^{14}C beta decay and the ^{14}N $M1$ form factors

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The description of p-shell orbitals in the $A=14$ nuclear system in terms of relativistic wave functions, which are solutions to the Dirac equation with strong scalar and vector potentials, provides a good description of the $M1$ form factors in ^{14}N and the greatly suppressed ^{14}C beta decay rate.

I. INTRODUCTION

Nuclei in the p shell have long provided testing grounds for nuclear wave functions and thereby the nuclear shell model. Many ground state and low-lying excited state properties throughout the p shell can be described as admixtures of $p_{3/2}$ and $p_{1/2}$ particle or hole states such as in the global fits of Cohen and Kurath.[1] The $A=14$ system, particularly the ground and first excited states of ^{14}N and the ground state of ^{14}C, has attracted a great deal of attention.[2-4] The elastic and inelastic $M1$ form factors of ^{14}N have been measured over a large range of momentum transfer which provides considerable constraint on possible nuclear wave functions, while the severely retarded ^{14}C beta-decay rate has long been a puzzle, and in conventional analyses seems to force a constraint on the wave functions which is inconsistent with fits to data obtained from the $A=14$ system with other probes.[2] As discussed in detail by Huffman *et al.*,[2] best fits to the form factors alone produce wave functions which have too large a Gamow-Teller matrix element and thus do not describe the beta-decay rate. Furthermore, analysis of the photopion reaction ^{14}N $(\gamma,\pi^+)^{14}$C$_{\text{g.s.}}$ is well fitted with the Gamow-Teller matrix element coming from the Huffman analysis which ignores the beta-decay constraint on the wave functions.[4] It has been suggested[5] that the vanishingly small beta-decay rate might be due to a destructive interference between a small one-body Gamow-Teller matrix element and higher-order exchange current terms, but a conclusive calculation of this possibility has not yet been carried out.

All of the discussions of Huffman and others who have investigated the $A=14$ system have been carried out in terms of nonrelativistic wave functions, and usually in terms of harmonic oscillator wave functions characterized only by a range parameter b. Following the empirical success of Clark and collaborators in fitting intermediate energy proton scattering with relativistic wave functions,[6] the large relativistic effects predicted in kaon photoproduction from nuclei,[7] and the theoretical underpinnings of the sigma-omega model,[8] we choose to investigate the $A=14$ system using relativistic single-particle wave functions. In particular, we calculate the elastic and inelastic $M1$ form factors of ^{14}N and the beta decay of ^{14}C, using single-particle wave functions which are bound-state solutions to the Dirac equation with scalar and timelike vector spherically symmetric potentials. We will comment on the limitations of this approach in Sec. III.

Following Huffman *et al.* and others, we assume that isospin is a good quantum number (that is, we neglect the Coulomb potential in the $A=14$ system and the proton-neutron mass difference) and represent the ground states of ^{14}N and ^{14}C, and the first excited state of ^{14}N at 2.313 MeV, in terms of two p-shell holes in the doubly closed $A=16$ system. In j-j coupling, the ground state $(J=1^+,T=0)$ and first excited state $(J=0^+,\ T=1)$ of ^{14}N can be written in standard notation as

$$\begin{aligned}|J^\pi=1^+,\ T=0\rangle &= a\,|1p_{1/2}^{-2}\rangle + b\,|1p_{3/2}^{-1},1p_{1/2}^{-1}\rangle + c\,|1p_{3/2}^{-2}\rangle\ ,\\ |J^\pi=0^+,\ \ T=1\rangle &= m\,|1p_{1/2}^{-2}\rangle + n\,|1p_{3/2}^{-2}\rangle\ .\end{aligned} \tag{1}$$

The two-hole configurations are normalized by

$$a^2+b^2+c^2=m^2+n^2=1\ ,$$

and when we consider a Dirac basis, a better label for the state is the Dirac quantum number κ. Recall the $l(\kappa)=|\kappa+\frac{1}{2}|-\frac{1}{2}$ and $j(\kappa)=|\kappa|-\frac{1}{2}$.

We are interested in comparing the nonrelativistic and relativistic description of the elastic and inelastic $M1$ form factors of ^{14}N and the ^{14}C beta-decay rate. That is, we want to compare Schrödinger single-particle wave functions

$$\psi_{nlj}(\mathbf{r})=\frac{R_{nl}(r)}{r}\sum_{m_s m_l} C^{l\ \ 1/2\ j}_{m_l\, m_s\, m}\, Y_l^{m_l}(\hat{\mathbf{r}})\,\xi_{1/2}^{m_s}\ , \tag{2}$$

where $\xi_{1/2}$ is a Pauli spinor, to Dirac single-particle wave functions

$$\psi_{n\kappa\mu}(\mathbf{r})=\begin{bmatrix} f_{n\kappa}(r) & \chi_\kappa^\mu(\hat{\mathbf{r}}) \\ i g_{n\kappa}(r) & \chi_{-\kappa}^\mu(\hat{\mathbf{r}})\end{bmatrix}\ , \tag{3}$$

where the spin-angle functions $\mathcal{X}_\kappa^\mu$ are given by

$$\mathcal{X}_\kappa^\mu(\hat{\mathbf{r}}) = \sum_{m_l m_s} C_{m_l\, m_s\, \mu}^{l\ 1/2\, j} Y_l^{m_l}(\hat{\mathbf{r}}) \xi_{1/2}^{m_s} .$$

Usually nonrelativistic shell-model analyses make use of a harmonic oscillator potential without a spin-orbit term and, therefore, the wave functions are only characterized by a range parameter b even though a potential with the Woods-Saxon shape

$$h(r) = \frac{1}{1+\exp\left[\dfrac{r-R}{a}\right]} , \tag{4}$$

where R and a are the radius and diffuseness parameters might furnish a better description for high-momentum-transfer process. Note, $R = r_0 A^{1/3}$.

The Dirac radial functions $f_\kappa(r)$ and $g_\kappa(r)$ in Eq. (3) satisfy the following coupled differential equations which contain the strong scalar $S(r)$ and timelike vector $V(r)$ potentials:

$$\begin{aligned} \frac{df_\kappa}{dr} &= \frac{-\kappa-1}{r} f_\kappa + [M+S(r)+E-V(r)] g_\kappa , \\ \frac{dg_\kappa}{dr} &= \frac{\kappa-1}{r} g_\kappa + [M+S(r)-E+V(r)] f_\kappa . \end{aligned} \tag{5}$$

The potentials $S(r)$ and $V(r)$ are written in terms of the Woods-Saxon shape $h(r)$ with strengths S_0 and V_0, respectively. The strengths are of order of 400 MeV, but of opposite sign ($S_0 < 0$, $V_0 > 0$). We choose the potential parameters S_0, V_0, R, and a to give the correct separation energy and spin-orbit splitting for ^{16}O and the correct rms for the $A = 14$ system. The resulting scalar and vector potentials are qualitatively similar to what one obtains with a Hartree calculation using the σ-ω model, but lack the slight bump near the origin. In addition, we have constrained the scalar and vector potential to have the same radius and diffuseness parameters.

II. RELATIVISTIC MATRIX ELEMENTS

The transverse magnetic form factors squared between initial and final nuclear shell-model states of spin (isospin) $J(T)$ is given by[9,10]

$$F_T^2(q) = \frac{1}{2J_i+1} \frac{f_{\mathrm{SN}}^2 f_{\mathrm{c.m.}}^2}{Z^2} \sum_{J\geq 1} \left| \sum_T \begin{pmatrix} T_f & T & T_i \\ -N_f & N & N_i \end{pmatrix} \langle J_f;T_f \| -i\hat{T}_{J,T}^{\mathrm{mag}} \| J_i;T_i \rangle \right|^2 , \tag{6}$$

where f_{SN} is the single-nucleon form factor, and $f_{\mathrm{c.m.}}$ is the correction for lack of translational invariance of the shell model, and the isospin projections are denoted by N. We are using the convention where the Mott cross section contains a factor of Z^2. In terms of the four-momentum-transfer squared q_μ^2 and the three-momentum-transfer squared q^2, they are given by

$$f_{\mathrm{SN}}(q_\mu^2) = [1 - q_\mu^2/(855\ \mathrm{MeV})^2]^{-2}$$

and

$$f_{\mathrm{c.m.}} = \exp(b^2 q^2 / 4A) ,$$

where A is the atomic number and b is the harmonic oscillator range parameter. We use the notation and conventions of Bjorken and Drell,[11] so the four-momentum squared is negative. The reduced nuclear matrix element can be written in terms of the matrix elements for single-particle states (labeled a,a') (Ref. 10)

$$\langle J_f;T_f \| -i\hat{T}_{J,T}^{\mathrm{mag}} \| J_i;T_i \rangle = \sum_{a',a} \psi_{JT}(a,a') \langle a' \| -i\hat{T}_{J,T}^{\mathrm{mag}} \| a \rangle \tag{7}$$

The magnetic form factor operator is given by

$$\hat{T}_{JM,TN}(q) = j_J(qr) \mathbf{Y}_{JJ}^M(\hat{\mathbf{r}}) \cdot \mathbf{J} \hat{I}_T^N , \tag{8}$$

where $\hat{I}_T^N$ takes into account the isospin change in the matrix element and is defined by

$$I_T^N \equiv \frac{1}{2} \begin{cases} 1 & T=0, \quad N=0 \\ \tau_3 & T=1, \quad N=0 \\ \tau_{\pm 1} & T=1, \quad N=\pm 1 \end{cases} . \tag{9}$$

The ψ_{JT} are nuclear structure matrix elements reduced in spin and isospin. For our case we restrict a,a' to be states in the p shell, and we make use of j-j coupling since in a relativistic calculation L is not a good quantum number. The spin and isospin reduced nuclear structure matrix elements for the transitions $(J_i T_i) \to (J_f T_f)$ are given in terms of the wave function coefficients a, b, c, m, and n of Eq. (1) by

$$\begin{aligned} &(1\,0) \to (1\,0) , \\ &\psi_{10}(\tfrac{1}{2},\tfrac{1}{2}) = \sqrt{2}a^2 - b^2/\sqrt{8} , \\ &\psi_{10}(\tfrac{3}{2},\tfrac{3}{2}) = c^2/\sqrt{5} + \sqrt{5}b^2/4 , \\ &\psi_{10}(\tfrac{3}{2},\tfrac{1}{2}) = -(\sqrt{10}bc)/4 + (ab)/2 , \\ &\psi_{10}(\tfrac{1}{2},\tfrac{3}{2}) = -\psi_{10}(\tfrac{3}{2},\tfrac{1}{2}) , \\ &(1\,0) \to (0\,1) , \\ &\psi_{11}(\tfrac{1}{2},\tfrac{1}{2}) = -am , \\ &\psi_{11}(\tfrac{3}{2},\tfrac{3}{2}) = -cn/\sqrt{2} , \\ &\psi_{11}(\tfrac{3}{2},\tfrac{1}{2}) = -bm/\sqrt{2} , \\ &\psi_{11}(\tfrac{1}{2},\tfrac{3}{2}) = bn/2 . \end{aligned} \tag{10}$$

For the nucleon current operator we make the conventional choice

$$J^{\mu}=F_1\gamma^{\mu}+\frac{i\mu_T}{2M}F_2\sigma^{\mu\nu}q_{\nu}\ , \tag{11}$$

where q^{μ} is the four-momentum transfer, μ_T is the isoscalar ($T=0$) or isovector ($T=1$) anomalous magnetic moment, and M is the nucleon mass. We equate the Dirac and anomalous form factors F_1 and F_2 to the single-nucleon form factor $f_{\rm SN}$ given earlier. With this choice, the four-current can be written in zero and three-component form in terms of γ_0, q_0, $\boldsymbol{\gamma}$, $\mathbf{q}$, and

$$\boldsymbol{\Sigma}=\begin{bmatrix}\boldsymbol{\sigma} & 0\\ 0 & \boldsymbol{\sigma}\end{bmatrix}$$

as

$$J^{\mu}=f_{\rm SN}\left[\gamma_0+\frac{\mu_T}{2M}\gamma_0\boldsymbol{\gamma}\cdot\mathbf{q},\ \boldsymbol{\gamma}+\frac{\mu_T q_0}{2M}\gamma_0\boldsymbol{\gamma}+\frac{i\mu_T}{2M}\boldsymbol{\Sigma}\times\mathbf{q}\right]. \tag{12}$$

Inserting this result into Eq. (8) and multiplying by γ_0, we obtain

$$\gamma_0\hat{T}^{\rm mag}_{JM,TN}(q\mathbf{r})=\left[j_J(qr)\left[\boldsymbol{\alpha}\cdot\mathbf{Y}^{M}_{JJ}+\frac{\mu_T q_0}{2M}\boldsymbol{\gamma}\cdot\mathbf{Y}^{M}_{JJ}\right]+\frac{iq\mu_T}{2M[J]}[-J^{1/2}j_{J+1}(qr)\mathbf{Y}^{M}_{J,J+1}+(J+1)^{1/2}j_{J-1}(qr)\mathbf{Y}^{M}_{J,J-1}]\cdot\gamma_0\boldsymbol{\Sigma}\right]\hat{I}^{N}_{T}, \tag{13}$$

where $[J]=(2J+1)^{1/2}$ and $\boldsymbol{\alpha}$ is the standard Dirac matrix.

Inserting the operator in Eq. (13) between initial and final Dirac wave functions, we obtain the reduced single-particle matrix element between relativistic states labeled by the Dirac quantum numbers κ and κ',

$$\begin{aligned}\langle\kappa'||-i\hat{T}^{\rm mag}_{J,T}||\kappa\rangle=&\frac{(-1)^{j-1/2}[j][j'][J]}{\sqrt{4\pi J(J+1)}}\begin{pmatrix}j' & J & j\\ -\frac12 & 0 & \frac12\end{pmatrix}\left[\frac{1+(-1)^{l+l'+J+1}}{2}\right]\frac{[T]}{\sqrt{2}}\\ &\times\int_0^{\infty}r^2dr\left\{(\kappa+\kappa')j_J(qr)\left[(f_{\kappa'}g_{\kappa}+g_{\kappa'}f_{\kappa})+\frac{\mu_T q_0}{2M}(f_{\kappa'}g_{\kappa}-g_{\kappa'}f_{\kappa})\right]\right.\\ &+\frac{\mu_T q}{2M(2J+1)}\{J(J+1)[j_{J+1}(qr)+j_{J-1}(qr)](f_{\kappa'}f_{\kappa}-g_{\kappa'}g_{\kappa})\\ &\left.+(\kappa+\kappa')[Jj_{J+1}(qr)-(J+1)j_{J-1}(qr)](f_{\kappa'}f_{\kappa}+g_{\kappa'}g_{\kappa})\}\right\}. \end{aligned} \tag{14}$$

Note that $\langle\kappa'\mu'|=\bar{\psi}^{\dagger}_{\kappa'\mu'}$, $\gamma_0^2=1$, and we have included the usual factor of $-i$ to make the matrix element real.

To obtain the nonrelativistic reduction of the single-particle matrix element in Eq. (14), we replace the "small" component g_{κ} by the free relation for a particle with $E=M$

$$g_{\kappa}(r)=\frac{1}{2M}\left[\frac{df_{\kappa}}{dr}+\frac{\kappa+1}{r}f_{\kappa}\right], \tag{15}$$

which is obtained in the limit that all potential and kinetic energy terms can be ignored in comparison to M, discard terms of order $1/M^2$, and interpret $f_{\kappa}(r)$ as the normalized radial solution of the Schrödinger solution. Carrying out these steps we obtain

$$\begin{aligned}\langle l'j'||-i\hat{T}^{\rm mag}_{J,T}||lj\rangle_{\rm nonrel}=&\frac{(-1)^{j-1/2}[j][j'][J]}{\sqrt{4\pi J(J+1)}}\begin{pmatrix}j' & J & j\\ -\frac12 & 0 & \frac12\end{pmatrix}\left[\frac{1+(-1)^{l+l'+J+1}}{2}\right]\frac{[T]}{\sqrt{2}}\frac{q}{2M}\\ &\times\left\{[(\kappa+\kappa')(\kappa+\kappa'+1)-J(J+1)]\left\langle l'j'\left|\left|\frac{j_J(qr)}{qr}\right|\right|lj\right\rangle\right.\\ &+\frac{\mu'_T}{2J+1}[J(\kappa+\kappa'+J+1)\langle l'j'||j_{J+1}(qr)||lj\rangle\\ &\left.+(J+1)(J-\kappa-\kappa')\langle l'j'||j_{J-1}(qr)||lj\rangle]\right\}, \end{aligned} \tag{16}$$

where the nonrelativistic moment $\mu'_T=\mu_T+1$ and

$$\kappa=(-1)^{j+l+1/2}(j+\tfrac12)\ .$$

This result appears to be considerably simpler in its angular momentum structure than the result given in Donnelly and Sick.[10] However, the two results can be shown to agree apart from an overall phase by using the following relation between $6j$ and $3j$ symbols (which we believe to be given here for the first time)

$$[J(J+1)l(l+1)]^{1/2}\begin{Bmatrix} l' & j' & \frac{1}{2} \\ j & l & J \end{Bmatrix}\begin{pmatrix} l & l' & J \\ 1 & 0 & -1 \end{pmatrix}=\frac{(-1)^{l+l'+J+1}}{2[l][l']}[(\kappa'+\kappa+1)(\kappa'+(-1)^{J+l+l'+1}\kappa)-J(J+1)] \times\begin{pmatrix} j' & J & j \\ -\frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix} . \tag{17}$$

It has been our experience that the angular momentum matrix elements work out more easily in the relativistic formulation.

The ground-state magnetic moment μ is given in terms of the elastic form factor in the $q\to 0$ limit

$$\mu=\lim_{q\to 0}\left[\frac{2M}{q}\left[\frac{6\pi J_i}{(J_i+1)(2J_i+1)}\right]^{1/2}\langle J_i,T_i\,||\,-i\hat{T}^{\text{mag}}_{J=1,T=0}\,||\,J_i,T_i\rangle\right] . \tag{18}$$

For the ground state of ^{14}N, we obtain

$$\mu_{\text{rel}}=\sum_{a'a}\psi_{10}(a,a')(-1)^{j-1/2}[j][j']\frac{\sqrt{3}M}{2}\begin{pmatrix} j' & 1 & j \\ -\frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}\left[\frac{1+(-1)^{l+l'}}{2}\right] \times\int_0^\infty r^2dr\left[\frac{r}{3}(\kappa+\kappa')(f_{\kappa'}g_\kappa+g_{\kappa'}f_\kappa)+\frac{\mu_0}{3M}[(1-\kappa-\kappa')f_{\kappa'}f_k-(1+\kappa+\kappa')g_{\kappa'}g_\kappa]\right] . \tag{19}$$

Unlike the nonrelativistic case, the magnetic moment in a Dirac picture depends on the wave functions, and since the integral involving the product fg dominates for isoscalar moments, and since in the nuclear interior $g\approx g^{\text{free}}/0.7$, one obtains the relativistic enhancement which has been extensively discussed.[12] Repeating the same limiting process for the nonrelativistic matrix in Eq. (16), inserting the nuclear matrix element for $(10)\to(10)$ from Eq. (10), and using $\mu_0'=\mu_p+\mu_n=0.8798$, we obtain

$$\mu_{\text{nonrel}}=\tfrac{1}{3}(1.12a^2+0.76ab+2.07b^2-1.20bc+1.88c^2) . \tag{20}$$

The other quantity of interest is the ^{14}C beta-decay rate. We use the standard V-A model for beta decay, but evaluate the nuclear matrix element fully relativistically. The weak nucleon current operator is $\hat{J}_\lambda=\gamma_\lambda(1-g\gamma_5)$ with the renormalized value of $g=1.253$ arising from the nonpointlike character of the nucleon and $\gamma_5=i\gamma^0\gamma^1\gamma^2\gamma^3$. The Q value for the ^{14}C decay is less than an MeV, so we evaluate the beta-decay matrix element in the zero-momentum-transfer limit. Thus we need only consider the Gamow-Teller nuclear matrix element arising from the operator $\hat{T}^{\beta}_{J,T}$ with $J=T=1$, $N=-1$ which is given by

$$\hat{T}^{GT}_{1,1}=\gamma_0\alpha\gamma_5\hat{I}_1^{-1} , \tag{21}$$

with I_1^{-1} being defined in Eq. (9). The beta-decay operator is a one-body operator, so, as in the case of the magnetic form factor, we define the Gamow-Teller matrix element as

$$G^{GT}=\frac{\langle J_f^\pi,T_f\,||\,\hat{T}^{GT}_{1,1}\,||\,J_i^\pi,T_i\rangle}{[J][T]}=\tfrac{1}{3}\sum_{a',a}\psi_{JT}(a,a')\langle a'\,||\,\hat{T}^{GT}_{1,1}\,||\,a\rangle . \tag{22}$$

Using results from Ref. 13, the single-particle spin and isospin reduced matrix elements are readily evaluated to be

$$\langle\kappa'\,||\,\hat{T}^{GT}_{1,1}\,||\,\kappa\rangle=\frac{(-1)^{j-1/2}[j][j']\sqrt{3}}{\sqrt{2}}\begin{pmatrix} j' & 1 & j \\ -\frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}\left[\frac{1+(-1)^{l+l'}}{2}\right]\int_0^\infty r^2dr[(1-\kappa-\kappa')f_{\kappa'}f_\kappa+(1+\kappa+\kappa')g_{\kappa'}g_\kappa] . \tag{23}$$

Unlike the magnetic form factors, the relativistic matrix element is not directly proportional to the "small" component g_κ, so, unless the leading contribution accidentally cancels, one would expect small relativistic effects in beta decay. The nonrelativistic limit is taken using Eq. (15) to obtain

$$\langle l'j'\,||\,\hat{T}^{GT}_{1,1}\,||\,lj\rangle_{\text{nonrel}}=\frac{(-1)^{j-1/2}[j][j']\sqrt{3}}{\sqrt{2}}\begin{pmatrix} j' & 1 & j \\ -\frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}\left[\frac{1+(-1)^{l+l'}}{2}\right](1-\kappa-\kappa')\int_0^\infty R_{l'j'}(r)R_{lj}(r)dr , \tag{24}$$

where

$$\kappa=(-1)^{j+l+1/2}(j+\tfrac{1}{2})$$

as before. Finally, using the wave functions of Eq. (1), the Gamow-Teller matrix element for the beta decay of ^{14}C is

$$G^{GT}=-\tfrac{1}{3}[amN_{\frac{1}{2},\frac{1}{2}}+(\sqrt{2}bn+2bm)N_{1/2,3/2}-\sqrt{5}cnN_{3/2,3/2}]\ , \tag{25}$$

where the "normalization" integrals $N_{a',a}$ are

$$\begin{aligned} N_{1/2,1/2}&=\int_0^{\infty}(f_1^2-3g_1^2)r^2dr\ , \\ N_{1/2,3/2}&=\int_0^{\infty}f_1f_{-2}r^2dr\ , \\ N_{3/2,3/2}&=\int_0^{\infty}(f_{-2}^2-\tfrac{3}{5}g_{-2}^2)r^2dr\ , \end{aligned} \tag{26}$$

where the subscripts on f and g in Eq. (26) are kappa values for a' and a.

In the nonrelativistic limit with g^2 terms omitted and with no spin-orbit terms in the Schrödinger equation, all the N's are unity. Experimentally, the Gamow-Teller matrix element for ^{14}C is quite small ($\approx 10^{-3}$) which requires extensive cancellation between the four terms in Eq. (25). In such a situation, the deviation of the $N_{a',a}$ factors from unity can play a significant role. Clearly, with larger Gamow-Teller matrix elements, the relativistic effects are negligible since the $N_{a',a}$ only differ from unity by about 10%.

III. RESULTS AND CONCLUSIONS

All the formalism given earlier is based on a single-particle approach which is known to be incomplete for certain nuclear properties such as magnetic moments where the response of the nuclear medium makes a significant contribution, particularly at low-momentum transfer.[12,14,15–19] As discussed in detail by Furnstahl and Serot,[19] these many-body contributions have not been evaluated for nonclosed-shell nuclei, although their contributions to the isoscalar form factors for one particle outside a closed shell have been evaluated in an approximate way. The effects at low-momentum transfer are large, and basically reduce the enhancement of the magnetic moment in the single-particle relativistic calculations back to the nonrelativistic values. Furnstahl and Serot[19] modify the Dirac part of the current operator for isoscalar magnetic form factors by a factor depending on the nucleon Fermi motion as modified by the local baryon density. While the theoretical justification for this procedure appears on solid footing for low-momentum components of shell-model wave functions, it is not obvious that a result based on nuclear matter calculations is valid for high-momentum components. We are testing an admittedly intuitive notion that medium effects such as "backflow" cannot respond at larger momentum transfer and, therefore, the single-particle response will be revealed. In any case, we attempt to fit the magnetic form factors at momentum transfers greater than 200 MeV/c and do not attempt to fit the magnetic moment or the $M1$ transition strength. Of course, the beta-decay rate of ^{14}C is a low-momentum transfer process. However, as noted earlier, the Gamow-Teller decay rate does not depend strongly on the small component of the relativistic wave function and is an isovector transition. While at the present time it is only speculation, we suspect that low-momentum transfer processes which depend sensitively on the "small" Dirac component are those where the relativistic many-body effects will make significant contributions. Thus, we will also consider constraining our fit to the magnetic form factors to the beta-decay rate.

In Fig. 1 we show our best fit to the elastic and inelastic magnetic form factors[2,20–23] of ^{14}N as compared to the nonrelativistic harmonic oscillator fit ($H1$) of Huffman *et al.*[2] The fit, using relativistic single-particle wave functions which we obtained by numerically solving Eq. (5) with the parameters shown in Table I, is clearly superior at higher-momentum transfer for both form factors, although at very high-momentum transfer, the transition form factor still falls beneath the experimental data. In Table II, we give the coefficients of our single-particle wave functions for this fit (labeled $J1$) and compare them to two fits of Huffman *et al.* ($H1$ and $HF2$).

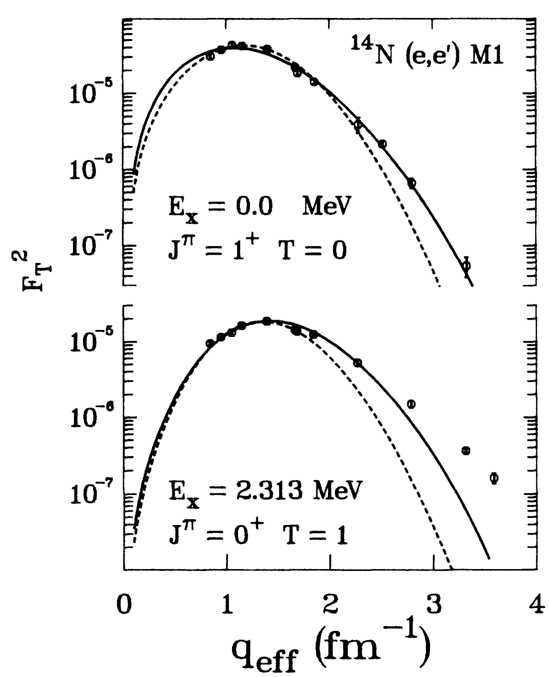

FIG. 1. The ground state and transition $M1$ form factors of ^{14}N. The dashed line shows the harmonic oscillator fit ($H1$) of Huffman *et al.*, (Ref. 2) while the solid line shows our fit ($J1$) using relativistic bound-state wave functions. The experimental data are from References 2, 20–23.

TABLE I. Potential parameters and resulting single-particle energies and rms radii of the Dirac bound state wave functions $V_0=317$ MeV, $S_0=-433$ MeV, $r_0=0.80$ fm, $a=0.70$ fm.

State	Energy (MeV)	rms radius (fm)
$1p_{1/2}$	-12.2	2.60
$1p_{3/2}$	-19.0	2.40
$1s_{1/2}$	-48.4	1.70

The $H1$ wave function does not fit the beta decay while the $HF2$ does. Clearly, the $J1$ wave function is quite different from either $H1$ or $HF2$. In particular, our results for the first excited state of ^{14}N is much closer to the j-j coupling limit. We also give the magnetic moment of the ^{14}N ground state in Table I but, as argued earlier, do not expect the relativistic single-particle calculation to reproduce this low-q result which depends sensitively on the small component of the Dirac wave function.

In Fig. 2 we show our best fit to the magnetic form factors when we constrain the relativistic Gamow-Teller matrix element given in Eq. (19) to agree with the experimental lifetime of ^{14}C. The fit to the form factors remains quite good and, more significantly perhaps, the wave function obtained with this fit (labeled $J2$ in Table III) differs much less from the fit without the beta-decay constraint ($J1$) than the two equivalent fits of Huffman *et al.* ($H1$ and $HF2$). Of course, the beta-decay fit should not be taken too seriously since in this approach the cancellation in the Gamow-Teller matrix element is very large and clearly many normally neglected effects such as "exchange currents" and/or many-body effects may be important. For example, the "normalization" integrals given in Eq. (26) cause the matrix element to change from -12.7×10^{-3} when set equal to 1 to -0.83×10^{-3} when their values of 0.899, 0.970, and 0.968 are used.

Part of the reason for our improved fit to the form factors at higher-momentum transfer is the use of Woods-Saxon wave functions rather than harmonic oscillator wave functions. In Fig. 3 we show our best fit to the magnetic moment and the two form factors obtained in a Schrödinger picture but with Woods-Saxon, rather than harmonic oscillator wave functions. The shape parameters are $r_0=2.65$ fm, $a=0.60$ fm, and $V_0=66.5$ MeV, and the resulting configuration amplitudes differ somewhat from $H1$ of Huffman *et al.*,[2] and are given by $a=0.944$, $b=0.302$, $c=-0.131$, $m=0.500$, and $n=-0.866$. The beta-decay matrix element for this fit is -50.4×10^{-3}. We also tried to fit the form factors with

TABLE II. Configuration amplitudes ($1p$ shell) $H1$ and $HF2$ from Ref. 2, $J1$ and $J2$ from this analysis.

	$H1$	$HF2$	$J1$	$J2$
a	0.978	0.974	0.486	0.4874
b	0.071	-0.228	-0.431	-0.2743
c	-0.194	0.000	-0.761	-0.8290
m	0.553	0.526	0.978	0.9977
n	-0.833	0.851	0.208	0.0677
μ/μ_N	0.407	0.334	0.542	0.603
$10^3\times G^{\rm GT}$	-58.1	0.84	56.9	-0.83

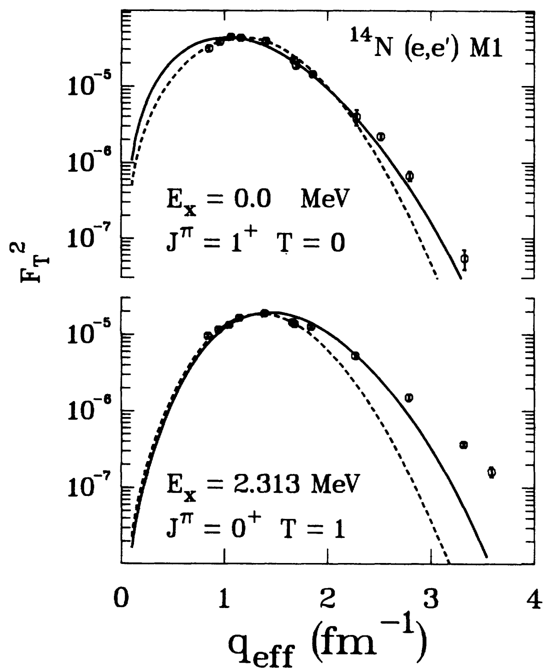

FIG. 2. Same as in Fig. 1, except that the solid line shows our fit ($J2$) constrained to fit the experimentally determined beta-decay rate of ^{14}C.

Woods-Saxon wave functions and the beta-decay constraint. We found configuration amplitudes close to the $HF2$ fit of Huffman *et al.*,[2] but the fit to the form factors is considerably worse than the fit without the beta-decay constraint. Clearly Woods-Saxon wave functions provide a superior description of the large momentum com-

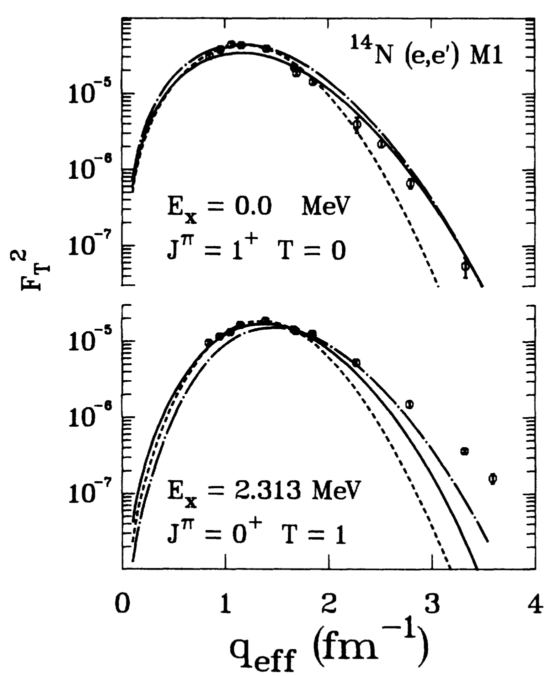

FIG. 3. Same as in Fig. 1, except that the solid line shows the relativistic fit with the modification of the Dirac current in the isoscalar form factor and the dashed-dotted curve is the Schrödinger best fit with Woods-Saxon wave functions.

TABLE III. Spin and isospin reduced nuclear matrix elements $\psi_{JT}(j,j')$ for $A=14$.

	$H1$	$HF2$	$J1$	$J2$
$\psi_{10}(\frac{1}{2},\frac{1}{2})$	1.351	1.323	0.268	0.309
$\psi_{10}(\frac{3}{2},\frac{3}{2})$	0.020	0.029	0.363	0.349
$\psi_{10}(\frac{3}{2},\frac{1}{2})$	0.046	−0.111	−0.364	−0.247
$\psi_{10}(\frac{1}{2},\frac{3}{2})$	−0.046	0.111	0.364	0.247
$\psi_{11}(\frac{1}{2},\frac{1}{2})$	−0.541	−0.512	−0.475	−0.486
$\psi_{11}(\frac{3}{2},\frac{3}{2})$	−0.114	0	0.112	0.040
$\psi_{11}(\frac{3}{2},\frac{1}{2})$	−0.028	0.085	0.298	0.194
$\psi_{11}(\frac{1}{2},\frac{3}{2})$	−0.030	−0.097	−0.045	−0.009

ponents in the wave functions as compared to the harmonic oscillator, but imposing the beta-decay constraint still modifies the configuration amplitudes by a considerable amount.

We also show in Fig. 3 our best fit to the form factors using the modification of the Dirac current in the isoscalar form factor as proposed by Serot and Furnstahl.[19] The fit is not bad and, of course, now produces the correct magnetic moment for the ground state of ^{14}N. The beta-decay matrix element for this fit is -24.1×10^{-3}, and we were unable to find a reasonable fit if we also constrained the beta-decay matrix element. The configuration amplitudes for this fit are reasonably similar to $H1$ and to those obtained with the Schrödinger Woods-Saxon wave functions discussed earlier. They are $a=0.953$, $b=0.273$, $c=-0.128$, $m=0.467$, and $n=-0.884$.

Huffman *et al.*, in their search for the $A=14$ wave function, also considered other reactions such as photopion production, (p,p'), and (p,n). Based on systematics of nuclear structure calculations in the p shell and on the analysis of the available data from these reactions, they concluded that most likely there was some other reason for the suppression of the ^{14}C beta decay other than extreme cancellation of the single-particle contributions to the Gamow-Teller matrix element. A particularly strong piece of evidence was the excellent fit obtained for the reaction ^{14}N $(\gamma,\pi^+)^{14}$C with the $H1$ wave function which has a considerable Gamow-Teller matrix element.[4,24] Furthermore, at low-momentum transfer the nonrelativistic calculation performed in LS coupling shows that the dominant term in the pion photoproduction from ^{14}N is the Gamow-Teller matrix element. However, the momentum transfer for this reaction is not zero ($q=0.5$ fm^{-1} for the lowest-momentum transfer point measured[24]), and preliminary calculations do find some relativistic effects (of order of 20%) in pion photoproduction from p-shell nuclei.[13] These relativistic effects may affect a sensitive cancellation of one-body matrix elements greatly, and work is underway currently to carry out a reanalysis of pion photoproduction from ^{14}N with relativistic Woods-Saxon rather than nonrelativistic harmonic oscillator nuclear wave functions, and use of the complete pion photoproduction operator.

In conclusion, we find that quite good fits to the magnetic dipole form factors in ^{14}N at high-momentum transfer can be obtained with relativistic $1p$-shell wave functions and the single-particle Dirac current. Furthermore, the greatly suppressed beta-decay rate of ^{14}C can be fitted simultaneously without great changes in the wave function. On the other hand, we also find that a Schrödinger analysis with Woods-Saxon wave functions, as well as a Dirac analysis using a modified current operator, also provide reasonable fits to the form factors. In the case of the Schrödinger calculation, the beta-decay lifetime can also be fit, but the configuration amplitudes must be modified considerably. We suggest that additional nuclear properties be investigated with relativistic single-particle wave functions, being careful to avoid low-momentum transfer properties which depend sensitively on the "small" Dirac component where nuclear many-body effects play a large role. Furthermore, a relativistic Hartree calculation should be carried out for non-closed-shell nuclei to investigate the importance of medium modifications of the isoscalar current at large momentum transfer.

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