Core-plus-alpha-particle states of ^{20}Ne and ^{16}O in terms of vibron models

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The SU(3) dynamical symmetries of the vibron and the nuclear vibron models have been applied to the ^{20}Ne and ^{16}O nuclei, respectively. Well established cluster bands with core-plus-alpha-particle structure and high-lying states populated as resonances in alpha scattering have been considered. Many resonances fit to the classification schemes of the energy spectra which have parameters largely determined by the well-known cluster bands. A procedure is proposed to take the Pauli principle into account, at an approximate level, when these models are applied to nuclei.

I. INTRODUCTION

Phenomenological algebraic models have proven to be useful in various fields of nuclear physics in describing collective motion. The core-plus-alpha-particle-type molecular excitations of light nuclei are considered as special examples of nuclear collectivity, so the question arises whether or not group theoretical models can be useful in their description also. The vibron model[1] (which is also applied in molecular physics) and its extended version called the nucleon vibron model[2] are meant to be proper tools to deal with this problem. Here we report simple calculations in which their dynamical symmetries have been used to describe some states of ^{20}Ne and ^{16}O. We have performed these analyses with a twofold aim in mind.

On the one hand, we wanted to see if it is possible to find some systematics among the experimental data with the help of the dynamical symmetries. In addition to the well-established cluster bands[3] these nuclei have large numbers of states with considerable alpha-particle reduced widths populated in alpha scattering.[4,5] These states are located mainly at high excitation energies of the compound systems. The question is whether these resonances, or some of them, fit into the classification scheme provided by the algebraic models when their parameters are determined largely by the well-known cluster bands. On the other hand, the application of a new cluster model to well-studied cluster nuclei seems to be a straightforward task. The comparison with the description of other models may shed some light on the relation between different approaches and can deepen our understanding on this new algebraic cluster model as well.

Some other algebraic description of alpha clustering of light nuclei have also been published recently. Chen suggested a model[6] based on bosons which are considered as nucleon pairs. Lomnitz-Adler and Van Isacker have proposed[7] a microscopic treatment based on nucleon degrees of freedom. Their approach starts from the shell model and can be considered as an extention of Elliott's SU(3) model, but in a way it turns out to be similar to the alpha-cluster models.

Unlike these models, the present approach (based on the vibron model[1,2]) starts from a purely phenomenological standpoint, without reference to the microscopic nature of the nuclear states. Nevertheless, the experience obtained from these applications helped us to reveal somewhat the connection to the shell model, and to find the way how the Pauli principle can be taken into account, at least approximately, in these phenomenological models.

Another application of a phenomenological algebraic model for the description of molecular states in light nuclei has been published recently by Yang and Hwang.[8] We note that their model is different from the ones we have applied. They used a dynamical symmetry characterized by the $U(5)\supset O(5)\supset O(3)$ group chain, while we applied the $U(4)\supset U(3)\supset O(3)$ and $U(6)\otimes U(4)\supset SU_a(3)\otimes U_b(3)\supset SU_a(3)\otimes SU_b(3)\supset SU(3)\supset O(3)$ symmetries, as it is discussed below.

In what follows, first we briefly review the basic characteristics of the applied models in Sec. II, and then present the applications to the ^{20}Ne and ^{16}O cases in Secs. III and IV.

II. THE VIBRON MODELS

The simplest version of the vibron models[1] describes the collective motion of two structureless clusters. An example for this kind of behavior is the rotational vibrational motion of a diatomic chemical molecule. The characteristic degree of freedom of this relative motion is dipole type. The spectrum of the system is generated by taking into account a finite number of bosons with one- and two-body interactions. They can occupy single-particle levels with $l=0$ and $l=1$ angular momenta, and they are called s' and p bosons, respectively. The general Hamiltonian of this model has U(4) group structure, and there are two dynamical symmetries, corresponding to simpler forms of the Hamiltonian. The first one is labeled by the group chain

$$U(4)\supset U(3)\supset O(3)\ . \tag{1}$$

From the end of this and other group chains we have omitted the O(2) group which is important only in the presence of an external field. The quantum numbers characterizing the states are given by the representation

indices

$$M,\ n_p,\ L\ , \tag{2}$$

where M means the total (finite) number of bosons, n_p is the number of bosons with $l=1$, and L is the angular momentum of the state. Their relations are $n_p=M$, $M-1,\ldots,0$, and $L=n_p,n_p-2,\ldots,1$ or 0 for $n_p=$odd or even. The Hamiltonian is obtained as an analytic formula

$$E=F+\epsilon n_p+\alpha n_p(n_p+3)+\beta L(L+1)\ . \tag{3}$$

This symmetry describes an anharmonic soft vibrator and the corresponding description based on the use of a potential would require an attractive well such as the Pöschl-Teller or Woods-Saxon potentials. The other dynamical symmetry is labeled by the group chain

$$\mathrm{U}(4)\supset\mathrm{O}(4)\supset\mathrm{O}(3) \tag{4}$$

and the quantum numbers and energy formula are given by simple expressions in this case, too. This limit describes the motion of a rigid system of the two clusters with finite equilibrium distance. This model has been applied to chemical molecules,[1] as well as to nuclear quasimolecular states populated in light heavy-ion reactions.[9]

The nuclear vibron model[2] not only describes the relative motion of the clusters, but also their collective internal excitations. When there are only two clusters, and only one of them can be excited (which is typical for a core-plus-alpha-particle system), then the model has a group structure of U(6)⊗U(4). The U(4) part describes the relative motion, while the U(6) part deals with the collective quadrupole excitations[10] of the heavier cluster. Now the spectrum is generated by two sets of bosons, both of them with conserved numbers. In addition to the s' and p vibrons there are s ($l=0$) and d ($l=2$) bosons of the quadrupole collectivity, which are also in one- and two-body interactions with each other. The interaction between the members of the two sets are taken into account also up to the two-body terms. One of the possible dynamical symmetries of this model is worked out in detail in Ref. 2. It is characterized by the group chain and the corresponding representation labels as follows:

$$\begin{array}{lllll} \mathrm{U}(6)\otimes\mathrm{U}(4)\supset \mathrm{SU}_a(3)\otimes\mathrm{U}_b(3)\supset \mathrm{SU}_a(3)\otimes\mathrm{SU}_b(3)\supset & \mathrm{SU}(3)\supset\mathrm{O}(3) \\ N,\quad M,\quad (\lambda,\mu)_a, & (\lambda,\mu),\quad L\ . \end{array} \tag{5}$$

Here N is the total number of the s and d bosons, and (λ,μ) is the pair of indices for the representation of the SU(3) groups. The energy formula is

$$E=F'+\epsilon_p n_p+\alpha_p n_p(n_p+3)+\kappa_d C(\lambda_a,\mu_a) +\kappa C(\lambda,\mu)+\kappa' L(L+1)\ , \tag{6}$$

where

$$C(\lambda,\mu)=\lambda^2+\mu^2+\lambda\mu+3\lambda+3\mu\ . \tag{7}$$

This symmetry describes a system in which the heavier cluster has axially symmetric deformation and the relative motion of the two clusters is anharmonic vibration. The nuclear vibron model has so far been mainly applied to heavy nuclei.[11]

III. ^{16}O+α STATES IN TERMS OF THE SIMPLE VIBRON MODEL

The ^{20}Ne is one of the best known cluster nuclei.[3] It has several bands consisting of cluster states, i.e., states having structure, in which the cluster configuration is important. As for the core-plus-alpha-particle configuration it appears with the largest weights in the $K^\pi=0^-$ band, containing states with J^π and E_x (MeV), as follows: 1^-,5.785; 3^-,7.156; 5^-, 10.261; 7^-,15.336; 9^-,22.87; and in the $K^\pi=0_4^+$ band as follows: 0^+,8.3; 2^+,8.8; 4^+, 10.79; 6^+,12.582; 8^+,17.591. The $K^\pi=0_1^+$ and 0_2^+ bands have a bigger part of the shell model (i.e., no cluster) configuration, while in the states of the 0_3^+ band the ^{12}C+2α configuration seems to be present with large weights.

We have considered the states of the 0^- and 0_4^+ bands as core-plus-alpha-particle states, and, since both the ^{16}O and the ^{4}He are double closed shell nuclei, we treated them as structureless clusters and applied the simple vibron model.

A. The spectrum

We have used the U(3) dynamical symmetry for the description of the spectrum. The reason for the choice between the two symmetries of the U(4) model was that while the collective bands in the O(4) limit involve both positive and negative parity states, the bands with fixed n_p values in the U(3) limit contain only either positive or negative parity states. Since in the 0^- and 0_4^+ bands the states are separated according to their parity, it is straightforward to apply the U(3) limit. As for the resonances of the alpha scattering[4] we have considered, in the first step, only the ones with $\theta_\alpha^2\geq 5\%$, i.e., $\gamma_\alpha^2\geq 0.05\times$(Wigner limit). In the second step we took into account the states with $\theta_\alpha^2\geq 1\%$.

The parameters of the model spectrum have been determined by a least-squares fitting procedure in which the θ_α^2 values were used as weights. Not all of the known resonances are involved in our experimental spectrum. Whether a state fits to this classification scheme or not was decided by performing the parameter search in many steps with various trials on the sets of resonances. The result is shown in Fig. 1. The energies of the model spectrum are given by (3) with the parameters $F=-15.616$ MeV, $\epsilon=3.506$ MeV, $\alpha=-0.088$ MeV, and $\beta=0.126$ MeV. We have chosen the total number of bosons $M=18$ in order to have the model spectrum as large as it

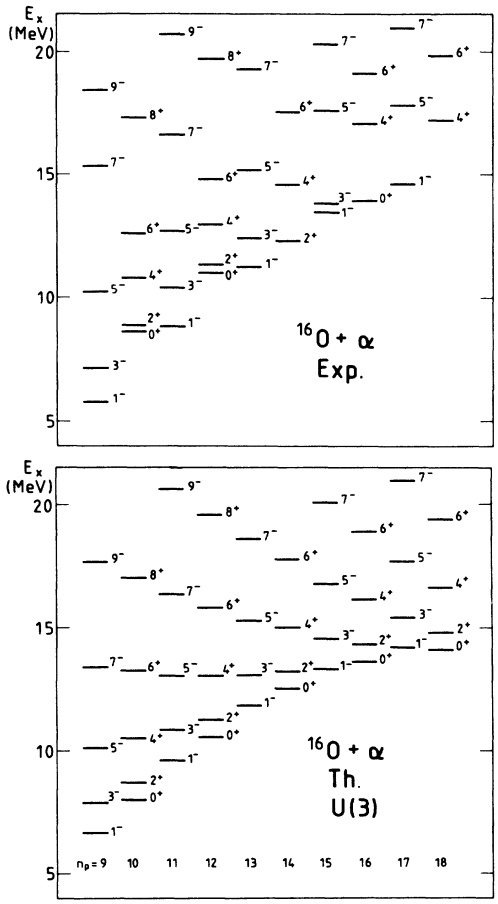

FIG. 1. Energies of experimental and model states in ^{20}Ne.

is required by the experimental one. The $n_p=9$ value for the lowest-lying negative-parity band was chosen as the smallest possible value that contains angular momenta up to $L=9$.

The spectrum of the U(3) dynamical symmetry with $M=18$ is larger than the one shown in Fig. 1. The plotted part is a result of truncations of the whole model spectrum along both axes. As for the excitation energy we did not display the states above $E_x \approx 21$ MeV, because that is the upper end of the available experimental spectrum. As for the n_p quantum number, we do not have the small n_p part ($n_p<9$) of the spectrum, and this is thought to be a consequence of the Pauli principle, and is discussed in more detail in Sec. III B.

As we mentioned before, the experimental spectrum does not contain all the known states. It involves more than half of the resonances with $\theta_\alpha^2 > 1\%$, but there is a considerable number of states which are not shown in Fig. 1. So, what we can say is that a good portion of the high-lying low-spin resonances can be classified by the same scheme which is used for the description of the well-established cluster bands with core-plus-alpha-particle structure, but there are many other resonances which do not fit this picture. This, however, is not surprising. In the structure of the low-lying states, configurations other than that of the core-plus-alpha-particle are known to be important, and that is obviously true for the high-lying resonances, too.

B. Exclusion of small n_p values, and the Pauli principle

As it was mentioned before, when applying the U(3) dynamical symmetry for the description of the well-known cluster bands in ^{20}Ne, we had to exclude the states with $n_p<9$ quantum numbers, in order to have the lowest-lying negative-parity cluster band long enough. This exclusion turns out to be a consequence of the Pauli principle. Or, conversely, it can be said that by truncating the spectrum properly from the small n_p side, the Pauli principle can be taken into account, at an approximate level. This can be understood qualitatively by considering the relation between the U(3) limit of the vibron model and the shell model. In its general form this is not known and rather complicated. However, in the harmonic oscillator (HO) limits, their relation is simple and very transparent. First of all it should be mentioned that the U(3) dynamical symmetry of the vibron model corresponds to a description based on the anharmonic oscillator,[12] so it has a harmonic oscillator limit. As for the connection between the two models, it is known from the early cluster studies[13] that the Hamiltonian of the HO shell model can be rewritten by introducing collective coordinates, as a Hamiltonian of an HO cluster model. In the HO cluster model the two clusters are described by the HO shell model, and their interaction is represented by an HO potential. The parameters of these oscillators are equal to the one of the shell-model oscillator. Since the Hamiltonians of the two models have the same eigenvalues, the number of the excitation quanta is the same in both descriptions of any state. When in the lighter cluster each nucleon is in the lowest-lying orbit then the excitation quanta carried by these nucleons in the HO shell model description are transferred into the excitation quanta of the relative motion in the cluster-model picture. This is the case when one of the clusters is, for instance, an alpha particle in its ground state. So, the procedure is to require at least as many excitation quanta in the relative motion of the two clusters as obtained in the shell-model description by putting the nucleons of the lighter cluster into orbits above the Fermi level of the core:

$$n_p \geq \sum_{i=1}^{A_{\text{cluster}}} 2n_i + l_i \ , \tag{8}$$

where n_i and l_i are the radial node number and angular momentum quantum number of the shell model. In the harmonic oscillator limit this rule guarantees that the Pauli principle is not violated. In the anharmonic case it is valid only approximately.

In our description of the ^{20}Ne, the exclusion principle requires $n_p \geq 8$; thus for a negative-parity band the smallest possible n_p value is 9.

This procedure of the inclusion of the Pauli principle is exactly the same as the one used in another phenomenological cluster model, in which the description of the relative motion is based on an attractive potential. That is the model developed by Buck, Dover and Vary,[14] and applied successfully by several authors[15] to the description of two-cluster configurations. The similarity is not

surprising if we recall the fact that the U(3) dynamical symmetry also corresponds in the potential picture to the use of an attractive well.[1]

Another consequence of the correspondence between the HO limits of the shell model and this dynamical symmetry is an indication on the microscopic content of the p bosons. On the macroscopic level they are known to be phonons of the relative motion of the two clusters.[12] The relation to the shell model suggests that microscopically they mean shell-model excitation quanta carried by the nucleons of the light cluster.

Since the U(3) dynamical symmetry is a limiting case of the vibron model, the next question addresses how we might deal with the Pauli principle in the general case of the model or in the case of the other dynamical symmetry. It can be taken into account by using the U(3) basis in the calculations, and truncating it as described above. It works also for the O(4) limit, but for this case it can be done without carrying out the numerical diagonalization, by applying the transformation brackets between the two bases.[16]

By adding the exclusion principle to the vibron model its application becomes, in a certain respect, very similar to the application of the U(3,1) model by Lomnitz-Adler and Van Isacker.[7] In their model the spectrum is truncated from below by the rules of the group representations and from above by physical considerations. In the vibron model the spectrum is truncated from below by physical considerations (Pauli principle via shell model connection), and from above by the rules of the group representations ($n_p \leq M$).

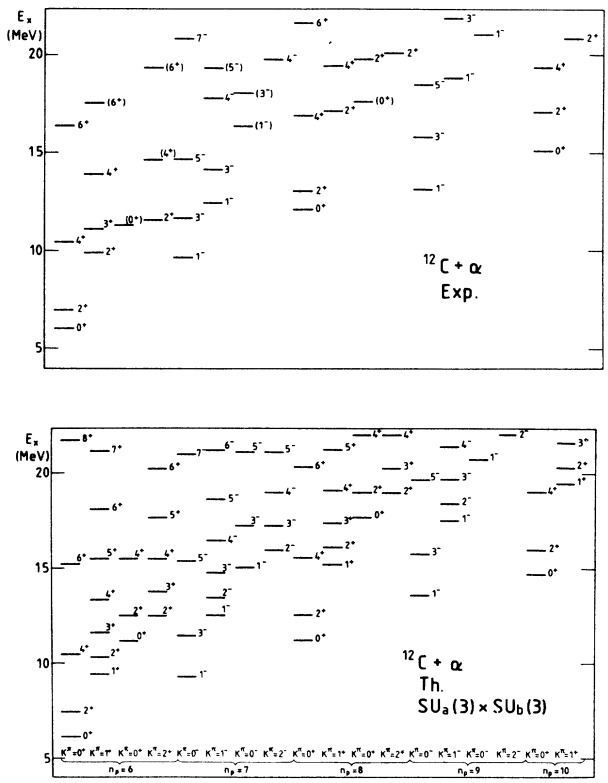

FIG. 2. Energies of experimental and model states in ^{16}O.

IV. ^{12}C+α STATES IN TERMS OF THE NUCLEAR VIBRON MODEL

The ^{16}O nucleus is also known to have well-established cluster bands with core-plus-alpha-particle configuration. They are the $K^\pi=0_1^+$ band (with 0^+,6.049; 2^+,6.911; 4^+,10.355; 6^+,16.275) and the $K^\pi=0^-$ band (with 1^-,9.632; 3^-,11.60; 5^-,14.66, 7^-,20.057). There are also resonances of alpha scattering;[5] so the question is, again, whether or not they fit to the same scheme as the well-known cluster states. The ^{12}C is not a closed-shell nuclei, it can undergo collective excitations, so for the description of this system we have applied the nuclear vibron model which describes this possibility.

It is worth mentioning that the ^{16}O nucleus played an important role in the testing of the potential cluster model, too. In the first paper[14] on that model the authors described the 0_1^+ and 0^- bands, and after nine years of successful applications, the model has been extended to involve internal excitations of a cluster and was applied again to the ^{12}C+α system.[17] In this respect the algebraic approach has the advantage of the easy application, at least in the first step, when the dynamical symmetries are used. Having analytic solutions then the coupled system can be treated almost as easily as the single-channel problem.

Thinking of the ^{12}C as a deformed core we have applied the SU(3) dynamical symmetry (5)–(7), and, since we wanted to take into account only one excited state of the core (the first 2^+), we have chosen $N=1$. The $M=10$ value was taken again to have large-enough spectrum, while the $n_p \geq 6$ truncation is required by the states of the lowest-lying bands on the one side, and is in agreement with the exclusion principle on the other side.

Here all the elastic alpha-scattering resonances have been considered below $E_x \approx 22$ MeV. To guess good initial values for the parameters we have determined them based on a few states as follows. The moment of inertia was deduced from the two known bands. Then there are only four more parameters, because κ_d and F' in (6) can be combined to a constant F'' due to the $N=1$ choice. They were determined from the energies of four band heads: 0^+,6.05; 1^-,9.651; 1^-,12.443; 0^+,12.048. Having these parameters the whole spectrum is given, and that initial spectrum proved to be almost as similar, to the experimental one, as the final result obtained from a least-squares-fitting procedure. In the fit the θ_α^2 values were used again as weight factors. Figure 2 shows the experimental and the model spectrum; the latter one was calculated with the parameters $F''=-17.540$ MeV, $\epsilon_p=6.440$ MeV, $\alpha_p=-0.082$ MeV, $\kappa=-0.120$ MeV, and $\kappa'=0.217$ MeV.

In this case we have left out only nine resonances which do not fit to the scheme. There are some model states without experimental correspondents. Most of them have unnatural spin parity. Since they cannot be populated in alpha-particle induced reactions, which pro-

vided most of the available data, it is understandable that many of them are missing. However, the low-energy part of the spectrum is better known, and the fact that the lowest-lying 1^+ state is approximately 3.5 MeV higher than predicted by this scheme shows that this simple approach is not quite correct. Combining, this failure with some overall similarity found between the experimental and model spectra, one gets the impression that more refined versions of the model might be able to give a better description.

V. CONCLUSIONS

We have applied the SU(3) dynamical symmetries of the vibron and the nuclear vibron models for the description of certain states in ^{20}Ne and ^{16}O which contain the core-plus-alpha-particle configuration with considerable weights. Many high-lying resonances found in elastic alpha scattering could be classified with the same scheme as the well-known cluster bands. The overall agreement between the experimental and model spectra is remarkable, but not complete. The extension of this kind of analysis to involve more nuclei and more elaborate versions of the models seems to be promising.

The experience of this application led us to the conclusion that the model spectra have to be truncated from the small n_p side, as shown by (8), in order to take the Pauli principle into account. The explanation for this rule is provided by the relation between the harmonic oscillator limits of the $U(4)\supset U(3)\supset O(3)$ dynamical symmetry and the shell model.

The U(3) limit of the vibron model turns out to be similar, in some respects, to the potential cluster model introduced by Buck, Dover, and Vary.[14] For a detailed comparison between the two models more extensive application of the vibron model is needed. Here we concentrated on the energy spectra of two nuclei. Several of their states have been described also by the potential model;[14,17] however, only a few were treated in both descriptions (the $K=0^-$ band in ^{20}Ne and $K=0^\pm$ bands in ^{16}O). Their energies are in good agreement, both with each other and with the experimental data. Obviously, these examples are not enough to reveal the relative advantages of the two approaches. What one could expect is that in trying to find some systematics among a great deal of experimental data, the vibron model and especially its dynamical symmetries might be the more suitable tools. Actually, this was our aim here. Another useful feature of the algebraic model can be that it involves both purely attractive potential and a potential with repulsive core, together with the continuous interplay between these two limits. On the other hand, this is a model of bound states, so it is not possible to describe the widths of the resonant states. Also, it is not clear, at present, whether it can be easily extended to involve as many internal excitations of the clusters as was done in some applications[18] of the potential model.

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