-i\eta)/2]\Gamma[(l-w+1-i\eta)/2]}\times\exp[i(2\ln 2)\eta]\,. \tag{1}$$

Here, η is the Sommerfield parameter, $\eta=Z_1Z_2e^2\mu/(\hbar^2k)$, and w is defined as

$$w=v-\tfrac{1}{2}\,. \tag{2}$$

The complex number v labels the SO(2) part of the scattering states involved and, therefore, is related to the interaction. For an actual application, v has to be specified as a function of l and k.

The crucial point for a better understanding of the AAS is its connection with the traditional description of scattering by a local or nonlocal potential. Up to now this connection is not known except for the case of $v=\frac{1}{2}$, i.e., $w=0$, which corresponds to pure Coulomb scattering.[1,2] A first attempt to obtain a potential has been performed by Amado and Sparrow,[3] who assumed a local interaction and applied an inversion procedure for the case of peripheral scattering (i.e., $w,\eta\ll l$) using the eikonal approximation. The necessary conditions for the high-energy eikonal approximation and peripheral scattering are not fulfilled in the particular case of low-energy heavy-ion scattering, which is of special interest in this publication. Very recently, an approach comparable to Ref. 3 has been performed by Hussein, Pato, and Iachello,[4] who semiclassically analyzed an absorption-free S matrix based on the dynamical symmetry SO(3,1) and obtained the underlying potential for large internuclear separations. SO(3,1) scattering amplitudes also can be used in the framework of the AAS to describe elastic heavy-ion collisions.[5]

In this paper we show the result of an inversion of the AAS phases resulting from the SO(3,2) S matrix (1) for the elastic scattering of ^{16}O on ^{28}Si at $E_{\text{c.m.}}=20.12$ and 22.30 MeV. This scattering system was described by Alhassid[2] with the AAS theory. The following parametrization was used:[2]

$$w=w_0/\{1+\exp[(l-l_0)/\Delta]\}\,. \tag{3}$$

The values of the parameters w_0, l_0, and Δ were determined by fitting the angular distribution calculated from Eq. (1) to the measured elastic angular distribution,[6] which yielded the following parameters:[2]

$$w_0=6-17i,\quad \Delta=1,\quad l_0(20.12\text{ MeV})=8.9\,,\quad l_0(22.30\text{ Mev})=11.7\,. \tag{4}$$

The real phase shifts and reflection coefficients resulting from these parameters and Eq. (1) are shown for $E_{\text{c.m.}}=20.12$ MeV by the solid curve in Fig. 1.

INVERSION METHOD

The inversion is performed at fixed energy with the modified Newton method[7] which is an exact inversion scheme for an infinite number of given phase shifts. The method assumes a local spherical potential which is equal to the Coulomb potential for $r\geq R_c$, where R_c is larger than the sum of the nuclear radii of both nuclei. In this limited class of potentials we get a unique and model-independent result from the phase shifts. The potential is, in general, contingent upon the scattering energy but not on angular momentum.

We start with a Povzner-Levitan-like representation of the solutions $\phi_l^U(r)$ of the Schrödinger equation with the unknown potential $U(r)=V(r)+iW(r)$ in terms of the known solutions $\phi_l^{U_0}(r)$ of a given reference potential $U_0(r)$:

$$\phi_l^U(r)=\phi_l^{U_0}(r)-\sum_{l'=0}^{l_{\max}}c_{l'}L_{ll'}(r)\phi_{l'}^U(r) \tag{5}$$

with known coefficients

$$L_{ll'}(r)=\int_0^r dr'\phi_l^{U_0}(r')\phi_{l'}^{U_0}(r')/r'^2\,. \tag{6}$$

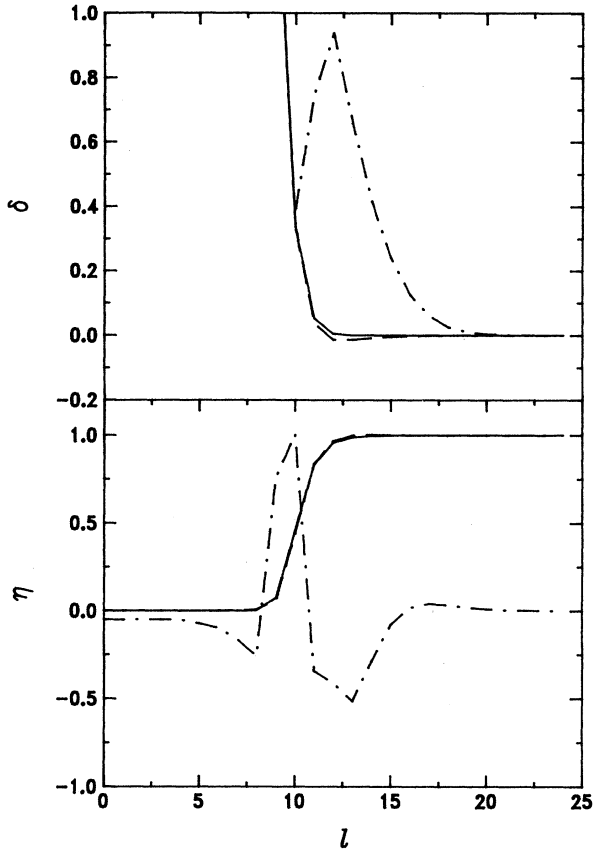

FIG. 1. The AAS real phase shifts and reflection coefficients for the elastic scattering of ^{16}O on ^{28}Si at $E_{c.m.}=20.12$ MeV are shown by the solid curves. The dashed curves represent the real phases and reflection coefficients of the dashed potential in Fig. 2. The differences of the solid and dashed curves multiplied by a factor of 50 are given by the dashed-dotted curves.

Then we end up with the following expansion of the potential:

$$U(r)=U_0(r)-\frac{2}{r}\frac{d}{dr}\left[\sum_{l=0}^{l_{max}} c_l \phi_l^{U_0}(r)\phi_l^{U}(r)/r\right]. \quad (7)$$

The coefficients c_l are determined in the asymptotic region $r \geqq R_c$ by using the analytical representation of ϕ_l^U in terms of Coulomb functions and the given phase shifts. The evaluation is done by solving the system of Eq. (5) at certain radii $r_i \geqq R_c$ $(i=1,2,\ldots \geqq 2)$ via a least-squares fit to obtain a best mean solution in the asymptotic region. The same Eq. (5) with the known coefficients c_l serves to calculate ϕ_l^U for $r < R_c$. Finally, we obtain the potential in the expansion (7).

For simplicity we chose the radii r_i to be equidistant with $r_{i+1}-r_i=\lambda\!\!\!^{-}$ and $r_1=R_c$, where $\lambda\!\!\!^{-}$ is the wavelength $\lambda\!\!\!^{-}=\hbar[2m(E_{c.m.}-V_0)]^{-1/2}$. Here, V_0 is the constant reference potential $U_0(r)=V_0=Z_1Z_2e^2/R_c$. The number and values of the radii r_i, which are connected to R_c and $\lambda\!\!\!^{-}$ and therefore to the incident energy $E_{c.m.}$ in the given simple way, and the quantum number l_{max} of the phase shift with highest angular momentum taken into account constitute the technical parameters of the inversion scheme. They have, in principle, no influence, at least on the physics of the resulting optical potential.[7]

DISCUSSION OF RESULTS

The real and imaginary parts of the optical potential obtained with the AAS phases and the inversion scheme are shown in Figs. 2 and 3 for $E_{c.m.}=20.12$ and 22.30 MeV, respectively. The different potentials in each figure result from different sets of the parameters r_i and l_{max}, given in the figure captions. The solid and dashed curves in Fig. 2 show the result of the inversion with three radii r_i and a quantum number $l_{max}=24$ and 28, respectively. The potentials in Fig. 3 are obtained with four radii and $l_{max}=24$ (solid curve), 26 (dashed curve), and 28 (dashed-dotted curve). These figures and further calculations reveal the independence of the calculated optical potential of the special choice of the parameters at least for radii $r \geqq 3$ fm. The special structure of the oscillations in the potential is stable, the exact position of the extrema deviates by not more than 0.3 fm for different values of l_{max}. For an increasing number of contributing phase

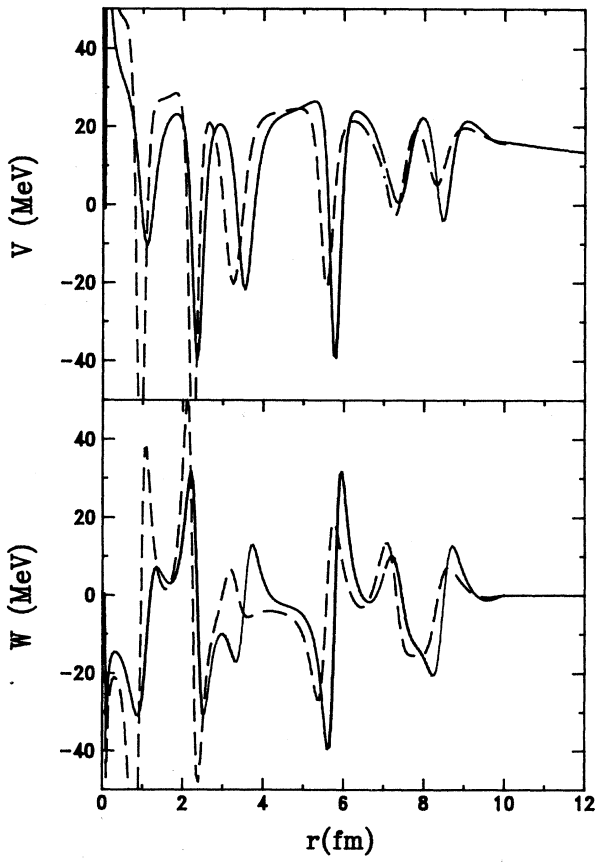

FIG. 2. The real and imaginary parts of the potential, V and W, calculated with the AAS phases of Fig. 1 for $E_{c.m.}=20.12$ MeV. The parameters of the inversion procedure are $r_1=10$ fm, $r_2=10.717$ fm, $r_3=11.434$ fm, and $l_{max}=24$ (solid curves) and 28 (dashed curves).

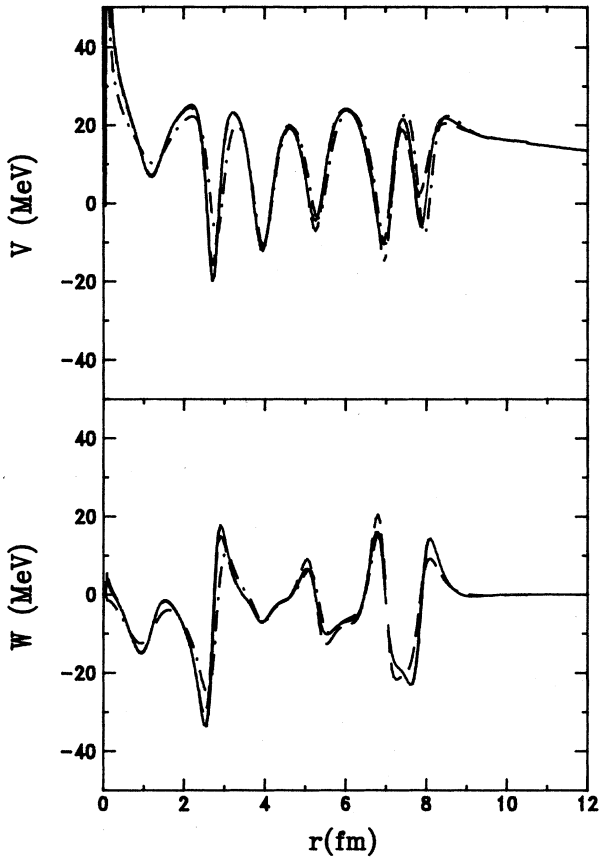

FIG. 3. The real and imaginary parts of the potential, V and W, for $E_{c.m.}=22.30$ MeV. The parameters of the inversion procedure are $r_1=10.5$ fm, $r_2=11.044$ fm, $r_3=11.588$ fm, $r_4=12.132$ fm, and $l_{max}=24$ (solid curves), 26 (dashed curves), and 28 (dashed-dotted curves).

shifts ($l > 35$) the calculations suffer from the larger numerical error and the results become poor.[7] For radii $r \geqq 9.5$ fm the inverted potential is a pure Coulomb potential with a vanishing imaginary part, even if the radius R_c is pushed out as far as 12 fm.

In Fig. 1 (dashed curves) we show the phases determined with an optical model code using the dashed potential given in Fig. 2. The dashed-dotted curves in Fig. 1 depict the differences of the analytical AAS phases and the calculated real phases and reflection coefficients multiplied by a factor of 50. An error of less than 2% is a very satisfactory result. For small angular momenta ($l \leqq 9$) the real parts of the phase shifts become arbitrary because the corresponding reflection coefficients tend to zero.

The applicability and uniqueness of the modified Newton method for nuclear heavy-ion potentials was exhaustively proven and demonstrated in Ref. 7. Consequently, the oscillations in the potentials are of physical nature and not of numerical origin. This is also revealed by a comparison of the phases of these potentials and of smoothed potentials in which the oscillations are averaged out. Whereas the true oscillating potentials exactly yield the phases used for the inversion method (cf. Fig. 1), the averaged smooth potentials show very different phases and, therefore, wrong differential cross sections.

Comparing Figs. 2 and 3 we find an energy dependence of the optical potential. This fact and the oscillations of the potential give rise to the supposition that the potential is of nonlocal nature.[8] A transformation of a local potential to a nonlocal one has been carried out recently by Fiedeldey *et al.*[9] by assuming a nonlocal potential of the Frahn-Lemmer-type. This method could be applied to the present problem.

SUMMARY

To summarize, for the first time we have given a connection between an AAS interaction and a potential by inverting AAS elastic phase shifts for two different energies. The potentials obtained are local for a given energy due to the inversion procedure applied, but in the context of different energies they show the special features of an underlying nonlocal interaction.

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