PHYSICAL REVIEW C VOLUME 38, NUMBER 1 JULY 1988

Coupling of the giant dipole resonance to gamma vibrations

A. M. Nathan
Nuclear Physics Laboratory and Department of Physics, University of Illinois at Urbana-Champaign, Champaign, Illinois 61820
(Received 15 January 1988)

Existing photonuclear data for ^{166}Er are analyzed in the framework of both the dynamic collective model and the interacting boson approximation. In particular, the cross section for the inelastic scattering of photons populating the γ-vibrational band is reanalyzed in order to infer information about the coupling of the giant dipole resonance to γ vibrations in deformed nuclei. These data are found to be in agreement with the predictions of both models and therefore do not discriminate between them.

I. INTRODUCTION

It has long been thought that the giant dipole resonance (GDR) is strongly coupled to low-lying collective degrees of freedom, such as quadrupole surface vibrations.[1] The consequences of this coupling are the mixture of surface vibrational components into the GDR state, the fractionation of the GDR into vibrational satellite peaks, and the acquisition of often substantial branching ratios for photon emission from the GDR to low-lying vibrational levels. Photon scattering is an ideal reaction for probing the coupling of the dipole and quadrupole modes, since inelastic scattering to vibrational states provides a direct measure of the strength of vibrational components in the wave function of the GDR.

Historically, the first attempt to describe the coupling quantitatively was the hydrodynamic model and its extension, the dynamic collective model (DCM),[2] where the coupling is a consequence of the hydrodynamic result that the frequency of the dipole mode of a liquid drop is proportional to the inverse of the radius of the drop. For nuclei with a vibrating surface, this leads to an almost classical problem of the coupling between high-frequency dipole modes and low-frequency surface modes, leading to an admixture of surface vibrational components into the GDR state. Qualitatively, for nuclei that are "soft" vibrators (i.e., low frequency and large amplitude) one expects a strong coupling between the dipole mode and surface vibrations, a large splitting of the dipole strength into satellite peaks, and a substantial photon decay branch of the GDR to low-lying vibrational levels. The opposite is expected for "stiff" vibrators (i.e., high frequency and small amplitude). The DCM has met with considerable qualitative and quantitative success in predicting both the distribution of dipole strength[3] and photon decay modes[4] for a variety of spherical and transitional nuclei. In deformed nuclei, on the other hand, the distribution of dipole strength is largely determined by the static deformation, and the principal signatures for coupling to vibrations are the photon decay branches to low-lying γ- and β-vibrational levels. Several years ago, we reported the only definitive test of the DCM in a deformed nucleus ^{166}Er.[5] Experimentally, we measured the inelastic photon scattering cross section leaving the nucleus in the $K=2$, 2^+ γ-vibrational band head at 0.786 MeV excitation. At 90° the ratio R_γ of this cross section to the cross section populating the 0^+ and 2^+ members of the ground-state band was typically $\lesssim 3\%$, whereas the DCM (Refs. 5–7) predicted $R_\gamma \simeq 12-18\%$. We concluded that the DCM greatly overpredicted the coupling of the GDR to γ vibrations. The apparent success of the DCM in spherical and transitional nuclei, and failure in deformed nuclei, has remained a mystery.

More recently, the interacting boson approximation (IBA) has been expanded to include the GDR and its coupling to the low-lying collective levels.[8,9] The IBA has enjoyed considerable success over the last decade in predicting the properties of these low-lying levels in a wide range of nuclei throughout the periodic table. Among its more recent accomplishments is the successful prediction of the shape of the photoabsorption cross section in the GDR region of transitional and deformed nuclei.[10] However, there have been few comparisons between theory and experiment with regard to the photon decay branches. The most detailed comparison to date has been for the inelastic photon scattering into the γ band of ^{166}Er, where it has been found that the apparent coupling of the GDR to γ vibrations is significantly less than that predicted by the DCM and essentially in agreement with experiment.[9] In fact, with a relatively simple version of the IBA, it has been shown that excellent agreement with all photonuclear data on ^{166}Er can be obtained, especially the inelastic photon scattering into the γ band. This has led to the suggestion that the IBA provides a more correct description of the coupling of the GDR to low-lying collective states than does the DCM.[9]

In this paper we critically reexamine the photonuclear data for ^{166}Er in the framework of both the DCM (Sec. II) and the IBA (Sec. III). We will show that, after adjusting the input parameters of each model to agree with the known properties of the low-lying levels, both models can adequately account for *all* the existing photonuclear data on ^{166}Er. In particular, the mystery as to why the DCM had previously overpredicted the scattering into the γ band is resolved. We suggest that before one can draw conclusions as to which model provides a better description of the GDR, better experimental data are needed. These results are discussed in Sec. IV, and our conclusions are summarized in Sec. V.

II. DCM ANALYSIS

A. Formalism

The reinterpretation of the ^{166}Er(γ,γ') data utilizes the formalism of Semenko[11] to describe the coupling of the GDR to γ vibrations in deformed nuclei. We briefly review that formalism here. The starting point is the semiclassical Danos-Okamoto relation[12] for the energies of the dipole modes E_μ corresponding to oscillations along the principal axes of a spheroidal nucleus

$$E_\mu(\beta,\gamma)=\bar{E}\frac{\bar{R}}{R_\mu}\left[1+0.08\frac{R_\mu-\bar{R}}{\bar{R}}\right], \tag{1}$$

where for small deformations

$$R_\mu=\bar{R}[1+(-\tfrac{1}{2})^{|\mu|}\sqrt{5}/4\pi(\beta+\sqrt{6}\mu\eta)] . \tag{2}$$

The index μ is the Cartesian label for the principal axis, whose length is R_μ; the parameter β is the overall deformation, and the parameter η measures the deviation from axial symmetry

$$\eta=\frac{\beta\gamma}{\sqrt{2}} . \tag{3}$$

This leads to the following Hamiltonian for the GDR:

$$H_{\rm dip}=\sum_{\mu=-1}^{\mu=1}E_\mu(\beta,\gamma)Q_\mu^\dagger Q_\mu , \tag{4}$$

where $Q_\mu^\dagger, Q_\mu$ are the creation and annihilation operators for the dipole modes along the Cartesian axes in the intrinsic frame. For a nonstatic nuclear surface, one invokes the adiabatic approximation, since the frequency of the dipole vibration is much larger than that of the surface vibrations. For axially symmetric, deformed nuclei, one sets $\beta=\beta_0+\xi$ and $\eta=0+\eta$, and expands the GDR energies about the equilibrium values β_0 and 0. The parameters ξ and η are the amplitudes for β and γ vibrations, respectively, and are treated as dynamic variables. For the present analysis, the coupling of the GDR to β vibrations is expected to be small, and in any case it is not of interest. We therefore set $\xi\equiv 0$ and only explicitly consider the coupling to γ vibrations. We also switch to a spherical basis, with $q_\Omega^\dagger, q_\Omega$ being the creation and annihilation operators of the dipole mode with projection of angular momentum along the intrinsic symmetry axis $\Omega=0,\pm1$. Finally, we assume small amplitude vibrations and, therefore, only retain terms linear in η. The GDR Hamiltonian then separates into a part $H_{\rm dip}^0$ depending only on the equilibrium deformation β_0 and a part $H_{\rm int}$ depending on the dynamic variable η

$$H_{\rm dip}=H_{\rm dip}^0+H_{\rm int} , \tag{5}$$

where

$$\begin{aligned} H_{\rm dip}^0&=\sum_{\Omega=-1}^{\Omega=+1}E_\Omega(0)q_\Omega^\dagger q_\Omega , \\ H_{\rm int}&=(q_1^\dagger q_{-1}+q_{-1}^\dagger q_1)E_{\pm1}(0)G_{\pm1}\sqrt{6}\eta . \end{aligned} \tag{6}$$

We have defined the quantities

$$E_\Omega(0)=\bar{E}[1+0.08(-\tfrac{1}{2})^{|\Omega|}\tilde{\beta}_0][1+(-\tfrac{1}{2})^{|\Omega|}\tilde{\beta}_0]^{-1} , \tag{7}$$

$$G_\Omega=(-\tfrac{1}{2})^{|\Omega|}(4\pi/5)^{-1/2}\{-[1+(-\tfrac{1}{2})^{|\Omega|}\tilde{\beta}_0]^{-1}+0.08[1+0.08(-\tfrac{1}{2})^{|\Omega|}\tilde{\beta}_0]^{-1}\} , \tag{8}$$

and

$$\tilde{\beta}_0=(4\pi/5)^{-1/2}\beta_0 . \tag{9}$$

The full DCM Hamiltonian is

$$H=H_{\rm rot}+H_\gamma+H_{\rm dip}^0+H_{\rm int} , \tag{10}$$

where $H_{\rm rot}$ accounts for the rotational motion normal to the symmetry axis. In the harmonic approximation the Hamiltonian for the γ vibrations H_γ is given by[7]

$$H_\gamma=-\frac{\hbar^2}{4B}\frac{d^2}{d\eta^2}+C\eta^2+\frac{\hbar^2}{16B\eta^2}[(K-j_3)^2-1] . \tag{11}$$

The first term is the translational kinetic energy associated with the γ vibrations, the second term is the vibrational potential energy, and the third term is the kinetic energy of rotation along the symmetry axis, where K is the total angular momentum along the symmetry axis and j_3 is the projection along the symmetry axis of the GDR angular momentum operator. The eigenvalue of j_3 is Ω. The parameters B and C are, respectively, the inertial parameter and the restoring force constant associated with the γ vibrations and are uniquely determined by the properties of the low-lying $K=2$ γ band.

In the absence of $H_{\rm int}$, the diagonalization of H is straightforward and the energy levels relevant to ^{166}Er are shown in Fig. 1. The lowest dipole states are the $K=\Omega=0$ longitudinal mode, corresponding to dipole vibrations along the symmetry axis with energy

$$E_0(0)\approx\bar{E}(1-0.58\beta_0) ,$$

and the $K=\Omega=1$ transverse mode, corresponding to dipole vibrations normal to the symmetry axis with energy

$$E_{\pm1}\approx\bar{E}(1+0.29\beta_0) .$$

This is the familiar deformation splitting of the GDR. The structure of $H_{\rm int}$ shows that only the transverse GDR couples to the γ vibrations, so we focus the discussion on this mode. The uncoupled spectrum has a sequence of transverse modes at the energies $E_{\pm1}+nE_\gamma$, where $E_\gamma=\hbar\sqrt{C/B}$, corresponding to the simultaneous excitation of the transverse GDR and n γ-vibrational phonons (i.e., the transverse GDR built on the n phonon

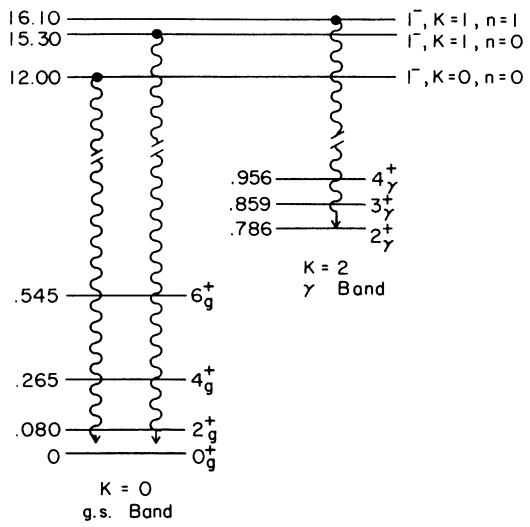

FIG. 1. The relevant energy levels for ^{166}Er, including the ground-state band, the γ band, and the uncoupled components of the giant dipole resonance. The wavy lines show which dipole levels are connected to which low-lying levels via a dipole transition.

γ vibration). In particular, the $n=0$ and $n=1$ states contain all the transverse dipole strength built on the ground state and the γ-vibrational band head, respectively. Since $H_{\rm int}$ is linear in the γ-vibrational amplitude, the states with n differing by ± 1 are mixed, resulting in a fractionation of the ground-state ($n=0$) dipole strength and a mixing of components with different n into the resulting eigenstates. The simultaneous presence of both $n=0$ and $n=1$ components in a particular eigenstate gives rise to a nonvanishing amplitude for inelastic photon scattering to the γ-vibrational band head. In first-order, two-level perturbation theory, the strength with which the $n=1$ and $n=0$ components are mixed depends on the ratio of the matrix element $\langle n=1 \mid \eta \mid n=0\rangle$ to the energy separation E_γ. The matrix element is related to the zero-point amplitude for γ vibrations[7] $\bar{\gamma}$

$$\langle n=1 \mid \eta \mid n=0\rangle = \frac{\beta_0 \bar{\gamma}}{\sqrt{2}} = \frac{\hbar}{\sqrt{2BE_\gamma}} .$$

"Soft" vibrations ($\bar{\gamma}$ large or B small) imply strong mixing whereas "stiff" vibrations ($\bar{\gamma}$ small or B large) imply weak mixing. Therefore, the inelastic scattering cross section into the γ-vibrational band head is expected to be a sensitive function of $\bar{\gamma}$; we demonstrate this point explicitly in the following.

The actual calculation proceeds along the lines outlined by Semenko.[11] One diagonalizes the full Hamiltonian in the basis of eigenstates of the uncoupled Hamiltonian $H_{\rm rot}+H_\gamma+H^0_{\rm dip}$, for which all the matrix elements can be computed analytically. We have determined that stable results for the $n=0$ and $n=1$ components (the only ones of importance for the inelastic scattering experiment) are achieved if multiphonon γ-vibrational states up to $n=7$ are included. The matrix diagonalization yields both the dipole energies, E_n, and the wave functions for both the low-lying, $|I_j^+\rangle$, and dipole states, $|1_n^-\rangle$. These are then used to calculate dipole transition strengths connecting the dipole and low-lying states, $\langle 1_n^- \| D \| I_f^+\rangle$. These quantities are related to the cross sections as follows:[4] (i) photoabsorption,

$$\sigma_\gamma = \frac{4\pi\hbar c}{\sqrt{3}E} \mathrm{Im} P_0 , \tag{12}$$

(ii) photon scattering,

$$\frac{d\sigma_{0^+\to I_f^+}(E,\theta)}{d\Omega} = \frac{E'}{E} \sum_j |P_j|^2 g_j(\theta)\delta_{jI_f} , \tag{13}$$

where the angular distributions are given by

$$g_0(\theta) = \tfrac{1}{6}(1+\cos^2\theta) ,$$

$$g_2(\theta) = \tfrac{1}{12}(13+\cos^2\theta) .$$

Here, θ is the scattering angle and E and E' are the energies of the incident and scattered photons, respectively. The polarizabilities are given by

$$P_j = \delta_{jI_f} \frac{1}{\sqrt{3(2I_f+1)}} \frac{EE'}{(\hbar c)^2} \sum_n \langle I_f^+ \| D \| 1_n^- \rangle \langle 1_n^- \| D \| 0^+ \rangle \left[\frac{1}{E_n + E' + \frac{1}{2}i\Gamma_n} + \frac{1}{E_n - E - \frac{1}{2}i\Gamma_n} \right] - \delta_{j0}\delta_{f0}\sqrt{3}\frac{(Ze)^2}{AMc^2} . \tag{14}$$

The DCM does not specify the widths of the GDR states Γ_n. We parametrize these widths according to the phenomenological prescription of Danos and Greiner,[13]

$$\Gamma_n = \Gamma_0 (E_n/\bar{E})^\delta , \tag{15}$$

and adjust Γ_0 and δ to fit some body of data, such as the photoabsorption or the elastic scattering cross section.

Before proceeding, we point out that the formalism just described is not identical to that described both by Arenhövel *et al.*[6] and by Eisenberg and Greiner[7] and used by us in our previous analysis.[5] These latter treatments solve the eigenvalue problem for the γ vibrations in an approximate manner rather than exactly as we do here. The two formalisms do not appear to be equivalent. This was pointed out long ago by Semenko[11] but has been largely ignored in the literature.

B. Results

We now describe in detail our results. The input parameters[14] include the deformation $\beta_0=0.30\pm0.05$, the rotational energy $\epsilon=E_{2_g^+}/3=0.0269$ MeV, the width parameters Γ_0 and δ; the mean energy $\bar{E}$ of the GDR, and the two parameters characterizing the γ vibrations B and C (or, equivalently, E_γ and $\bar{\gamma}$). The energy $E_\gamma=E_{2_\gamma^+}-\epsilon=0.759$ MeV. The calculation was done for various fixed values of $\bar{\gamma}$ ranging from 0° to 20°; for each value, the width parameters and $\bar{E}$ were adjusted to fit the photoabsorption cross section[15] σ_γ. The results are summarized in Figs. 2–7.

Figure 2 shows the distribution of ground-state dipole photoabsorption strength, or equivalently, the distribution of elastic scattering amplitudes as a function of $\bar{\gamma}$. The $K=0$ mode is near 12 MeV and is not affected by the coupling to γ vibrations. The $K=1$ mode is concentrated in the 15–17-MeV region and splits into several peaks depending on the size of $\bar{\gamma}$, separated roughly by E_γ. The larger the value of $\bar{\gamma}$, the larger the fractionation of strength. Figure 3 shows the distribution of amplitudes for inelastic scattering to the 2_γ^+ level, consisting of one positive component and one or more negative components. Both the absolute size of the amplitudes and the separation between the centroids of the positive and negative components increase with increasing $\bar{\gamma}$, resulting in a concentration of scattering cross section to the 2_γ^+ level which peaks in the region of the $K=1$ strength and which increases with increasing $\bar{\gamma}$. This is shown in Figs. 5 and 6, which we discuss later. Figures 4 and 5 show σ_γ and the scattering cross sections, respectively. The points in Fig. 4 are from Ref. 15 and the curve in both figures was calculated with $\bar{\gamma}=12°$. However, for both σ_γ and the scattering into the ground-state band, the curves look virtually identical for all values of $\bar{\gamma}$ up to 20°. The reason is that these cross sections are mainly determined by the relative strengths and energy separation of the $K=0$ and $K=1$ components, which are not a sensitive function of $\bar{\gamma}$. In the calculation, the increased splitting of the $K=1$ mode with increasing $\bar{\gamma}$ is compensated by a decrease in the damping widths in order to obtain an acceptable fit to σ_γ. In the absence of any theoretical guidance for these damping widths, we conclude that these cross sections are completely insensitive to the coupling to γ vibrations. The energy dependence of the scattering to the 2_γ^+ level is equally insensitive to $\bar{\gamma}$: It always peaks in the 15–17-MeV region. However, the overall magnitude of that cross section is very sensitive to $\bar{\gamma}$. This is shown in Fig. 6, in which the peak ratio R_γ of the scattering to the 2_γ^+ level to the scattering to the ground-state band is plotted as a function of $\bar{\gamma}$. In Fig. 7, the experimental data for R_γ as a function of energy are presented along with calculations for $\bar{\gamma}=12°$ and $\bar{\gamma}=18°$.

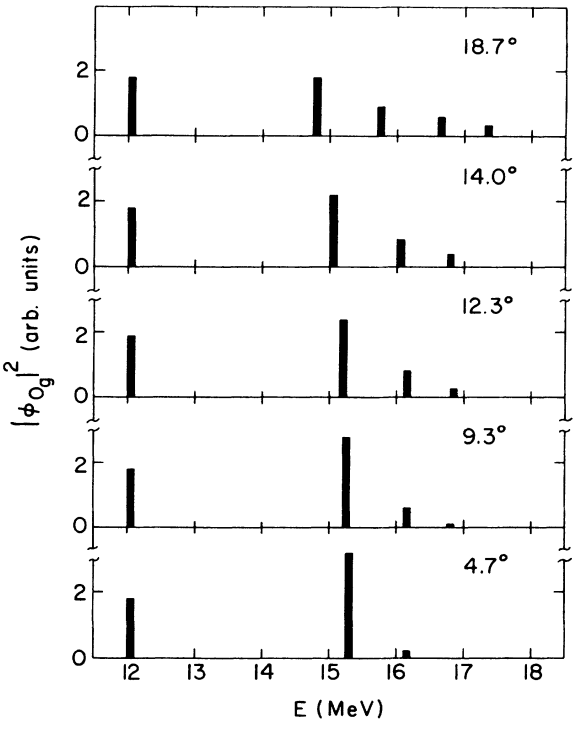

FIG. 2. DCM results for the distribution of dipole photoabsorption strength (or, equivalently, the distribution of elastic scattering amplitudes) for ^{166}Er, calculated as a function of the zero-point vibrational amplitude $\bar{\gamma}$.

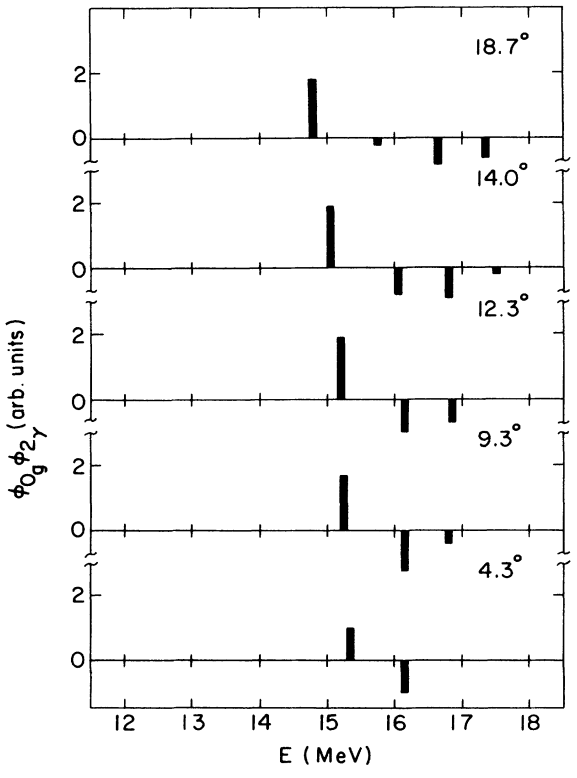

FIG. 3. DCM results for the distribution of amplitudes for inelastic scattering into the $K=2$, γ-vibrational band head in ^{166}Er, calculated as a function of the zero-point vibrational amplitude $\bar{\gamma}$.

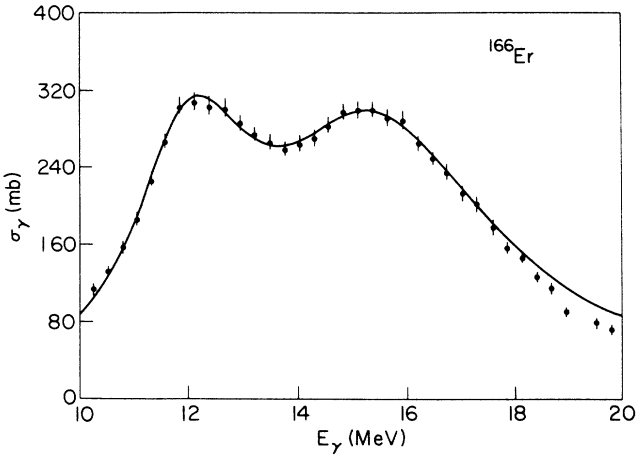

FIG. 4. The photoabsorption cross section for ^{166}Er. The points are the data of Ref. 15 and the curve is the present DCM calculation, assuming $\bar{\gamma}=12°$.

We now must decide on the appropriate value of $\bar{\gamma}$ for ^{166}Er.

We first define the ratio R_{E2}:

$$R_{E2}\equiv\frac{B(E2;0_g^+\rightarrow 2_\gamma^+)}{B(E2;0_g^+\rightarrow 2_g^+)}\ , \tag{16}$$

and note that R_{E2} is related to $\bar{\gamma}$ by[7]

$$R_{E2}=\bar{\gamma}^2\left[\frac{1-0.72\beta_0}{1+0.036\beta_0}\right]^2 . \tag{17}$$

Experimentally,[14] $R_{E2}=0.022$, thereby implying $\bar{\gamma}=12°$ and $R_\gamma\simeq 4\%$, in reasonable agreement with experiment, as shown in Fig. 7. On the other hand, in the pure liquid drop model (LDM),[7] $\bar{\gamma}$ is completely determined by the parameters ϵ and E_γ: $(\bar{\gamma})_{\rm LDM}=\sqrt{3\epsilon/E_\gamma}$, thereby implying $\bar{\gamma}=18°$ and $R_\gamma\simeq 10\%$. Our previous calculations[5] as well as those of Refs. 6 and 7 utilized the LDM value of $\bar{\gamma}$, thereby resulting in an overprediction of R_{E2} by a factor of 2.3 and an overprediction of R_γ by a factor of 3. This reduction of R_{E2} from its LDM value is a systematic occurrence in the rare-earth deformed nuclei, and it results in a significant reduction in the coupling between the GDR and the γ vibrations. We conclude that, using input parameters in agreement with the known properties of the low-lying levels, the DCM calculation is able to ac-

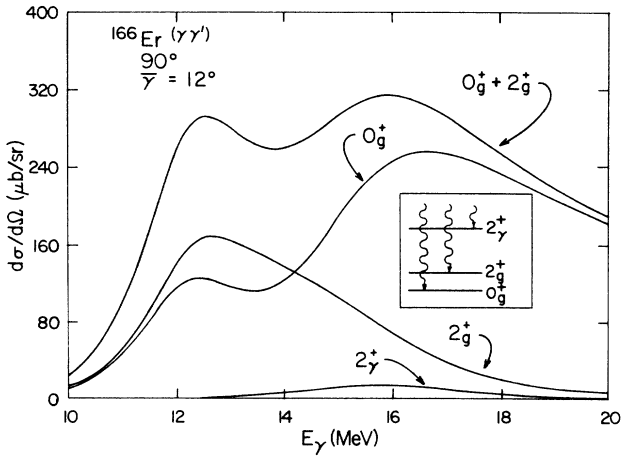

FIG. 5. DCM calculations of the scattering cross sections into the ground-state band (g) or γ band (γ) of ^{166}Er, assuming $\bar{\gamma}=12°$ and other parameters determined from the properties of the low-lying levels. The inset show the relevant levels in the decay scheme.

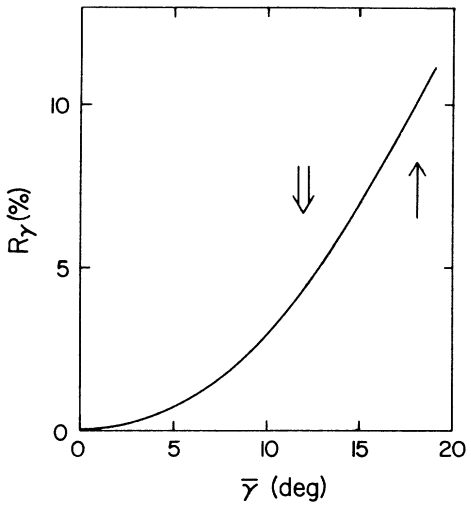

FIG. 6. The DCM result for the peak ratio R_γ of cross section populating the γ-band head to that populating the ground-state band, calculated as a function of $\bar{\gamma}$. The single arrow points to the value of $\bar{\gamma}$ specified by the liquid drop model, and the double arrow points to that value determined by the experimental value of R_{E2}. Other parameters are determined by the properties of the low-lying levels of ^{166}Er.

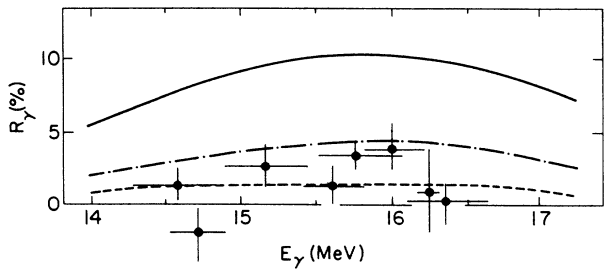

FIG. 7. The value of R_γ plotted as a function of incident photon energy. The points are the data of Ref. 5. The solid and dashed-dotted curves are DCM calculations with $\bar{\gamma}$ equal to 18° and 12°, respectively. The dashed curve is the IBA calculation with the value of $\chi_p\equiv\chi$ adjusted to reproduce the experimental R_{E2}. The dashed and dashed-dotted curves are the only ones that should be compared to the data or to each other, as discussed in the text.

count for all the known photonuclear data for the GDR of ^{166}Er, including both photoabsorption and photon scattering cross sections, and especially the inelastic photon scattering into the γ band.

III. IBA ANALYSIS

An analysis of the photonuclear data for the GDR of ^{166}Er in the framework of the IBA has been presented by Scholtz and Hahne.[9] Our treatment is similar to theirs in many respects, but different in one important respect, which we discuss later. We first review the IBA formalism.

In the IBA, the low-lying states are described as interacting bosons of spins 0 and 2, or s,d bosons. The giant dipole resonance is described as spin-1 p bosons. The Hamiltonian of the low-lying states and their interaction with the dipole states are constructed using general group-theoretical techniques. In the usual second quantized formalism,[10] the Hamiltonian assumes the general form

$$H=H_{sd}+H_p+H_{\rm int} \ . \tag{18}$$

The low-lying collective levels are eigenstates of H_{sd}:

$$H_{sd}=\epsilon\hat{n}_d+a_0(\hat{P}^\dagger\cdot\hat{P})+a_1(\hat{L}\cdot\hat{L})+a_2(\hat{Q}\cdot\hat{Q}) +a_3(\hat{T}_3\cdot\hat{T}_3)+a_4(\hat{T}_4\cdot\hat{T}_4) \ , \tag{19}$$

where

$$\hat{n}_d=(d^\dagger\cdot\tilde{d}) \ ,$$
$$\hat{P}=\tfrac{1}{2}(\tilde{d}\cdot\tilde{d})-\tfrac{1}{2}(\tilde{s}\cdot\tilde{s}) \ ,$$
$$\hat{L}=\sqrt{10}(d^\dagger\times\tilde{d})^{(1)} \ ,$$
$$\hat{Q}=(s^\dagger\times\tilde{d}+d^\dagger\times\tilde{s})+\chi(d^\dagger\times\tilde{d})^{(2)} \ ,$$
$$\hat{T}_3=(d^\dagger\times\tilde{d})^{(3)} \ ,$$
$$\hat{T}_4=(d^\dagger\times\tilde{d})^{(4)} \ .$$

The Hamiltonian describing the giant dipole resonance is

$$H_p=\epsilon_p(p^\dagger\cdot\tilde{p}) \ , \tag{20}$$

and the interaction Hamiltonian,

$$H_{\rm int}=b_0(d^\dagger\times\tilde{d})^{(0)}\cdot(p^\dagger\times\tilde{p})^{(0)}+b_1(d^\dagger\times\tilde{d})^{(1)}\cdot(p^\dagger\times\tilde{p})^{(1)}+b_2[(s^\dagger\times\tilde{d}+d^\dagger\times\tilde{s})^{(2)}+\chi_p(d^\dagger\times\tilde{d})^{(2)}]\cdot(p^\dagger\times\tilde{p})^{(2)} \ . \tag{21}$$

The parameters appearing in front of the various operators in the Hamiltonian are not yet constrained by any microscopic theory. In principle, there is complete freedom to adjust the a_i in H_{sd} to fit the known properties of the low-lying states. There is less guidance in choosing the b_i in $H_{\rm int}$, and it is quite easy to demonstrate that, in the absence of any constraints, important features of the GDR, such as the scattering cross section into the γ band, are completely unconstrained by the model. Therefore, in order to facilitate a meaningful comparison with both the photonuclear data and the DCM calculations, we have chosen to simplify the IBA Hamiltonian as much as possible. Ultimately, of course, the extent to which this simplification is valid must be answered experimentally.

Since ^{166}Er is far from closed shells, one expects the IBA Hamiltonian to have approximate SU(3) symmetry. Following Scholtz and Hahne, we first investigate the consequences of pure SU(3) symmetry. This requires that only the $\hat{L}\cdot\hat{L}$ and $\hat{Q}\cdot\hat{Q}$ terms in H_{sd} are nonvanishing, and that the χ parameter in the definition of the quadrupole operator Q exactly equals $-\sqrt{7}/2$. Furthermore, motivated by the geometrical picture of the DCM, we assume that the chief microscopic mechanism for the coupling of the GDR to the low-lying collective states is through the quadrupole degree of freedom. Accordingly, we retain only the b_2 term in $H_{\rm int}$ and we require that χ_p is identically equal to χ.

With these assumptions, the spectrum of low-lying and GDR states has the following features.

(1) The moment of inertia of the rotational bands, as well as the excitation energy of the γ band, is determined by the parameters a_1 and a_2, which can be adjusted to reproduce the experimental values.

(2) The energies of the lowest γ and β bands are degenerate.

(3) The $E2$ transition rate connecting either the β or γ bands to the ground-state band is identically zero.

(4) The GDR splits into exactly two modes, which can be identified with the longitudinal and transverse modes in a geometrical picture. The mean GDR energy is ϵ_p and the energy splitting is determined entirely by b_2.

(5) The electric dipole transition rate connecting the GDR to the γ band is identically zero.

Clearly, items (2)–(5) are not in accord with experiment, so it is necessary to deviate from the pure SU(3) symmetry. Warner and Casten have shown that by allowing the parameter χ to deviate from its SU(3) value, one can simultaneously remove the β-γ degeneracy as well as reproduce the $E2$ transition rates connecting the γ band and the ground-state band.[16] This is called the "consistent Q formalism" because the same quadrupole operator is used in the Hamiltonian as is used to calculate the $E2$ transition rates. Motivated by this, we have extended the formalism so that the same quadrupole operator is also used in $H_{\rm int}$; i.e., $\chi_p\equiv\chi$. With this ansatz, the same number of free parameters is needed to do the IBA calculation as was needed to do the DCM calculation. These are the parameters a_1 and a_2, which are adjusted to

reproduce the 2_g^+ and 2_γ^+ energies (equivalent to ϵ and E_γ in the DCM); the parameter χ, which is adjusted to reproduce the $E2$ ratio R_{E2} in Eq. (16) (equivalent to $\bar{\gamma}$ in the DCM); the parameter b_2, which is adjusted to reproduce the deformation splitting of the GDR (equivalent to β_0 in the DCM); and the parameter ϵ_p, which is adjusted to reproduce the mean energy of the GDR (equivalent to $\bar{E}$ in the DCM). Of course, the width parameters must be adjusted to fit σ_γ, just as in the DCM. However, there are no further free parameters and, in particular, there is no further freedom in predicting the branching ratio R_γ for decay to the γ band. It should be noted that without the restriction $\chi_p \equiv \chi$, one can achieve a wide range of R_γ while still reproducing the remaining photonuclear data.

The actual calculation utilizes the computer codes GR and GRT,[17] which we have adapted to calculate photoabsorption and photon scattering cross sections. Assuming a value of $N=15$ bosons appropriate to ^{166}Er, the remaining input parameters, chosen as discussed earlier, were $a_1=0.0075$ MeV, $a_2=-0.0175$ MeV, $\chi=-0.4919$, $\epsilon_p=14.26$ MeV, and $b_2=0.3520$ MeV. The resulting cross sections for photoabsorption and photon scattering into the ground-state band are virtually identical to those obtained with the DCM and are in excellent agreement with the existing data. The calculated curve for R_γ is plotted in Fig. 7; this is also in good agreement with the data, although not in agreement with that of the DCM. Scholtz and Hahne[9] also achieved good agreement with the photonuclear data. They assumed pure SU(3) symmetry for H_{sd} retained the terms b_0 and b_2 in $H_{\rm int}$, without the restriction $\chi_p=\chi$. We prefer our own approach in part because we are able to account for all existing data with two fewer parameters, and in part because we believe it is the more consistent approach, as discussed in the following.

IV. DISCUSSION

We have shown that both the DCM and the IBA are able to account for the existing photonuclear data for ^{166}Er, particularly those data relevant to the coupling of the GDR to the γ band. Each calculation was done with a restricted form of the respective model: For the DCM we assumed the harmonic approximation for the γ vibrations, while for the IBA we assumed approximate SU(3) symmetry. Both calculations require a single parameter to describe the coupling of the GDR to the surface degrees of freedom. This parameter is fixed in the DCM by the Danos-Okamoto relation [Eq. (1)] and in the IBA by the parameter b_2. In the DCM one is able to successfully predict the deformation splitting of the GDR, whereas in the IBA the parameter b_2 must be adjusted to reproduce the splitting. In each model, once the coupling parameter is fixed and the model has been adjusted to reproduce the properties of the low-lying levels, there is no additional freedom in predicting the coupling of the GDR to the γ vibrations. Both models agree with the existing scattering data, although they disagree with each other by a factor of about 2.

Scholtz and Hahne[9] have carried out an extensive comparison between the DCM and the IBA, and have applied it to the case of ^{166}Er. They conclude that there is a fundamental difference between the two models in their predictions for the coupling of the GDR to the low-lying vibrational bands. They demonstrate that this difference is not just due to a choice of parameters, but is inherent in the structure of the two models. While we do not dispute this conclusion (indeed, the present calculation seems to reinforce it), we do dispute part of the reasoning that leads to it. The conclusion is based in part on the observation that in one form of the IBA, namely, pure SU(3) symmetry, the coupling of the GDR to γ vibrations exactly vanishes, whereas there is no limit of the DCM that achieves the same result. However, they have used the liquid drop parameters as input into the DCM, thereby removing one degree of freedom from the model. If one is really to compare the DCM to the SU(3) limit of the IBA, one needs to let $\bar{\gamma}\rightarrow 0$ or, equivalently, the mass parameter $B\rightarrow\infty$, since both models would then predict $R_{E2}\equiv 0$. Under these conditions, the DCM would also predict a vanishing of the coupling between the GDR and γ vibrations. In our own calculations, the comparison is done in a self-consistent manner in that the two parameters governing the coupling to the γ band, namely, $\bar{\gamma}$ and $\chi_p\equiv\chi$, are both controlled by the experimental value of R_{E2} and nothing else. We show that under these conditions there is indeed an asymmetry between the two models, and we suggest that it would be a worthwhile goal to try to understand the fundamental origin of this asymmetry. Clearly, it would also be desirable to have scattering data that would better discriminate between the two models.

V. CONCLUSIONS

We have reanalyzed the existing photonuclear data for ^{166}Er in the framework of both the dynamic collective model and the interacting boson model. Using input parameters consistent with known properties of the low-lying levels, and using a restricted form of the IBA Hamiltonian, both models adequately account for all the existing data, in particular the inelastic photon scattering cross section into the γ vibrational band head. This latter point disagrees with our earlier conclusion about the DCM. However, better data will be needed before one can further refine the models or decide which model works better.

ACKNOWLEDGMENTS

I thank P. van Isacker for providing me with his IBA codes and excellent documentation on how to use them, and S. Hoblit for his help in adapting these codes to calculate scattering cross sections. Several informative discussions with F. Iachello, F. Hahne, and R. Casten are also gratefully acknowledged. This research was supported by the National Science Foundation under Grant No. NSF-PHY-86-10493.

[1]A. Bohr and B. Mottleson, *Nuclear Structure, Vol. II* (Benjamin, Reading, Massachusetts, 1975), p. 453ff.
[2]M. Danos and W. Greiner, Phys. Rev. **134**, B284 (1964).
[3]P. Carlos, H. Beil, R. Bergère, A. Leprêtre, A. de Minaic, and A. Veyssière, Nucl. Phys. **A225**, 171 (1974).
[4]T. J. Bowles, R. J. Holt, H. E. Jackson, R. M. Laszewski, R. D. McKeown, A. M. Nathan, and J. R. Specht, Phys. Rev. C **24**, 1940 (1981); S. D. Hoblit, Ph.D. thesis, University of Illinois, 1988 (unpublished).
[5]A. M. Nathan and R. Moreh, Phys. Lett. **91B**, 38 (1980).
[6]H. Arenhövel, M. Danos, and W. Greiner, Phys. Rev. **157**, 1109 (1967).
[7]J. Eisenberg and W. Greiner, *Nuclear Theory* (North-Holland, Amsterdam, 1975) Vol. I, Ch. 11, pp. 331–385 and 127–167.
[8]I. Morrison and J. Weise, J. Phys. G **8**, 687 (1982); D. J. Rowe and F. Iachello, Phys. Lett. **130B**, 231 (1983).
[9]F. G. Scholtz and F. J. W. Hahne, Phys. Lett. **123B**, 147 (1983); Phys. Rev. C **34**, 693 (1986); Nucl. Phys. **A471**, 545 (1987).
[10]G. Maino, A. Ventura, P. van Isacker, and L. Zuffi, Phys. Rev. C **33**, 1089 (1986); G. Maino, A. Ventura, L. Zuffi, and F. Iachello, *ibid.* **30**, 2101 (1984); Phys. Lett. **152B**, 17 (1985); F. G. Scholtz, *ibid.* **151B**, 87 (1985).
[11]S. F. Semenko, Phys. Lett. **10**, 182 (1964); Yad. Fiz. **1**, 414 (1965) [Sov. J. Nucl. Phys. **1**, 295 (1965)].
[12]M. Danos, Nucl. Phys. **5**, 23 (1958); K. Okamoto, Phys. Rev. **110**, 143 (1958).
[13]M. Danos and W. Greiner, Phys. Rev. **138**, B876 (1965).
[14]Nucl. Data Sheets **14**, 471 (1975).
[15]R. Bergère, H. Beil, P. Carlos, and A. Veyssière, Nucl. Phys. **A133**, 417 (1969).
[16]D. D. Warner and R. F. Casten, Phys. Rev. Lett. **48**, 1385 (1982).
[17]P. van Isacker (private communication).