Shell model predictions for $^{19}N(\beta^-)^{19}O$

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The energy spectra and wave functions of ^{19}N and ^{19}O are calculated using a modification of the Millener-Kurath interaction. The results are used to predict the half-life and decay modes of both the allowed and first-forbidden β^- decays of ^{19}N. Comparison is made to recent experiments.

I. INTRODUCTION

This study of $^{19}N(\beta^-)^{19}O$ is part of a continuing effort to utilize β^- decay information on very neutron-rich nuclei towards an understanding of the underlying nuclear structure. The cases considered involve parent nuclei near $A=16$ with $N>8$ (Ref. 1) and near $A=40$ with $N>20$ (Refs. 2–5). Typically, the decays proceed by Gamow-Teller (GT) transitions to non-normal parity (i.e., $1\hbar\omega$ excitations) in the daughter nucleus or by first-forbidden transitions to the lowest $(0\hbar\omega)$ configuration. The shell-model calculations for these configurations are performed with cross-shell interactions developed specifically to describe levels with valence nucleons active in several major shells simultaneously. A modification[1] of the Millener-Kurath interaction[6] (designated MKIII) is used near $A=16$ and the WBMB interaction[5] is used near $A=40$.

The main source of experimental information on the decays considered is the β^- delayed γ spectra of Dufour *et al.*[7] Dufour *et al.*[7] have tabulated results for $^{19}N(\beta^-)^{19}O$ and these we will consider in the present study.

II. CALCULATION AND RESULTS

A. Predicted energy spectra

The calculations—carried out with the shell-model code OXBASH (Ref. 8)—have been fully described elsewhere[1–5] and will only be briefly summarized here. The low-lying even-parity states of ^{19}O were assumed to arise from the $(0s)^4(0p)^{12}(2s,1d)^3$ model space and the wave functions, etc. were calculated with either the Chung-Wildenthal[9] or "universal" $(2s,1d)$ interaction (designated USD).[10,11] The ^{19}N states were taken to arise from $(0s)^4(0p)^{11}(2s,1d)^4$. In order to avoid incomplete separation of spurious states, the model space for the odd-parity states of ^{19}O must include all possible $1\hbar\omega$ excitations of the $0\hbar\omega$ model space. This requirement results in $(0s)^4(0p)^{12}(2s,1d)^2(0f,1p)^1$ components as well as the main $(0s)^4(0p)^{11}(2s,1d)^4$ ones. In a similar study of ^{17}N (Ref. 1) it was found that the $(0f,1p)$ admixtures were of order 1% but the effect on $E1$ matrix elements could be considerable.

For ^{19}N the predicted energy spectrum has a $J^\pi=\frac{1}{2}^-$ ground state and excited states below 4-MeV excitation, in keV, at $1377(\frac{1}{2}^-)$, $2269(\frac{5}{2}^-)$, $2788(\frac{3}{2}^-)$, $3336(\frac{1}{2}^-)$, $3488(\frac{7}{2}^-)$, $3667(\frac{5}{2}^-)$, and $3854(\frac{3}{2}^-)$. The $\frac{1}{2}^-$ ground state is as expected from weak-coupling expectations, i.e., $^{15}N\otimes{}^{20}O$ and we shall assume it in the beta decay calculation.

The energy spectrum of ^{19}O is considered in Fig. 1. The experimental spectrum is from Ajzenberg-Selove[12] with the changes for the 3067-, 3232-, and 3945-keV levels considered in the Appendix. Note that the "3945-keV level" is most probably a complex of at least two levels with one member having $J^\pi=\frac{3}{2}^-$ with unknown γ-decay modes.

The 3p-0h spectrum shown is that of the USD interaction. For this interaction, the calculation of binding energies is absolute; the calculation places the $\frac{5}{2}^+$ ground state 81 keV above experiment. Except for $\frac{9}{2}{}_2^+$, the identification of the model states with experiment is the same as proposed by Wildenthal.[10,11] From weak-coupling considerations and calculations in a truncated model space, it is expected that the lowest-lying $\geq 2\hbar\omega$ intruder state in ^{19}O lies at 3–4 MeV and has $J^\pi=\frac{3}{2}^+$. We identify this state with the 3067-keV level which is marked with an asterisk in Fig. 1.

No attempt was made to obtain meaningful absolute binding energies for the odd-parity states calculated with the MKIII interaction, i.e., the calculated binding energy of the $\frac{1}{2}{}_1^-$ state is uncertain to order 1–2 MeV. The placement of the 4p-1h states in Fig. 1 corresponds to the minimum value of the rms deviation of the energy differences between model and experiment for the states connected by dashed lines on the left-hand side of Fig. 1.

The placement of the 4p-1h spectrum shown in Fig. 1 relies heavily on the identification of a level at 3945 keV as $\frac{3}{2}{}_1^-$ and location of further $\frac{3}{2}^-$ spectroscopic strength in the $^{18}O(d,p)^{19}O$ reaction at 4584 keV, assigned $\frac{3}{2}{}_2^-$ (Ref. 12). Then the only plausible candidate for the $\frac{1}{2}{}_1^-$ model state is the 3232-keV level. The $\frac{5}{2}{}_1^-$ state is very tentatively associated with the "3945-keV level" complex, it could as well lie at higher excitation.

Supporting evidence for the proposed odd-parity scheme of Fig. 1 is provided by the β^- delayed singles γ-ray spectrum of Dufour *et al.*[7] The reported results of Dufour *et al.* consist of a tabulation of the three γ-ray energies and intensities shown in Fig. 2. We have placed these three γ transitions in the only possible way which does not demand any new ^{19}O levels below the $^{18}O+n$ threshold at 3957 keV. Note that the $^{19}N(\beta^-)^{19}O$ γ spectra of Dufour *et al.* has rather poor statistics. Thus, other γ transitions are certainly possible, although the ones shown should be the most intense. The results of Fig. 2 demand that the dominant β^- branch to γ-emitting states is to a level at 3943 keV with the possibility of weaker branches to the 3234-keV level and to the 96-keV level and/or other levels that γ cascade into the 96-keV level. Regarding the latter point, the 1472-keV level decays 100% to the 96-keV level (Ref. 12). It is expected on general grounds and indicated by our calculations (see Sec. II B) that the observed β^- decay into the 3943-keV level is allowed. Since there is a known $\frac{3}{2}^-$ level in the "3945-keV level" complex it is natural to identify the decay as proceeding to it.

The large uncertainties on the γ-ray intensities and the possibility of unobserved decay modes allows for the possibility of a relatively large β^- branch to the 3234-keV level. If this branch were significant then odd parity would be indicated for the 3234-keV level as well.

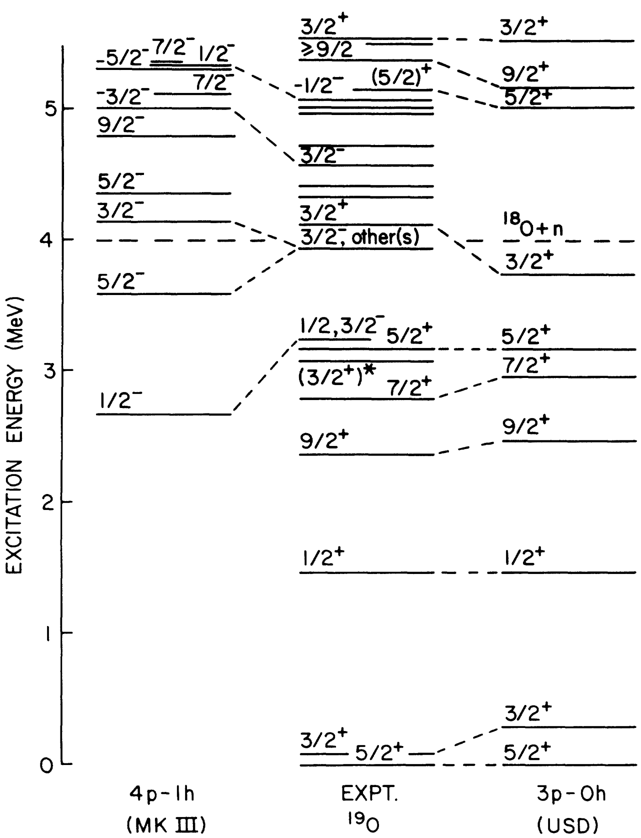

FIG. 1. Comparison of experimental and calculated excitation energies, spins, and parities for ^{19}O. The experimental spectrum is from Ref. 12 with the exceptions noted in the text. The calculated even-parity (3p-0h) and odd-parity (4p-1h) levels are the predictions of the USD interaction (Refs. 10 and 11) and MKIII interaction (Refs. 1 and 6), respectively. The level marked with an asterisk is our candidate for the lowest-lying $\geq 2\hbar\omega$ intruder state.

In our discussion of $^{19}N(\beta^-)^{19}O$ beta decay we shall assume the placement of odd-parity levels indicated in Fig. 1. However, we note that the proposed scheme is speculative and only further experiments can determine the actual ordering of low-lying odd-parity states.

B. $^{19}N(\beta^-)^{19}O$

Allowed β^- decay was calculated for all the calculated energetically accessible states of ^{19}O with the final ^{19}O spectra placed as shown in Fig. 1. The Q value was taken as 12540(19) keV (Ref. 13), the half-life as 0.235(32) sec (Ref. 14), and the excitation energies of the first four $\frac{1}{2}^-,\frac{3}{2}^-$ states were taken from experiment as indicated by the correspondences in Fig. 1. The Gamow-Teller transi-

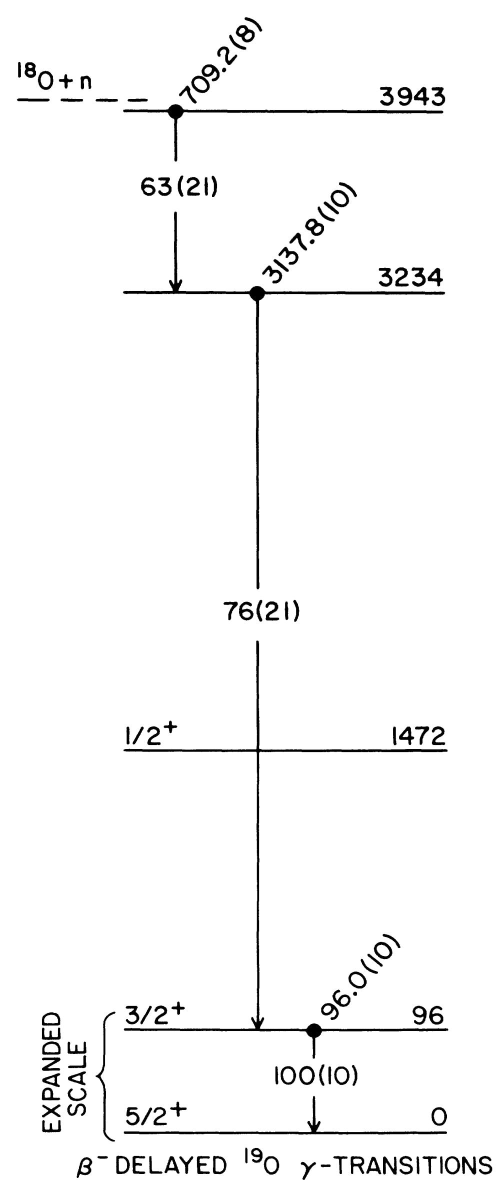

FIG. 2. The proposed β^- delayed γ decay scheme of ^{19}O as deduced from the γ-ray energies and intensities reported by Dufour *et al.* (Ref. 7).

TABLE I. First-forbidden beta decay of ^{19}N to the first $\frac{1}{2}^+$, $\frac{3}{2}^+$, and $\frac{5}{2}^+$ states of ^{19}O. No other first-forbidden branches are predicted to be greater than 0.1%. The partial half-life, t, is related to the β^- branching ratio (BR), via the total ^{19}N half-life of 0.235 sec by BR(%)$=23.5/t$. The predicted partial half-life for all first-forbidden decays is 3.6 sec.

J^π	E_x (expt.)	t (sec)	$\log f_0 t$	BR (%)
$\frac{5}{2}^+$	0	23.0	7.00	1.02
$\frac{3}{2}^+$	96	186.2	7.30	0.49
$\frac{1}{2}^+$	1472	4.81	6.06	4.88
				$\sum_{\text{all levels}} = 6.54\%$

tion strengths were calculated with free-nucleon operators and then multiplied by 0.6 to allow for quenching (Ref. 15). First-forbidden decays were calculated for the first ten final states of each allowed spin parity, i.e., for $\frac{1}{2}^+$, $\frac{3}{2}^+$, and $\frac{5}{2}^+$ states of ^{19}O. It was found that only the even-parity states below 4.5-MeV excitation (Fig. 1) had predicted beta branches $>10^{-2}\%$. For these five states the experimental excitations indicated in Fig. 1 were used in the calculation. The calculation also used effective operators designed to approximate the effects of ground-state correlations and meson exchange.[1,16]

Details of the calculations are given in Tables I and II. We first consider the first-forbidden decays of Table I. The considerably smaller $\log f_0 t$ value for the $\frac{1}{2}^- \to \frac{1}{2}^+$ branch is typical of the systematics of first-forbidden decays. It follows from the large $\Delta J=0$ matrix element of γ_5 which is not only kinematically favored but is also strongly enhanced by meson exchange.[16,17]

A marked feature of the allowed decays shown in Table II is the very weak decays to the states below 5-MeV excitation. In fact, only 7.3% of allowed decays is predicted to occur to γ-emitting states (those below 3957-keV excitation). The weakness of the decays to low-lying states can be traced to the different spatial symmetries of the low-lying $T=\frac{5}{2}$ and $T=\frac{3}{2}$ states of $(0s)^4(0p)^{11}(2s,1d)^4$. ^{19}N $\frac{1}{2}^-$ is mainly [4^2322] while the low-lying ^{19}O $\frac{1}{2}^-$ and $\frac{3}{2}^-$ states are mainly [4^321]. These differing spatial symmetries are a general property of neutron-rich light nuclei. It is discussed by Millener and Kurath[6] in reference to ^{14}B β^- decay. Since the Gamow-Teller operator cannot connect states of different spatial symmetries, allowed decay to low-lying states is generally weak. States in the daughter nucleus of the same symmetry as the parent state are expected near the analog of this state in the daughter nucleus. Thus, because of the Coulomb energy, they are not generally available for β^- decay. The total summed beta strength is then only a fraction of the sum-rule limit. In the present case, for instance, the sum-rule limit (Ref. 15) is

$$\sum_f B(\mathrm{GT}) = 0.6(g_A/g_V)^2 3(N_i - Z_i) = 14.3$$

TABLE II. Predictions for the allowed β^- decay of ^{19}N. The ^{19}N excitation energy, E_x, for the first four states is the experimental value indicated by the dashed lines in Fig. 1. For the remaining states it is the model energy with the model spectrum placed as in Fig. 1. The B(GT) values are the free-nucleon values multiplied by 0.6. $\log f_0 t$ is defined as $\log_{10}[6166/B(\mathrm{GT})]$. The branches shown are those $>1\%$ of the total allowed decays. The sum of the listed branches shown is 95.6%, the remaining 4.4% is distributed amongst many weak branches. The predicted partial half-life for allowed decays is 0.63 sec.

J_k^π	E_x (keV)	B(GT) ($\times 10^3$)	$\log f_0 t$	Allowed branching (%)
$\frac{1}{2}^-_1$	3232	1.0	6.77	1.1
$\frac{3}{2}^-_1$	3945	8.5	5.86	6.2
$\frac{3}{2}^-_2$	4582	6.6	5.97	3.4
$\frac{1}{2}^-_2$	5082	103.6	4.77	39.2
$\frac{3}{2}^-_6$	6755	109.3	4.75	12.7
$\frac{3}{2}^-_7$	7119	50.6	5.08	4.3
$\frac{3}{2}^-_8$	7509	200.1	4.49	12.2
$\frac{1}{2}^-_5$	7622	23.3	5.42	1.3
$\frac{3}{2}^-_9$	7843	82.7	4.87	3.7
$\frac{3}{2}^-_{10}$	8196	172.6	4.55	5.4
$\frac{1}{2}^-_6$	8505	166.9	4.57	3.6
$\frac{3}{2}^-_{11}$	8506	111.2	4.74	2.5

and the fraction available for β^- decay is found to be $\sum_{Q>0}B(\mathrm{GT})=2.78$ so only 20% of the sum-rule limit is available.

Of special interest for comparison to experiment is the predicted decays to the first two states of Table II since these two (or perhaps only one) are the only states expected to contribute to the β^- delayed γ spectra. From Table II we see that these β^- branches are 0.01 and 0.06% of the sum-rule limit. Our experience is that 0.4% of the sum-rule limit is about the smallest B(GT) that can be predicted reliable. To illustrate this we repeated the allowed β^- decay calculation with a modification of the MKIII interaction. Namely the $(2s,1d)$ part of the interaction was changed by replacing the Chung-Wildenthal 2BME and single-particle energies with the USD values. The result was a partial allowed half-life 30% smaller and—to the present point—B(GT) values for $\frac{1}{2}{}_1^-$ and $\frac{3}{2}{}_1^-$ 5.3 times larger and 2.3 times smaller, respectively. We note that we expect the GT matrix elements to be most sensitive to the cross-shell part of the interaction which happens to be the most poorly determined.

From Tables I and II we can see that 11.9% of the β^- decays is predicted to feed γ-emitting states and 42% of this intensity is due to first-forbidden decays to the $\frac{3}{2}{}_1^+$ and $\frac{1}{2}{}_1^+$ levels. The "missing" 1472→96 transition is then a major disagreement with experiment.

III. SUMMARY

The β^- decay of ^{19}N has been considered in the spherical shell model. The total half-life derived from partial half-lives for first-forbidden and allowed decays is 0.54 sec. This is in poor agreement with the experimental value of 0.235(32) sec. The unreliability of predictions for the allowed decays to the lowest-lying states is discussed. Taken at face value the prediction is that only 7.3% of allowed decays proceed to γ-emitting states. When combined with the first-forbidden calculation this leads to a prediction of $P_n=0.87$ for the fraction of β^- decays leading to neutron-emitting states (assuming decay modes other than γ and n are negligible). A measurement of P_n would be very valuable because without it we have no measure of the absolute decay strengths to the low-lying states.

We note that a 4.88% first-forbidden branch is predicted to the 1472-keV level. No γ decay of this state was observed by Dufour *et al.* This is indirect evidence that the percentage of allowed decay proceeding to γ-emitting states is considerably larger than 7.3%. That is, the most obvious resolution of the missing 1472→96 1376-keV γ ray and the too long prediction for the half-life is that the B(GT) values for the lower-lying ^{19}O states are underpredicted by roughly an order of magnitude. Increasing them by a factor of ~ 10 would lead to $P_n \sim 0.3$ and a small enough relative intensity for the 1472→0.96 γ ray so that it could have been overlooked.

We have speculated that the γ cascade of Fig. 2 is $\frac{3}{2}^- \rightarrow \frac{1}{2}^- \rightarrow \frac{3}{2}^+$. How does this compare to shell-model predictions? Using free-nucleon operators we find strengths of 0.07, 1.5×10^{-3}, 6.6×10^{-3}, and 2.0×10^{-3} Weisskopf units for the $M1$ $\frac{3}{2}{}_1^- \rightarrow \frac{1}{2}{}_1^-$ transition and the $E1$ $\frac{3}{2}{}_1^- \rightarrow \frac{1}{2}{}_1^+$ $\frac{3}{2}{}_1^- \rightarrow \frac{3}{2}{}_1^+$, and $\frac{3}{2}{}_1^- \rightarrow \frac{5}{2}{}_1^-$ transitions, respectively. Using the energies of Fig. 2, these strengths translate to branching ratios of 0.2%, 14%, 65%, and 21%, respectively. Clearly these predictions are at odds with our speculations. A definitive understanding of ^{19}N$(\beta^-)^{19}$O must await further experiments at which time it should prove a challenging test of cross-shell interactions.

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APPENDIX: THE ^{19}O 3067-, 3232-, AND 3945-keV LEVELS

The spin-parity assignments of Fig. 1 differ in some respects from those given in the compilation of Ajzenberg-Selove[12] and so some discussion is warranted. The differences involve the 3067- and 3232-keV levels. Both levels are formed in the ^{13}C(^{7}Li,p), ^{17}O(t,p), and ^{18}O(d,p) reactions. Fortune and Bingham[18] measured total cross sections in the ^{13}C(^{7}Li,p) reaction and found them to be closely proportional to $2J+1$. On this basis they gave most probable spin values of $\frac{3}{2}$ to both levels *assuming them to have even parity;* if the parity is odd their analysis would yield $\frac{1}{2}$ or $\frac{3}{2}$ for both levels. In any case, the best choice of J from an assumed $(2J+1)$ proportionality for the cross section cannot be said to constitute a definite assignment.

Proton angular distributions in the ^{17}O$(t,p)^{19}$O reaction were measured by Wiza and Middleton[19] and analyzed by Crozier *et al.*[20,21] Of the seven lowest-lying levels, the cross sections for the $\frac{1}{2}^+$ 1472-keV level and the 3067-keV were the lowest reported and the angular distributions for both levels were rather flat and featureless. Crozier *et al.* gave the 3067-keV level a *most probable* $\frac{3}{2}^+$ assignment on the basis of a fairly good fit to an $L=2+4$ distorted-wave analysis. However, the $L=2$ fit to the 1472-keV level was poor indicating competing second-order processes at the lower cross sections of these two levels. Thus, it seems clear that other explanations of the data for the 3067-keV level are possible. We emphasize that Crozier *et al.*[21] did not consider the $\frac{3}{2}^+$ assignment to be definite. The 3232-keV level was not observed in the ^{17}O$(t,p)^{19}$O reaction, it has a cross section less than 10% of that for the weakly formed 3067-keV level.

The ^{18}O$(d,p)^{19}$O reaction has been studied at $E_d=12$ MeV and relatively high proton resolution by Wiza and Middleton,[19] at $E_d=5$ MeV,[22] and with polarized deuterons at $E_d=14.8$ MeV.[23] The 3067-keV level has a very small cross section in the (d,p) reaction; its observation was not reported in any of these three studies and Wiza and Middleton[19] reported a cross-section limit of less than 25% of that for the very weakly formed $\frac{3}{2}^+$ 96-keV level. Wiza and Middleton observed the 3232-keV level with a strongly forward peaked (d,p) angular distribution. They show an $l=0$ fit to the data, but the $l=1$ curve shown for the nearby 3945-keV level would appear to provide even a better fit. From these results a definite $J=\frac{1}{2},\frac{3}{2}^-$ assignment can certainly be given to the 3232-keV level. For a $\frac{1}{2}^+$ assignment the spectroscopic strength is $(2J+1)S=0.084$ (Ref. 19), while for $l=1$ and $J^\pi=\frac{1}{2}^-$ or $\frac{3}{2}^-$ it is $(2J+1)S\sim0.012$. The 3232-keV level was also observed by Fintz *et al.*[22] but they only observed protons with poor statistics at four angles in the range $20°<\theta_{\rm c.m.}<64°$ as opposed to the high-quality comprehensive results of Wiza and Middleton.

Finally, we consider the available information on the γ decays of these two levels. ^{19}O γ decays have been observed via ^{18}O$(d,p\gamma)^{19}$O,[22,24] ^{2}H(^{18}O,$p\gamma$)^{19}O,[25] and ^{17}O$(t,p\gamma)^{19}$O.[24] No information is available on the 3232-keV level from any of these studies. For the 3067-keV level, there are two reported observations. Broude *et al.*[25] observed γ transitions in the ^{2}H(^{18}O,$p\gamma$)^{19}O reaction. This work consisted of singles γ-ray spectrum taken at 90° to the beam. Gamma-ray lines were identified by energy only. On this basis a 1597-keV γ transition was identified with the ^{19}O 3067$\rightarrow$1472 transition. In view of the known small cross section for the 3067-keV level in the (d,p) reaction [see the discussion of the (d,p) angular distributions above], this identification must be viewed with caution. Broude *et al.* do not comment on the possible existence of other decay modes. Hibou *et al.*[24] also reported the observation of γ decay of the 3067-keV level. They used (p,γ) coincidences with the protons detected at 180° to the beam in the ^{18}O$(d,p\gamma)$ and ^{17}O$(t,p\gamma)$ reactions. They reported a ^{19}O 3067$\rightarrow$96 transition (which of the reactions used is not stated). No comment is made on a possible 3067$\rightarrow$1472 transition. In view of these mildly conflicting results we view the γ decay of the 3067-keV level as a completely open question. In summary, our interpretation of the available experimental data leads us to $J^\pi=(\frac{3}{2}^+)$ and $(\frac{1}{2},\frac{3}{2}^-)$ for the 3067- and 3232-keV levels, respectively, with no available information on the γ decay or lifetimes of either level.

The evidence on the formation and decay of the 3945-keV complex of levels is also of direct interest to the present study. Fortune and Bingham[18] have presented convincing circumstantial evidence that the 3945-keV "level" observed via the three reactions discussed here is in actual fact at least a doublet. From the (d,p) studies a convincing $l=1$, $J^\pi=\frac{3}{2}^-$ assignment is obtained,[19,23] while the very large cross section in the (^{7}Li,p) reaction suggests a complex of levels with $\Sigma J\simeq\frac{11}{2}$. The γ decay of this "level" has been studied by the Strasbourg group[22,24] who reported branches (in %) of 33(8), 39(8), 24(4), <15, <15, <15 to the levels (in keV) at 0, 96, 1472, 2372, 2779, and 3154, respectively. These results were obtained in the $(d,p\gamma)$ with the protons observed at 180° to the beam. There is no way of ascertaining how these decays are to be distributed amongst the 3945-keV complex of levels.

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