Antisymmetrization correction for nucleon-nucleus elastic scattering

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Microscopic calculations of nucleon-nucleus elastic scattering which are based on multiple scattering theory usually do not account for the identical nature of the projectile nucleon and the nucleon constituents of the target nucleus, except at the two-body level through use of an effective interaction. In this work the leading three-body antisymmetrization correction is obtained and evaluated. The effects are very small at intermediate energies and are only significant for incident nucleon kinetic energies less than the Fermi energy of the nucleus.

It is customary in microscopic calculations of the intermediate energy nucleon-nucleus elastic scattering optical potential to base the model on multiple scattering formalisms[1,2] in which the identical nature of the incident nucleon and the constituent nucleons of the target has been ignored. The unsymmetrized projectile-target nucleon two-body operators in the final expressions for the optical potential are, however, replaced with antisymmetrized two-body nucleon-nucleon (NN) effective interaction t matrices[2] according to the prescription of Takeda and Watson (TW).[3] Although such procedures are only accurate to the lowest-order, two-body scattering terms in expansions of the optical potential, the authors of Ref. 3 estimate that this lowest-order treatment should be accurate for incident energies greater than 100 MeV.

The importance of antisymmetrization (e.g., Pauli blocking effects) in modifying the effective, two-body NN interaction for low-to-medium energy nucleon-nucleus scattering has been recognized for many years and a sizeable industry has emerged for incorporating this effect into microscopic models of the optical potential.[4–6] Furthermore, many authors have developed a number of scattering formalisms in which full $(A+1)$-body antisymmetrization is included,[7] A being the baryon number of the target nucleus. In light of the general importance of Pauli exclusion effects and the progress in theories which incorporate full $(A+1)$-body antisymmetrization, it is of interest to actually calculate the leading-order antisymmetrization correction and to determine in a quantitative manner the range in energy where the TW prescription is expected to be valid.

In this work the fully antisymmetrized, nucleon-nucleus optical potential formalism of Picklesimer and Thaler[7] is used and the leading correction to the Takeda and Watson prescription is obtained. This term is a three-body exchange potential which is linear in the NN effective interaction operator. Numerical results are given for proton scattering from ^{16}O and ^{40}Ca.

Picklesimer and Thaler[7] obtained the usual unsymmetrized and fully antisymmetrized nucleon-nucleus optical potentials for elastic scattering and represented them using the spectator expansion method.[8] In their notation the former is given by

$$PUP=\sum_{i=1}^{A}PU_iP+\sum_{i<j=1}^{A}P(U_{ij}-U_i-U_j)P+\cdots , \tag{1}$$

where

$$U_i=T_i(1-G_\alpha PU_i) , \tag{2}$$

$$U_{ij}=T_{ij}(1-G_\alpha PU_{ij}) , \tag{3}$$

$$T_i=v_i+v_iG_\alpha T_i , \tag{4}$$

and

$$T_{ij}=(v_i+v_j)(1+G_\alpha T_{ij}) . \tag{5}$$

The antisymmetrized optical potential is given by expressions analogous to Eqs. (1)–(3) where U, U_i, U_{ij}, T_i, and T_{ij} are replaced by $\hat{U}$, $\hat{U}_i$, $\hat{U}_{ij}$, $\hat{T}_i$, and $\hat{T}_{ij}$, respectively, and Eqs. (4) and (5) are replaced with

$$\hat{T}_i=v_i(1-E_{0i})(1+G_\alpha T_i)\equiv w_i(1+G_\alpha T_i) \tag{6}$$

and

$$\hat{T}_{ij}=(v_i+v_j)(1-E_{0i}-E_{0j})(1+G_\alpha T_{ij}) \equiv (w_i+w_j+X_{ij})(1+G_\alpha T_{ij}) . \tag{7}$$

In these expressions P projects the usual unsymmetrized elastic channel[7] where the target ground state is represented by an antisymmetrized A-body state function. The many-body propagator G_α is given by

$$G_\alpha=(E-h_0-H_A+i\varepsilon)^{-1} , \tag{8}$$

where h_0 is the projectile kinetic energy operator and H_A is the target nucleus Hamiltonian. The projectile-target nucleon interaction is given by v_i, E_{0i} is the exchange operator for the projectile nucleon (0) and target nucleon (i), and X_{ij}, defined in Eq. (7), is equal to $-(v_iE_{0j}+v_jE_{0i})$.

In usual applications of multiple scattering theory[9] the unsymmetrized optical potential is employed except that T_i is taken to be the same as $\hat{T}_i$ since v_i is assumed to exist only for the physical, antisymmetric NN states. This is equivalent in this order of expansion to evaluating $P\hat{U}P$ with X_{ij} in Eq. (7) replaced with zero. Therefore the necessary correction to the unsymmetrized optical potential is just

$$PU_AP \equiv P\hat{U}P - P\hat{U}(X_{ij} \to 0)P \ , \tag{9a}$$

where $F(X_{ij} \to 0)$ means replace X_{ij} with zero in the function F. In leading order this becomes

$$PU_AP \simeq \sum_{i<j} P[\hat{T}_{ij} - \hat{T}_{ij}(X_{ij} \to 0)]P \ . \tag{9b}$$

Using Eqs. (4) and (7), $\hat{T}_{ij}$ is expanded as

$$\hat{T}_{ij} = (T_i + T_j)(1 - E_{0i} - E_{0j})(1 + G_\alpha T_{ij}) - (T_i G_\alpha T_i + T_j G_\alpha T_j)(1 - E_{0i} - E_{0j}) + \cdots \ , \tag{10a}$$

and $\hat{T}_{ij}$ with X_{ij} set to zero becomes

$$\hat{T}_{ij}(X_{ij} \to 0) = [T_i(1 - E_{0i}) + T_j(1 - E_{0j})](1 + G_\alpha T_{ij}) - T_i G_\alpha T_i(1 - E_{0i}) - T_j G_\alpha T_j(1 - E_{0j}) + \cdots \ . \tag{10b}$$

From Eqs. (9b), (10a), and (10b) the leading-order antisymmetrization correction is

$$PU_AP \simeq -\sum_{i \neq j} PT_iE_{0j}P + \cdots \ . \tag{11}$$

Note that this is a three-body operator involving the projectile nucleon (0) and target nucleons (i) and (j). In applications of the unsymmetrized theory the proper symmetry between target nucleons (i) and (j) is included in the antisymmetrized target wave function while that involving particles (0) and (i) is correctly treated by way of the TW prescription using NN effective interactions. The term in Eq. (11) therefore corrects the unsymmetrized optical potential by including the exchange of particles (0) and (j) while particles (0) and (i) interact. This term is similar to that discussed in Ref. 3.

In momentum space the correction in Eq. (11) becomes

$$\tilde{U}_A^{00}(\mathbf{k},\mathbf{k}') \equiv \langle \mathbf{k}'\phi_{gs} | PU_AP | \phi_{gs}\mathbf{k} \rangle \ , \tag{12}$$

where ϕ_{gs} represents the target nucleus ground state and $|\mathbf{k}\rangle$ is the state function of the projectile in a plane-wave state with wave vector $\mathbf{k}$. Spin and isospin factors have been suppressed. Since the purpose here is to conduct an initial, quantitative investigation of this antisymmetrization effect, corrections were only obtained for the real part of the central, spin-independent optical potential which is the dominant component of the optical potential[10] in the energy range where $\tilde{U}_A^{00}$ is expected to be important.[3] Extension of this calculation to include the remaining terms in the NN interaction would be straightforward. Assuming, therefore, a local, spin-independent NN effective interaction for $\hat{T}_i$ yields[11]

$$\tilde{U}_A^{00}(\mathbf{k},\mathbf{k}') = -A(A-1) \int\int\int d^3r_0 d^3r_1 d^3r_2 e^{-i\mathbf{k}'\cdot\mathbf{r}_0} \rho_{(2)}(\mathbf{r}_1,\mathbf{r}_2;\mathbf{r}_1,\mathbf{r}_0) t^{\text{eff}}(|\mathbf{r}_0 - \mathbf{r}_1|) e^{i\mathbf{k}\cdot\mathbf{r}_2} \ , \tag{13a}$$

where

$$\rho_{(2)}(\mathbf{r}_1,\mathbf{r}_2;\mathbf{r}_1,\mathbf{r}_0) \equiv \int d^3r_3 \cdots d^3r_A \langle \phi_{gs} | \mathbf{r}_1\mathbf{r}_2\mathbf{r}_3 \cdots \mathbf{r}_A \rangle \langle \mathbf{r}_1\mathbf{r}_0\mathbf{r}_3 \cdots \mathbf{r}_A | \phi_{gs} \rangle \tag{13b}$$

is the two-body density matrix. This expression is Fourier transformed into coordinate space yielding the nonlocal potential.

$$U_A^{00}(\mathbf{r},\mathbf{r}') = (2\pi)^{-6} \int\int d^3k\, d^3k' e^{i\mathbf{k}'\cdot\mathbf{r}'} \tilde{U}_A^{00}(\mathbf{k},\mathbf{k}') e^{-i\mathbf{k}\cdot\mathbf{r}} \ . \tag{14}$$

The nonlocality of $U_A^{00}(\mathbf{r},\mathbf{r}')$ was examined for the cases considered here. The potential exhibited Gaussian-like dependence on the nonlocal coordinate with range ~ 2 fm, approximately independent of incident energy and target mass. Therefore the standard method[5] of obtaining the local, equivalent optical potential was employed here, yielding

$$U_{A,\text{local}}^{00}(\mathbf{r}) = \int d^3\Delta r\, U_A^{00}(\mathbf{r} + \Delta\mathbf{r}, \mathbf{r}) j_0[k(r)\Delta r] \ , \tag{15a}$$

where

$$k^2(r) = k_{\text{inc}}^2 - \frac{2\mu}{\hbar^2} V(r) \ , \tag{15b}$$

k_{inc} is the projectile nucleon incoming wave vector, μ is the nucleon-nucleus reduced mass, and $V(r)$ is the real part of the central, spin-independent empirical optical potential.[10]

The interaction t^{eff} was assumed to be a Gaussian with strength and range determined by requiring that the volume and root-mean-square (rms) radius of the folding model potential given by

$$U(r) = \int d^3r' t^{\text{eff}}(|\mathbf{r} - \mathbf{r}'|) \rho_m(r') \ , \tag{16}$$

equal those of optical model phenomenology.[10] The matter density ρ_m was assumed proportional to the empirical proton point density,[12] since only self-conjugate nuclear targets were considered. Reasonable strengths and ranges were thus obtained in comparison with those of the density-dependent, G-matrix effective interaction.[6,13]

The two-body density matrix was computed from a Slater determinant of single-particle states. These were assumed to be orthogonal eigenstates of a Woods-Saxon binding potential where j dependence of the radial wave functions was neglected for simplicity.

Calculations were performed for $p + {}^{16}\text{O}$ at incident

TABLE I. Optical potential volumes.

Reaction	$RE[\langle U_{\mathrm{cent}}\rangle]/A$ (MeV fm^3)	$RE[\langle U^{00}_{A,\mathrm{local}}\rangle]/A$ (MeV fm^3)	Absolute value of ratio (%)
$p+{}^{16}O$, 20.8 MeV	−483.4	11.8	2.4
$p+{}^{16}O$, 30.4 MeV	−449.2	4.7	1.05
$p+{}^{16}O$, 39.7 MeV	−454.5	0.027	0.006
$p+{}^{16}O$, 49.5 MeV	−419.7	−0.29	0.07
$p+{}^{40}Ca$, 9.86 MeV	−497.5	19.6	3.9
$p+{}^{40}Ca$, 20.6 MeV	−388.1	7.65	2.0
$p+{}^{40}Ca$, 30.3 MeV	−402.3	−1.56	0.39
$p+{}^{40}Ca$, 40 MeV	−392.5	−2.12	0.54
$p+{}^{40}Ca$, 49 MeV	−302.8	−0.94	0.31
$p+{}^{40}Ca$, 76 MeV	−240.2	−0.016	0.007

laboratory kinetic energies of 20.8, 30.4, 39.7, 49.5, 76, and 153 MeV, and for $p+{}^{40}Ca$ at 9.86, 20.6, 30.3, 40.0, 49.0, 76.0, and 153 MeV using empirical potentials from Ref. 10. Selected results are summarized in Table I which lists for each case the volume per nucleon of the empirical real, spin-independent proton+nucleus (pA) optical potential,[10] that of the local equivalent approximation of the antisymmetrization correction, and the absolute value of the ratio of the two in percent. The correction diminishes rapidly with increasing energy, however, below 30 MeV the effect is several percent. Optical model calculations at energies less than 30 MeV which include this term display appreciable effects in the differential cross sections.

To summarize, the antisymmetrization correction potential evaluated here is a three-body, exchange term which is linear in the NN t matrix. Due to its exchange nature this quantity is expected to be small at energies above the Fermi energy and such is the case found here. The correction was found to be completely negligible above 30 MeV incident nucleon energy, thus quantifying the original estimate of Takeda and Watson.[3] Therefore for incident energies of several tens to hundreds of MeV the TW antisymmetrization prescription for the unsymmetrized multiple scattering formalism is completely adequate. The dominant, residual effect of the Pauli exclusion principle in nucleon-nucleus scattering remains that due to Pauli blocking in the NN intermediate scattering states.[4–6] This effect contributes to the operator $\hat{T}_i$ as can be seen by evaluating Eqs. (4) or (6) with a complete set of antisymmetric A-body nuclear states utilized in the $(v_i G_\alpha T_i)$ term. It is important to remember, however, that the antisymmetrization correction calculated here contributes *in addition* to the medium effects one may include in evaluating $\hat{T}_i$ in Eq. (6) in multiple scattering theory. Of course at energies less than 30 MeV where this effect is nonnegligible, extensions[4–6] of Brueckner theory[14] are more often used which account for the full projectile-nucleus antisymmetry.

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