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Energy-dependent corrections to spin observables in nucleon-nucleus inelastic scattering

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The $0^+(\vec{p},\vec{p}\,')0^-$ reaction has been used as an example to elucidate the importance of the energy dependence of the nucleon-nucleon (*NN*) interaction on nucleon-nucleus (*NA*) spin observables. Because of its simplicity the $0^+ \rightarrow 0^-$ transition is specially suited for the study of energy-dependent corrections. For this transition inclusion of corrections induced by energy dependence results in large modifications in the behavior of certain spin observables. Thus we conclude for the $0^+ \rightarrow 0^-$ transition that calculations neglecting energy-dependent corrections are not only inconsistent but can lead to qualitatively incorrect conclusions. We argue that corrections arising from the energy dependence studied will be small for stretched states. Finally, we propose a simple procedure that will allow one to study the importance of corrections arising from the energy dependence of a proposed *NN* interaction in a systematic way.

I. INTRODUCTION

Standard descriptions of nucleon-nucleus scattering at intermediate energies often make use of an effective nucleon-nucleon interaction that closely resembles the interaction between the two nucleons in free space. Use of the free *NN* interaction, a prescription that defines the impulse approximation, is a reasonable first approximation in the medium-energy region where the reactive part of the reaction is known to be dominated by quasifree knockout.[1] Nucleon-nucleus scattering in this energy domain then offers a unique possibility towards understanding details of nuclear structure or subtle modifications to the free-*NN* interaction inside the nuclear medium.

One of the most widely applied effective *NN* interactions is due to Love and Franey.[2] The model, which explicitly treats the identity of the scattering particles, is based on an effective interaction that consists of a sum of terms with different spin-isospin structure. Strengths and ranges are chosen to reproduce, in Born approximation, *NN* observables for bombarding energies from $T_{\rm lab} \sim 50-1000$ MeV. Among the advantages of this prescription is the possibility of incorporating off-shell effects in a very simple way. The simplicity of this model and others of its type[3] means, of course, that there have been nontrivial approximations made. For example, one suppresses much of the energy dependence that is usually generated in the iteration of the interaction in a Lippmann-Schwinger equation. Furthermore, it is well known that a real potential generates, in Born approximation, a scattering amplitude that violates unitary. In order to remedy these deficiencies the *NN* pseudopotential includes complex strengths with explicit energy dependence. It is this explicit energy dependence in the effective *NN* interaction that is the main focus of the present work. A previous study of corrections arising from the explicit energy dependence of the pion-nucleon interaction has indicated the importance of this effect in pion-nucleus inelastic scattering.[4] We carry out the present study in the context of nucleon-nucleus inelastic scattering where the possibility of measuring spin observables adds a potentially new dimension to the imporatnce of corrections associated with the energy dependence of the effective *NN* interaction.

The paper is divided as follows. In Sec. II the Love-Franey model is briefly reviewed with special emphasis on the explicit energy dependence of the *NN* interaction. In Sec. III the $0^+(\vec{p},\vec{p}\,')0^-$ reaction is presented as an example that highlights the main points of the present work. An explicit calculation is discussed in Sec. IV. Finally, in Sec. V results and conclusions are summarized.

II. LOVE-FRANEY MODEL

In the Love-Franey model,[2] the interaction between a free nucleon and a nucleon bound in the nucleus is written as

$$t_{12}=\left\langle \mathbf{k}';\phi' \left| \sum_i t_i(1-P_{12}) \right| \mathbf{k};\phi \right\rangle = \int \frac{d\mathbf{p}'}{(2\pi)^3} \int \frac{d\mathbf{p}}{(2\pi)^3} \phi'^*(\mathbf{p}') \left\langle \mathbf{k}';\mathbf{p}' \left| \sum_i t_i(1-P_{12}) \right| \mathbf{k};\mathbf{p} \right\rangle \phi(\mathbf{p}) \; , \qquad (1)$$

where $\mathbf{k}$ ($\mathbf{k}'$) is the initial (final) momentum of the projectile and ϕ (ϕ') is the initial (final) bound-state wave function. The nucleon-nucleon interaction $\sum_i t_i(1-P_{12})$ consists of an explicitly antisymmetrized sum of terms with different ranges and spin-isospin structure. For example, a particular spin- and isospin-independent piece of the interaction is given by

$$\langle \mathbf{k}';\mathbf{p}'|t_{c0}(1-P_{12})|\mathbf{k};\mathbf{p}\rangle = \left[\frac{\alpha}{q^2+\mu^2}\right] - \left[\frac{\alpha}{Q^2+\mu^2}\right][\tfrac{1}{2}(1+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\tfrac{1}{2}(1+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)] , \tag{2}$$

where α and μ are the coupling strength and range of the interaction, respectively, $\mathbf{q}=\mathbf{k}-\mathbf{k}'=\mathbf{p}'-\mathbf{p}$ is the momentum transfer to the bound nucleon, $\mathbf{Q}=\mathbf{k}-\mathbf{p}'=\mathbf{k}'-\mathbf{p}$ is the exchange momentum, and we have introduced the spin exchange, $\frac{1}{2}(1+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)$, and isospin exchange, $\frac{1}{2}(1+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)$, operators.

It is important to note that, in contrast to the direct part of the interaction, the exchange part depends explicitly on the bound-state momentum distribution and must be folded together with the nuclear transition density. This point has long been appreciated by many authors and has been properly incorporated into many theoretical calculations. The inclusion of this momentum dependence leads to important changes in the operator structure of the scattering amplitude and leads to the observed[5] nonzero value of certain spin observables, (e.g., $P-A_y$ in inelastic scattering).

On the other hand, the explicit energy dependence present in the coupling strengths of the effective NN interaction has, to date, been treated by assuming a single c.m. energy for the two interacting nucleons. As we will show, energy-dependent corrections give rise to similar modifications to the operator structure of the scattering amplitude as the nonlocal part of the exchange term. Therefore, unless one can show that such energy-dependent corrections are negligible, there is no *a priori* justification for retaining the one piece due to exchange while at the same time neglecting the other.

One expects that energy-dependent corrections should be most important for observables completely dominated by nonlocal effects. Such observables include $P-A_y$ or, more generally, the spin-difference function $\Delta_s \equiv (Q-B)+i(P-A_y)$,[6] and two-particle correlation observables measured in certain coincidence experiments, e.g., $(p,p'\gamma)$ and (p,ne^+).[7] An example that highlights much of the interesting physics is the $0^+(\vec{p},\vec{p}')0^-$ reaction. This reaction is not only of current experimental interest[8] but is from a theoretical perspective one of the simplest inelastic transitions possessing nontrivial features. For example, as shown in the next section, for this transition interesting physics contained in $P-A_y$ will already be present in the most easily measured spin observable, the analyzing power.

III. THE $0^+(\vec{p},\vec{p}')0^-$ REACTION

The most general form of the amplitude, for a $0^+(\vec{p},\vec{p}')0^-$ reaction, consistent with rotational and parity invariance can be written as[9]

$$A(0^-)=[A_q(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}})+A_K(\boldsymbol{\sigma}\cdot\hat{\mathbf{K}})] , \tag{3}$$

where $\mathbf{K}=(\mathbf{k}+\mathbf{k}')/2$ is the average momentum of the projectile in the reaction, and A_q and A_K are invariant (complex) amplitudes which are functions of the energy and momentum transfer. If one neglects the Q value of the reaction then $\hat{\mathbf{n}}\equiv\hat{\mathbf{q}}\times\hat{\mathbf{K}}$, $\hat{\mathbf{q}}$, and $\hat{\mathbf{K}}$ form a right-handed orthonormal coordinate system. At the energy and momentum transfers of interest the corrections due to a finite Q value of the reaction are very small. For example, at $T_{\text{lab}}\sim 500$ MeV and for typical excitation energies of low-lying inelastic states ($Q\sim 10$ MeV), $\hat{\mathbf{q}}\cdot\hat{\mathbf{K}}<\frac{1}{10}$ for $q>\frac{1}{2}$ fm^{-1}. Therefore, in the following example we will neglect the Q value of the reaction.

Since the scattering amplitude is defined up to an overall phase, only three independent measurements are sufficient to completely determine the $0^+\rightarrow 0^-$ amplitude. As is customary in elastic scattering, we will concentrate on the differential cross section, the analyzing power A_y and the spin-rotation function Q. These observables are written in terms of A_q and A_K in the following way:[9]

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= |A_q|^2+|A_K|^2 , \\ \frac{d\sigma}{d\Omega}A_y &= 2I_m(A_q A_K^*) , \\ \frac{d\sigma}{d\Omega}Q &= 2R_e(A_q A_K^*) . \end{aligned} \tag{4}$$

All remaining spin observables can in turn be obtained from these three. In particular, we obtain the following well-known relations:[9,10]

$$\begin{aligned} D_{n0} &\equiv P = -A_y \equiv -D_{0n} , \\ D_{qK} &\equiv Q = -B \equiv D_{Kq}, \quad D_{nn}=-1 , \end{aligned} \tag{5}$$

written in terms of the polarization transfer coefficients $D_{\alpha\beta}$ defined by

$$\left[\frac{d\sigma}{d\Omega}\right]D_{\alpha\beta}=\tfrac{1}{2}T_r[\sigma_\alpha A\sigma_\beta A^\dagger], \quad \alpha,\beta=\{0,n,q,K\} . \tag{6}$$

Note that for the $0^+\rightarrow 0^-$ transition the deviations from the elastic scattering relations, $P=A_y$, $Q=B$, and $D_{nn}=+1$, are maximal. There has been considerable interest in understanding possible sources of $P-A_y$.[6,11,12] We point out that whatever these sources might be, for the $0^+\rightarrow 0^-$ reaction, they are already present in the analyzing power.

Another advantage of the $0^+\rightarrow 0^-$ transition is that spin observables are relatively insensitive to distortion effects. Qualitatively one can understand this as follows. It turns out that spin observables driven exclusively by those parts of the scattering amplitude proportional to $\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}$ and/ or $\boldsymbol{\sigma}\cdot\hat{\mathbf{K}}$ are relatively insensitive to spin-orbit distortions. Thus, even if in general the analyzing power might be strongly dependent on spin-orbit distortions, $P-A_y$, which is driven exclusively by $\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}$ and $\boldsymbol{\sigma}\cdot\hat{\mathbf{K}}$,[12] is not. We can gain some analytic understanding of this result by using an eikonal approximation in the evaluation of the distortions. The effect of distortions on the scattering amplitude can be written, schematically, in the following way:

$$A_{\mathrm{DWIA}} \sim e^{-\chi_c} e^{i(\chi_{\mathrm{so}}/2)\boldsymbol{\sigma}\cdot\hat{\mathbf{n}}} A_{\mathrm{PWIA}} e^{i(\chi_{\mathrm{so}}/2)\boldsymbol{\sigma}\cdot\hat{\mathbf{n}}} , \tag{7}$$

where $\chi_c(\chi_{\mathrm{so}})$ is the central (spin-orbit) part of the eikonal phase. Under distortions the individual amplitudes will then transform in the following way:

$$\begin{bmatrix} A_0 \\ A_n \\ A_q \\ A_K \end{bmatrix}_{\mathrm{DWIA}} \sim e^{-\chi_c} \begin{bmatrix} \cos\chi_{\mathrm{so}} & i\sin\chi_{\mathrm{so}} & 0 & 0 \\ i\sin\chi_{\mathrm{so}} & \cos\chi_{\mathrm{so}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_0 \\ A_n \\ A_q \\ A_K \end{bmatrix}_{\mathrm{PWIA}} . \tag{8}$$

We then observe that while elasticlike nonspin-flip, A_0, and spin-flip, A_n, amplitudes are sensitive to spin-orbit distortions, the $0^+\to0^-$-like amplitudes, A_q and A_K, are not. We therefore expect that, for the $0^+\to0^-$ reaction, distortions will not qualitatively change the conclusions reached in the study of energy-dependent corrections on spin observables and will therefore be neglected in the analytic example discussed in the next section.

IV. SCHEMATIC CALCULATION FOR THE $0^+\to0^-$ TRANSITION

We first consider a local nonrelativistic plane-wave calculation. All dependence of the interaction on the integration variable $\mathbf{p}$, contained in the exchange momentum and in the energy dependence of the coupling strengths, will therefore be neglected. We use an optimally factorized form[13] for the amplitude which evaluates the NN t matrix with Breit frame kinetmatics, i.e.,

$$\mathbf{P}\equiv\tfrac{1}{2}(\mathbf{p}+\mathbf{p}')=0,\quad \mathbf{p}=-\frac{\mathbf{q}}{2},\quad \mathbf{p}'=+\frac{\mathbf{q}}{2} . \tag{9}$$

The $0^+\to0^-$ plane-wave transition amplitude can be written, in terms of an on-shell parametrization of the NN t matrix,[14]

$$\begin{aligned} t_n = {} & A + B\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}_n + iqC(\boldsymbol{\sigma}+\boldsymbol{\sigma}_n)\cdot\hat{\mathbf{n}} \\ & + q^2 D(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}})(\boldsymbol{\sigma}_n\cdot\hat{\mathbf{q}}) + E(\boldsymbol{\sigma}\cdot\hat{\mathbf{K}})(\boldsymbol{\sigma}_n\cdot\hat{\mathbf{K}}) , \end{aligned} \tag{10}$$

evaluated at an effective value of the laboratory kinetic energy,[13]

$$T_{\mathrm{lab}}^{\mathrm{eff}}(q^2)=\frac{1}{m}\left\{\left[\left[K^2+\frac{q^2}{4}+m^2\right]\left[\frac{K^2}{A^2}+\frac{q^2}{4}+m^2\right]\right]^{1/2} + \left[\frac{K^2}{A}+\frac{q^2}{4}-m^2\right]\right\} , \tag{11}$$

as follows:

$$A(0^+\to0^-)=\left\langle 0^- \left| \sum_{n=1}^{A} e^{i\mathbf{q}\cdot\mathbf{x}_n} t_n \right| 0^+ \right\rangle . \tag{12}$$

One can infer some interesting features of this example without having to explicitly perform the calculation. We have already noted, using invariance, that the $0^+\to0^-$ scattering amplitude must be proportional to the spin operator $\boldsymbol{\sigma}$ of the projectile [see Eq. (3)]. Similarly, since the nucleus is initially in a 0^+ state and is excited to a 0^- final state, the nuclear transition density must be proportional to the spin operator $\boldsymbol{\sigma}_n$ of a target nucleon. Consequently, neither the central nor the spin-orbit part of the t matrix contributes to this reaction. Furthermore, all remaining pieces of the t matrix have the general form $(\boldsymbol{\sigma}\cdot\hat{\mathbf{e}})(\boldsymbol{\sigma}_n\cdot\hat{\mathbf{e}})$, where $\hat{\mathbf{e}}=\{\hat{\mathbf{n}},\hat{\mathbf{q}},\hat{\mathbf{K}}\}$. The restriction that the nuclear operator must be a pseudoscalar leads to a contribution from the term $e^{i\mathbf{q}\cdot\mathbf{x}_n}(\boldsymbol{\sigma}\cdot\hat{\mathbf{e}})(\boldsymbol{\sigma}_n\cdot\hat{\mathbf{e}})$ of the form

$$(\mathbf{q}\cdot\hat{\mathbf{e}})(\boldsymbol{\sigma}\cdot\hat{\mathbf{e}})(\boldsymbol{\sigma}_n\cdot\mathbf{x}_n) . \tag{13}$$

In the limit of a zero Q value for the reaction, the above expression vanishes for both $\hat{\mathbf{e}}=\hat{\mathbf{n}}$ and $\hat{\mathbf{e}}=\hat{\mathbf{K}}$, and thus the scattering amplitude takes the following form:

$$\begin{aligned} A(0^+\to0^-) = {} & (B+q^2D) \\ & \times\left\langle 0^- \left| \sum_n e^{i\mathbf{q}\cdot\mathbf{x}_n}(\boldsymbol{\sigma}_n\cdot\hat{\mathbf{q}}) \right| 0^+ \right\rangle (\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}) . \end{aligned} \tag{14}$$

We observe that the $0^+\to0^-$ reaction samples the longitudinal part of the nuclear spin density. More importantly, however, is the absence of the $\boldsymbol{\sigma}\cdot\hat{\mathbf{K}}$ amplitude. In particular, this model will predict zero for the analyzing power, and in general for all observables that require the interference between the A_q and A_K amplitudes. This results is in contradiction with experiment which finds a nonzero analyzing power.[15] As we mentioned before, spin-orbit distortions are not expected to significantly modify this result.

For the above example, however, nonlocal corrections result in a modification to the operator structure of the amplitude that leads to a nonzero value for the analyzing power. First we consider corrections associated with the exchange term in Eq. (1). One can gain some analytic understanding regarding the importance of nonlocal corrections by performing a Taylor series expansion of the exchange term around the optimal momentum $\mathbf{Q}=\mathbf{K}$:

$$\frac{1}{Q^2+\mu^2}\simeq\frac{1}{K^2+\mu^2}\left[1+\frac{2MK}{K^2+\mu^2}(\hat{\mathbf{K}}\cdot\mathbf{J}_n)\right] , \tag{15}$$

where $\mathbf{J}_n \equiv \mathbf{P}_n/M$ is the convection current of a bound nucleon. From this result we observe that nonlocal effects decrease in importance with increasing beam energy and/or when the reaction becomes dominated by the short-range pieces of the interaction. More importantly, the additional factor, $\hat{\mathbf{K}}\cdot\mathbf{J}_n$, modifies the structure of the scattering amplitude in a crucial way. Following the same steps as for the on-shell amplitude, we observe that the restriction of forming a nuclear pseudoscalar operator constrains the contribution from the term $e^{i\mathbf{q}\cdot\mathbf{x}_n}(\boldsymbol{\sigma}\cdot\hat{\mathbf{e}})(\boldsymbol{\sigma}_n\cdot\hat{\mathbf{e}})(\hat{\mathbf{K}}\cdot\mathbf{J}_n)$ to the form

$$(\boldsymbol{\sigma}\cdot\hat{\mathbf{e}})\sum_{l=0,2}[Y_l(\mathbf{q})\otimes(\hat{\mathbf{e}}\otimes\hat{\mathbf{K}})_l]_{0,0}[Y_l(\mathbf{x}_n)\otimes(\boldsymbol{\sigma}_n\otimes\mathbf{J}_n)_l]_{0,0}\ . \tag{16}$$

By examining the projectile part of the operator and remembering that $\hat{\mathbf{q}}\cdot\hat{\mathbf{K}}=0$, we observe that this part of the operator is proportional to $\hat{\mathbf{e}}\cdot\hat{\mathbf{K}}$ and therefore conclude that $\hat{\mathbf{e}}$ must be equal to $\hat{\mathbf{K}}$. The first nonlocal correction to the scattering amplitude, coming from exchange, can then be written as

$$A'(0^+\to 0^-)=(B'+E')\left\langle 0^-\left|\sum_n e^{i\mathbf{q}\cdot\mathbf{x}_n}(\boldsymbol{\sigma}_n\cdot\hat{\mathbf{K}})(\hat{\mathbf{K}}\cdot\mathbf{J}_n)\right|0^+\right\rangle(\boldsymbol{\sigma}\cdot\hat{\mathbf{K}}) \tag{17}$$

and generates a nonzero A_K amplitude that couples to the spin-convection current density of the nucleus. In the above expression B' and E' are the spin-spin and tensor exchange part of the NN t matrix modified according to Eq. (15). It is this term that is responsible in nonrelativistic calculations for a nonvanishing analyzing power for $0^+\to 0^-$ transitions.

Current relativistic models of nucleon-nucleus scattering are able to generate a nonzero value for A_K even in the absence of nonlocal corrections.[9] This result has its origin in the coupling between upper and lower components of relativistic wave functions and will therefore also generate an amplitude proportional to the spin-convection current density of the nucleus. Note that although a local relativistic calculation shows a richer spin structure than an equivalent nonrelativistic calculation, the differences may not be as dramatic once nonlocal corrections are included.

For the $0^+\to 0^-$ example under consideration we have discussed the importance of nonlocal effects in nonrelativistic calculations. So far, however, we have only included nonlocal corrections coming from the exchange part of the NN interaction. Momentum-dependent corrections associated with the energy dependence of the effective NN interactions have been ignored. Energy-dependent corrections, in the πN interaction, have been previously addressed and found to be of importance by Siciliano and Walker in πA scattering.[4] These authors noted that the first-order correction, due to energy-dependent coupling strengths, can be written in the following way:[4]

$$\alpha\simeq\alpha_0+\alpha_0'(\hat{\mathbf{K}}\cdot\mathbf{J}_n),\quad \alpha_0'\equiv -K\frac{\partial\alpha}{\partial T_{\rm lab}}\ , \tag{18}$$

and therefore leads to precisely the same modification to the operator structure of the amplitude as was obtained from exchange in NA scattering. Thus, unless in the present case energy-dependent corrections are negligible, it seems inconsistent to keep one effect while *a priori* neglecting the other. We saw previously for the case of exchange that only the spin-spin and tensor exchange part of the NN interaction contribute to A_K. However, in the case of energy-dependent corrections, modifications to the structure of the amplitude arise from both the exchange as well as the direct term. To illustrate this result we evaluate the contribution to the $0^+\to 0^-$ amplitude coming from the (isoscalar) spin-spin part of the NN interaction,

$$\begin{aligned}\langle \mathbf{k}';\mathbf{p}_n'|t_s(1-P_n)|\mathbf{k};\mathbf{p}_n\rangle &= \left[\frac{\alpha}{q^2+\mu^2}\right](\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}_n)-\left[\frac{\alpha}{Q^2+\mu^2}\right](\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}_n)P_{\sigma\tau}(n)\\ &\simeq \left[\frac{\alpha_0}{q^2+\mu^2}\right](\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}_n)-\left[\frac{\alpha_0}{Q^2+\mu^2}\right](\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}_n)P_{\sigma\tau}(n)\\ &\quad+\left[\left[\frac{\alpha_0'}{q^2+\mu^2}\right](\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}_n)-\left[\frac{\alpha_0'}{K^2+\mu^2}\right](\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}_n)P_{\sigma\tau}(n)\right](\hat{\mathbf{K}}\cdot\mathbf{J}_n)\ ,\end{aligned} \tag{19}$$

where $P_{\sigma\tau}(n) \equiv \frac{1}{2}(1+\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}_n)\frac{1}{2}(1+\boldsymbol{\tau}\cdot\boldsymbol{\tau}_n)$ is the spin-isospin exchange operator. Due to the smooth energy dependence of the NN interaction, energy-dependent corrections to the amplitude were treated to leading order in α_0'. To obtain the above result we have therefore used Eq. (18) to write the coupling strengths to leading order in α_0'. The second line in the above equation contains nonlocal corrections arising from exchange through the exchange momentum Q^2. As shown below, we have treated this term exactly by folding the exchange term of the NN interaction with the transition density. The third line in Eq. (19) contains the new feature presented in this work. It shows the contribution to the amplitude coming from the explicit energy dependence of the NN interaction. Nonlocal corrections coming from exchange were ignored in those terms already linear in α_0'. To determine α_0' for all coupling constants needed in the calculation (i.e., spin-spin, tensor, etc.), we have calculated their derivative with respect to the energy by performing a smooth interpolation between those values of the energy chosen by Love and Franey in their fits to the NN amplitude.[2]

We have performed nonrelativistic plane-wave impulse approximation (PWIA) calculations for $0^+(\vec{p},\vec{p}\,')0^-$ transitions to both the $T=0$ (10.957 MeV) and $T=1$ (12.797 MeV) states in ^{16}O induced by 200 MeV incident protons. The scattering amplitude without energy-dependent corrections was inferred from knowledge of the three observables, $(d\sigma/d\Omega, A_y, Q)$,[16] obtained by running the standard DW81 code in the plane-wave limit. Nonlocal corrections coming from the exchange part of the interaction were therefore treated exactly. In the top part of Fig. 1 results are shown for the analyzing power for the $T=0$ and the $T=1$ states in ^{16}O. The differences between the calculations with and without energy-dependent corrections are small. Results for the spin-rotation parameter $D_{s'l}$, which for the $0^+ \rightarrow 0^-$ reaction satisfies $D_{s'l} = -Q$, are, however, much more interesting. We observe that energy-dependent corrections dramatically change the predictions. Our results show that the elucidation of details of either the reaction mechanism or nuclear structure, without incorporating energy-dependent corrections is potentially fatally flawed.

The appearance of the rank-one, nuclear convection current operator $\mathbf{J}_n$ coming from momentum-dependent corrections enables one to form an operator with the same quantum numbers of the final state using one less power of q. The importance of energy-dependent corrections, as compared with the on-shell part of the amplitude, therefore scales as

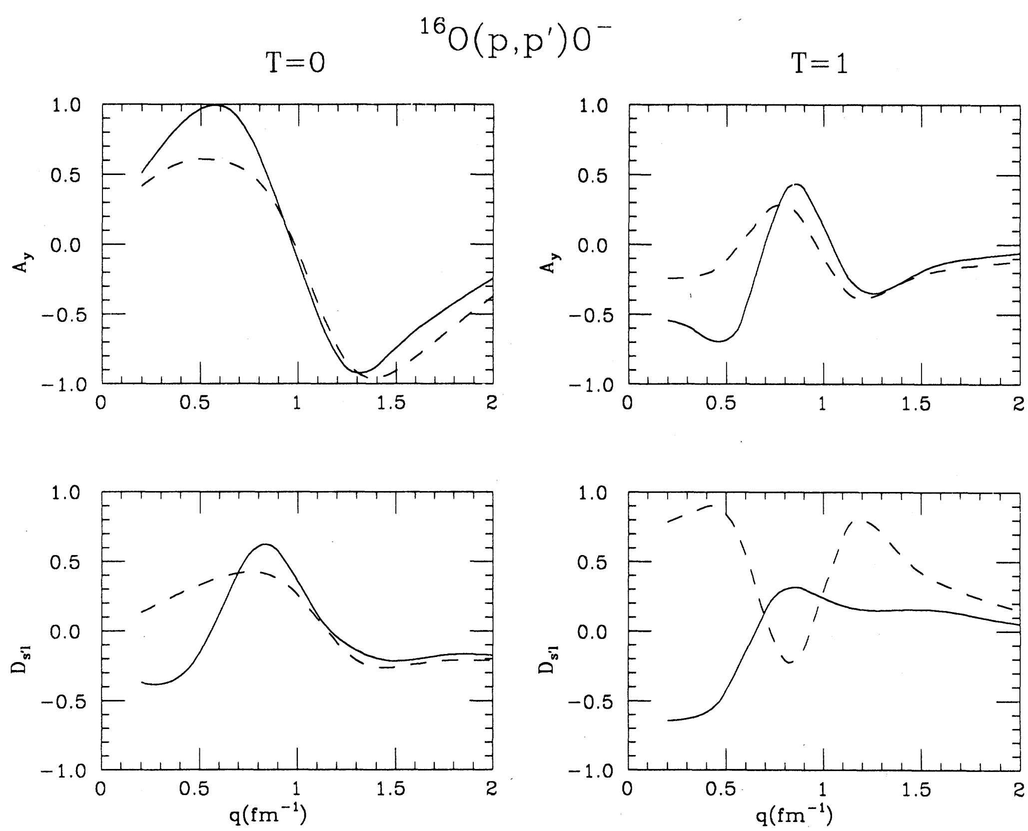

FIG. 1. Analyzing power and spin-rotation parameter $D_{s'l} = -Q$ for the 0^- $T=0$ (10.957 MeV) and 0^- $T=1$ (12.797 MeV) states in ^{16}O excited by 200 MeV incident protons. Solid (dashed) lines show results from a PWIA calculation with (without) energy-dependent corrections.

$$\frac{\alpha_0' j_L(qR)}{\alpha_0 j_{L+1}(qR)} \simeq (2L+3)\frac{1}{\alpha_0 R}\frac{\partial\alpha}{\partial T_{\rm lab}}\frac{K}{q} \tag{20}$$

and dominates at low-momentum transfer. In the above expression R is a nuclear length scale and L a typical value for the (orbital) momentum transfer in the reaction.

It has been shown that energy-dependent corrections are unimportant for the case of pion-induced transitions to stretched states.[4] We have performed calculations for proton-induced transitions to stretched states and found energy-dependent corrections to be very small for all spin observables at all values of momentum transfer.

Even though we have estimated the importance of energy-dependent corrections for several (p,p') transitions, a complete understanding can only be achieved by performing a systematic study of its effect on selected spin observables and for a variety of reactions. This calls for a more ambitious effort that should incorporate energy-dependent corrections into the standard technology in the same way that it was done previously for exchange. Fortunately, the task does not seem hopeless. Once the contribution from the two-body spin-orbit force has been evaluated using the standard procedure the additional contribution to the amplitude, due to energy-dependent corrections, involves the same matrix elements with only slight modifications in the angular-momentum algebra. The operator structure of the spin-orbit part of the NN interaction has the following form:

$$\hat{\mathbf{O}}_{\rm so} = (\boldsymbol{\sigma}+\boldsymbol{\sigma}_n)\cdot[\mathbf{q}\times(\mathbf{K}-\mathbf{P}_n)] \ . \tag{21}$$

If we define the nuclear convection current density $\rho^J_{sj}(\mathbf{q})$ as

$$\rho^J_{sj}(\mathbf{q}) \equiv \left\langle J \left|\left| \sum_n e^{i\mathbf{q}\cdot\mathbf{x}_n}[\sigma_s(n)\otimes\mathbf{J}_n]_j \right|\right| 0 \right\rangle, \quad \sigma_0 \equiv 1 \tag{22}$$

we observe that evaluation of the spin-orbit contribution requires knowledge of the spin-independent part, $\rho^J_{01}(\mathbf{q})$, as well as the spin-dependent part, $\rho^J_{11}(\mathbf{q})$, of the convection current density. Excluding velocity-dependent terms of the NN interaction, e.g., the spin-orbit contribution, the most general form of the NN interaction can be written as

$$t_n = d_0 + \mathbf{d}\cdot\boldsymbol{\sigma}_n \ , \tag{23}$$

where d_0 and $\mathbf{d}$ are operators independent of nuclear coordinates. The NN t matrix gets modified due to the first nonlocal correction in the following way:

$$t_n' = (d_0 + \mathbf{d}\cdot\boldsymbol{\sigma}_n)(\hat{\mathbf{K}}\cdot\mathbf{J}_n) \ . \tag{24}$$

We observe, after a straightforward angular-momentum recoupling, that the evaluation of energy-dependent corrections requires knowledge of the spin-independent piece $\rho^J_{01}(\mathbf{q})$ as well as the three spin-dependent pieces $\rho^J_{1j}(\mathbf{q})$, $(j=0,1,2)$ of the convection current density. Thus, aside from geometrical factors (i.e., Clebsch-Gordan coefficients) and excluding velocity-dependent terms, one can evaluate energy-dependent corrections to the amplitude from knowledge of the spin-orbit matrix elements.

V. CONCLUSIONS

We have shown that certain spin observables for the $0^+ \to 0^-$ transition are very sensitive to nonlocalities in nonrelativistic theories of NA inelastic scattering. The nonlocal corrections studied have their origin in the exchange part of the NN interaction as well as in the explicit energy dependence of the coupling strengths. We have used the $0^+(\vec{p},\vec{p}\,')0^-$ reaction to show that inclusion of energy-dependent corrections can lead to large modifications in the behavior of spin observables. We have also investigated energy-dependent corrections to other transitions. In particular, we have found negligible effects in transitions to stretched states. The full impact of these energy-dependent corrections, however, can only be assessed after a systematic study of different reactions and observables has been made. Although by no means trivial, we suggest that the inclusion of energy-dependent corrections into standard computer codes can, in certain limits, be related to already known matrix elements of the spin-orbit force.

In conclusion, the results obtained in the model calculation for the $0^+(\vec{p},\vec{p}\,')0^-$ reaction suggest that it is important to include corrections associated with the explicit energy dependence of effective nucleon-nucleon interactions.

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