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Reaction $\pi^- p \to \pi^0 n$ below the Δ resonance

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The reaction $\pi^- p \to \pi^0 n$ has been studied using a large NaI crystal to detect the decay γ rays of the π^0, at pion beam energies of 45.6, 62.2, 76.4, 91.7, 106.8, and 121.9 MeV. The NaI was set at nine different laboratory angles, but at almost every angle the angular distribution of the π^0 can be derived from the energy spectrum of the γ rays. The results are compared with recent phase-shift analyses and with various calculations. New values of the charge exchange scattering length are derived.

I. INTRODUCTION

It is surprising, but true, that there is still a great deal of confusion about the low-energy pion-nucleon interaction below 100 MeV. The last few years have seen a resurgence of interest because of the improved low-energy pion beams that have become available at the meson factories. It is becoming clear that many of the earlier experiments were in serious error, well outside their quoted uncertainties. The present difficulty, of course, is to separate the chaff from the wheat, without making the dangerous hypothesis that all old experiments are chaff.

This paper describes an experiment on the charge-exchange reaction, $\pi^- p \to \pi^0 n$, which has been studied at six pion beam energies from 45.6 to 121.9 MeV. This experiment was carried out on the *M*11 channel at TRIUMF and is an extension of an earlier experiment at 26.4 and 39.3 MeV on *M*13 which have already been reported.[1] Preliminary results of the present experiment has already appeared in various conference proceedings[2,3] and complete details are available in the Ph.D. thesis of one of us (A.B.) (Ref. 4).

Recently, there have been several experiments on the low-energy pion-nucleon interaction. From TRIUMF there is a measurement of the differential cross sections of $\pi^\pm p$ elastic scattering from 67 to 139 MeV (Ref. 5), whilst from LAMPF there is a measurement of the 0° cross section for $\pi^- p \to \pi^0 n$ from 101 to 147 MeV/*c* ($32.4 < T_\pi < 63.1$ MeV) (Ref. 6). From SIN a preliminary report[7] has been published on their $\pi^\pm p$ elastic scattering measurements at 55 MeV. At higher energies there has been a series of measurements by Nefkens and collaborators at LAMPF using a polarized target, and they have reported on charge-exchange scattering from 300 to 625 MeV/*c* ($190 < T_\pi < 500$ MeV) (Ref. 8) as well as $\pi^+ p$ elastic scattering[9] and $\pi^- p$ elastic scattering[10] from 350 to 560 MeV. A similar experiment on the polarization in charge-exchange scattering from 100 to 300 MeV has been reported from SIN by Alder *et al.*[11]

The standard way to parametrize these data is via phase-shift analyses. An ongoing effort at VPI by Arndt and Roper and collaborators[12] is continuously updated, and their SAID tapes are available at many major laboratories. In addition, a very valuable analysis which complements the SAID results is the so-called Karlsruhe-Helsinki analysis,[13] which uses more constraints from dispersion relations[14] and has been more stable over the years. There is also available a phase-shift analysis from Carnegie-Mellon University by Cutkosky and collaborators.[15,16]

An alternative description is via potentials, and Siegel and Gibbs[17] have recently reevaluated this method. Their particular interest is to develop a description of the elementary pion-nucleon amplitude in order to use this in calculations of the pion-nucleus interaction. The charge-exchange reaction has received a lot of attention recently because the single charge-exchange reaction in nuclei[18–20] exhibits a deep minimum in forward scattering which is observed in the elementary amplitude at about 45 MeV, whereas double charge exchange has a slight peaking at 0°, even though simple models indicate that it should have similar characteristics to the single charge-exchange reaction.[21–25] With such conundrums, it is essential to ensure that one's description of the elementary amplitude is as accurate as possible. Siegel and Gibbs have emphasized two important points; first, the mass differences between $\pi^\pm$ and π^0, and between n and p, change the position of the minimum in the scattering amplitude by several MeV; second, the existing $\pi^- p$ elastic scattering data around 60 MeV are totally incompatible with discrepancies of up to a factor of 2 in the shape when comparing the recent data of Frank *et al.*[26] from LAMPF with the older Berkeley measurements of Crowe *et al.*[27] Now the recent LAMPF data probably have a slight normalization problem because around 90 MeV they are incompatible with the total cross-section measurements of Carter *et al.*[28] and Pedroni *et al.*[29] Admittedly there are discrepancies of about 5% between these total cross-section measurements, but the LAMPF π^+p data need to be renormalized by over 20%. However, if the normalization is floated, then the π^+p results are in good agreement with other measurements. Thus, Siegel and Gibbs advocate the reasonable scenario that the problem lies with the π^-p data at 60 MeV and that there was an error in the measurements of Crowe *et al.* This is supported by the new data of Brack *et al.* and by the latest VPI phase-shift analysis SP86. They then seek to obtain a fit to the elastic and charge-exchange data. However, to describe the low-energy scattering data (including our low-energy results[1]), Siegel and Gibbs find it necessary to have a scattering length which is energy dependent or, to use another language, they believe that the data indicate a large effective range for the pion-nucleon interaction. They note that Källén[30] had pointed out this possibility in 1964, but it should be emphasized that the quality of the data then was fairly poor and many things could be, and were, "proved" with those results, including the violation of isospin invariance. It should be also pointed out that Siegel and Gibbs are a little unfair when they compare their calculation with the photoproduction data for the reaction $\gamma p \rightarrow \pi^+ n$. Their Fig. 14 exhibits a strong energy dependence for

$$\sigma'(q_\pi) = \frac{k_\gamma}{q_\pi}\sigma(\gamma p \rightarrow \pi^+ n) \tag{1}$$

but they plot the data only up to $q = 0.4\ \mathrm{fm}^{-1}$, whereas the complete data set[31] up to $1.2\ \mathrm{fm}^{-1}$ shows no convincing evidence for a rise at low energies. The only solution to all these problems is a concerted effort to remeasure π^-p elastic scattering below 100 MeV. This is actually fairly difficult (which is why there are such large discrepancies), but two efforts are underway, one at TRIUMF and one at SIN.

An important additional piece of information comes by measuring the πp scattering length using π^-p atoms. In theory this would be a very clean determination as the energy in this instance really is zero. The method is to determine the energy shift of the 1*s* state from the energy for a pure electromagnetic interaction. The experiment is very difficult because the shift is small and the π^- has to be stopped in a gas to ensure that it reaches the 1*s* state. The two existing measurements are important technically, but unfortunately do not have sufficient accuracy to be useful. Forster *et al.*[32] found $\Delta E = (-3.9 \pm 1.7)$ eV and

$$a(\pi^-p) = \tfrac{1}{3}(2a_1 + a_3) = (0.070 \pm 0.030)\ \mathrm{fm}\ ,$$

while Bovet *et al.*[33] measured

$$\Delta E = (-4.9 \pm 0.4 \pm 0.3)\ \mathrm{eV}$$

and

$$a(\pi^-p) = (0.084 \pm 0.009)\ \mathrm{fm}\ .$$

There has been some lively debate recently on the possible existence of πnn bound states.[34] The present indications are that they do not exist, but it is an interesting speculation. The low-energy πN interaction is a basic ingredient in the recipe for these hypothetical entities.

Because of its importance as a basic interaction, the pion-nucleon system has attracted much attention and many different models are tested against the data. (Would that the data were worthy of this dedication.) The cloudy bag model is a method of coupling pions to the nucleon and obviously it is a basic challenge of such calculations to describe pion-nucleon scattering; unfortunately the problem is somewhat refractory.[35–38] There have also been several attempts to describe the πN system in terms of the Skyrme model,[39–43] the Cheshire Cat model,[44,45] or chiral soliton models.[46,47] Other calculations have used *p*-wave πN scattering to obtain the πNN form factor via the Low equation.[48] The form factor can also be calculated by various methods including the Skyrme model.[49]

However, the most disconcerting problem relates to QCD. At low energies, chiral perturbation theory can make a fairly firm prediction for the σ term of πN scattering where

$$\sigma = \frac{m_u + m_d}{4m_N}\langle N \mid \bar{u}u + \bar{d}d \mid N \rangle\ , \tag{2}$$

which is a measure of how much chiral symmetry is broken in QCD via the quark mass term in the Lagrangian. Gasser and Leutwyler[50] have calculated $\sigma = (35 \pm 5)$ MeV from the hadronic mass spectrum. Experimentally, one can determine a related quantity, Σ, which can be found from the isoscalar spin-averaged amplitude $\bar{D}^{\,+}(\nu, t)$ evaluated at the Cheng-Dashen point, i.e.,

$$\Sigma = f_\pi^2 \bar{D}^{\,+}(\nu = 0, t = 2m_\pi^2)\ , \tag{3}$$

where the pion decay constant f_π is 93.2 MeV. Corrections can be estimated to relate Σ to σ, and the present belief is that

$$\Sigma = \sigma + (4.5 \pm 1.2)\ \mathrm{MeV}\ . \tag{4}$$

Now Koch[51] obtained $\sigma = (64 \pm 8)$ MeV from the Karlsruhe-Helsinki phase shifts which have been confirmed by the recent data of Wiedner *et al.*[7] Ericson[52] has recently studied this problem and has emphasized the critical importance of the $\bar{b}^{(+)}$ term in determining σ.

A possible solution to this inconsistency has been discussed by Donoghue and Nappi,[53] who suggested that the nucleon has a large component of $\bar{s}s$ among the sea quarks. Thus,

$$\sigma = \frac{35 + 5}{1 - y}\ \mathrm{MeV}\quad \text{where}\quad y = \frac{2\langle p \mid \bar{s}s \mid p \rangle}{\langle p \mid \bar{u}u + \bar{d}d \mid p \rangle}\ . \tag{5}$$

They argue that the matrix element for $\bar{s}s$ is about 0.21, which constitutes a radical departure from the naive picture of the proton. If we do not accept this proposal, we are left with a discrepancy which Gasser[54] reviewed recently and considered to be of major proportions. Now the prime data that is needed to settle this problem is isoscalar, so the charge-exchange information is less important. However as phase-shift analyses are used as an intermediate step, charge-exchange data are a useful stabilizing influence.

We see, therefore, that the pion-nucleon interaction is a keystone which links several aspects of the strong interaction and, then, many descriptions of other processes build on this structure, so a clear understanding of this elementary scattering amplitude is crucial to a complex edifice. Unfortunately, as we have indicated, the experimental situation is contradictory and confusing, especially in the $\pi^- p$ channel. It is, thus, essential to have data which are reliable, and we have, therefore, approached our experiment with caution, preferring a simple technique which presents few pitfalls than a more complex method.

II. THE EXPERIMENTAL METHOD

The technique that was used in this experiment was similar to our earlier measurement[1] and has already been described in a companion paper[55] on the reaction $\pi^- p \rightarrow n\gamma$, so we shall limit ourselves to the bare essentials apart from the features particularly important in this aspect of the experiment.

The measurement was made in the *M*11 beam line at TRIUMF using a defining telescope for the pion beam. By timing the beam particle's journey down the beam line one can determine the beam's composition. It should be emphasized that this is done continuously during the experiment with a pulse which selects random beam particles (of course the composition of the beam particles associated with events is quite different from that of the raw beam).

The target was liquid hydrogen with a guard ring to divert bubbles from the path of the beam. The target was a disk ≈4.4 cm thick ($\Delta E \approx 1.5$–2.2 MeV) and was oriented at about 30° to the beam. The target flask was changed between forward and backward positions of the detector so that the γ rays did not have to traverse any metal.

The detector was a large NaI crystal (TINA), cylindrical in shape, 457 mm in diameter, 508 mm long, with its axis pointing to the hydrogen target. The detector had a defining aperture of 254 mm in diameter for the lower beam energies but 152 mm in diameter for the higher beam energies in order to improve the energy resolution. The back surface of the collimator was placed 0.92 m from the target and acted as the defining aperture. This distance was chosen to give sufficient time of flight to distinguish γ rays from neutrons.

The data acquisition was performed by a PDP 11/34 and the information was written onto tape on an event-by-event basis for later analysis off-line. The cuts that were used were described in the companion paper. All the cuts were relatively loose and at most 1 or 2% of the events were lost (and a correction made).

III. DATA ANALYSIS

The NaI was oriented at many angles to the beam, but it is important to realize that at every angular setting of the detector one receives γ rays from pions at all angles. However, the charge-exchange reaction ($\pi^- p \rightarrow \pi^0 n$) is a two-body reaction and so the energy of the π^0 is uniquely related to its angle. The angle of the detector to the π^0 direction thus determines the energy of the decay γ ray via the Doppler shift.

Now because of these characteristics, the *energy* spectrum of the π^0 γ rays is simple related to the *angular* distribution of the π^0. This is most easily seen if one visualizes a geometry with the detector at 0°. If a π^0 leaves the interaction also at 0°, the only γ rays which can be detected will be those at 0°, which will be Doppler shifted up in energy to 300 MeV or so. If a π^0 leaves at 90°, the γ rays will approximately have the same energy as they do in the c.m. system ($\sim$67 MeV), whereas if a π^0 leaves at 180°, the γ rays will be Doppler shifted down to the lowest possible energy (20–30 MeV, depending on the beam energy). Thus, at the detector angle of 0°, the energy of the γ ray is uniquely related to the angle of the π^0 leaving the interaction. At other angles the effect is like smearing the resolution with a detector aperture covering twice the laboratory angle. It is difficult (though not impossible) to place the detector at 0° because even if a magnet were used to divert the pion beam, the bremsstrahlung from the electrons in the beam would cause an unpleasant background.

The total energy spectrum of the γ rays is illustrated in Fig. 1. The small peak at the highest energy is the radiative capture, and this was separately analyzed and then subtracted off. It gives, however, valuable guidance about the energy calibration and the energy resolution of the detector.

The resolution function for the detector is well represented by the following relations:

$$R(E_\gamma, A,B,C,D) = A\left[1-\operatorname{erf}\left[\frac{E_\gamma - B}{C}\right]\right] \times \exp[(E_\gamma - B)/D] , \quad (6)$$

where A is the amplitude, B is the peak position, C is the half-width of the high-energy edge, and D is the half-width of the low-energy tail.

The parameters C and D can be determined from a spectrum using stopped π^- in hydrogen, and we observe a resolution of about 4% FWHM for the monochromatic peak at 129 MeV. However, the resolution observed in this experiment was slightly larger (5–6%) due to the energy spread in the beam, energy loss of the pions in the hydrogen and finite aperture of the detector which causes kinematic broadening.

Now, to analyzer the π^0 γ rays we need to know how the resolution function varies with energy. We have traditionally used

$$C = C_0 E_\gamma^{0.5} \quad \text{and} \quad D = D_0 E_\gamma^{1.0} . \quad (7)$$

However, it was felt prudent to make a Monte Carlo study using the well-known EGGS program which simulates γ-ray showers in NaI crystal. A diverging beam of γ rays represented the situation better and the results are given in Table I. The EGGS results have been smoothed with a Gaussian proportional to $\sqrt{E_\gamma}$ in order to imitate the statistics of the photoelectrons in the phototube. (The normalization was chosen to best suit the observed

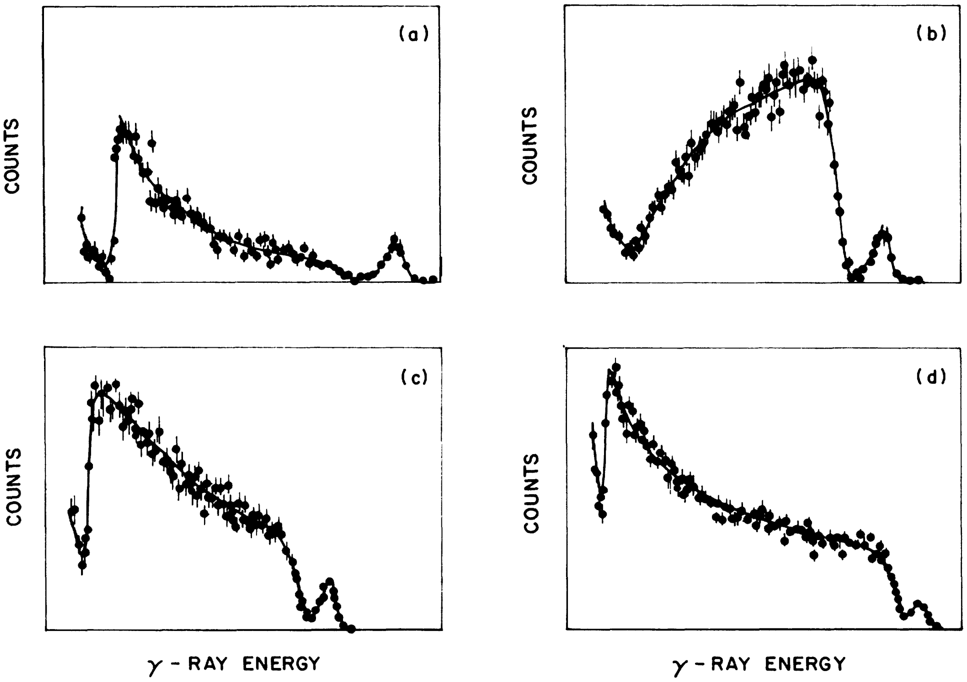

FIG. 1. Typical γ-ray spectra with all cuts applied. The line is a fit which is used to deduce the cross sections. The γ ray from radiative capture is clearly separated from the π^0 γ rays. The data were taken at (a) 45.6 MeV and 45°; (b) 45.6 MeV and 141°; (c) 76.4 MeV and 75°; and (d) 121.9 MeV and 60°.

resolution of the capture γ rays). The results were then fitted to an energy dependence for C and D; the parameters thereby obtained were

$$C=(0.055\pm0.007)E_\gamma^{0.84\pm0.10}\ , \tag{8a}$$

$$D=(0.130\pm0.020)E_\gamma^{0.69\pm0.08}\ . \tag{8b}$$

We did not have sufficient information to choose between the energy dependence of (7) and (8) so we analyzed several spectra with both forms. We even tried extremes such as keeping C and D constant. The effect on the determination of the total cross section was completely negligible. The only discernible effect was on A_1 and A_2, the coefficients of the Legendre polynomials for the π^0 angular distribution. Even here, however, the effect was less than 1%, and is dwarfed by fitting fluctuations which we shall describe later. The reason for the stability of A_l is simply that the energy resolution of the NaI crystal is much smaller than the structure in the system which is created by the angular distribution of the π^0.

The effect of the π^0 angular distribution can easily be seen in Fig. 1, for the spectra at 45.6 MeV. The spectrum [Fig. 1(a)] taken at 45° has few high-energy γ rays but many low-energy γ rays simply because the π^0 angular distribution is strongly peaked backwards; for the spectrum [Fig. 1(b)] taken at 141° most pions are heading towards the detector so there are many more high-energy γ rays. To describe this effect we have followed the procedure described by Kernan[56] and by Bodansky *et al.*[57] An alternative method has been discussed by Bayer *et al.*[58]

If one uses the standard description of the π^0 angular distribution in terms of Legendre Polynomials, viz.,

$$\frac{d\sigma}{d\Omega}=\sum_{l=0}^{m} A_l P_l(\cos\theta)\ , \tag{9}$$

where θ is the π^0 angle in the center of mass, then the γ-ray spectrum in the laboratory system is given by

TABLE I. Results of a Monte Carlo simulation for the detector response function.

E_γ (MeV)	C (MeV)	D (MeV)	D/C	R(%)
50	1.51±0.05	1.82±0.07	1.20±0.06	6.66
130	3.31±0.06	3.72±0.07	1.12±0.03	5.41
200	4.79±0.15	4.88±0.18	1.02±0.05	4.84
300	6.84±0.20	6.21±0.23	0.91±0.04	4.23

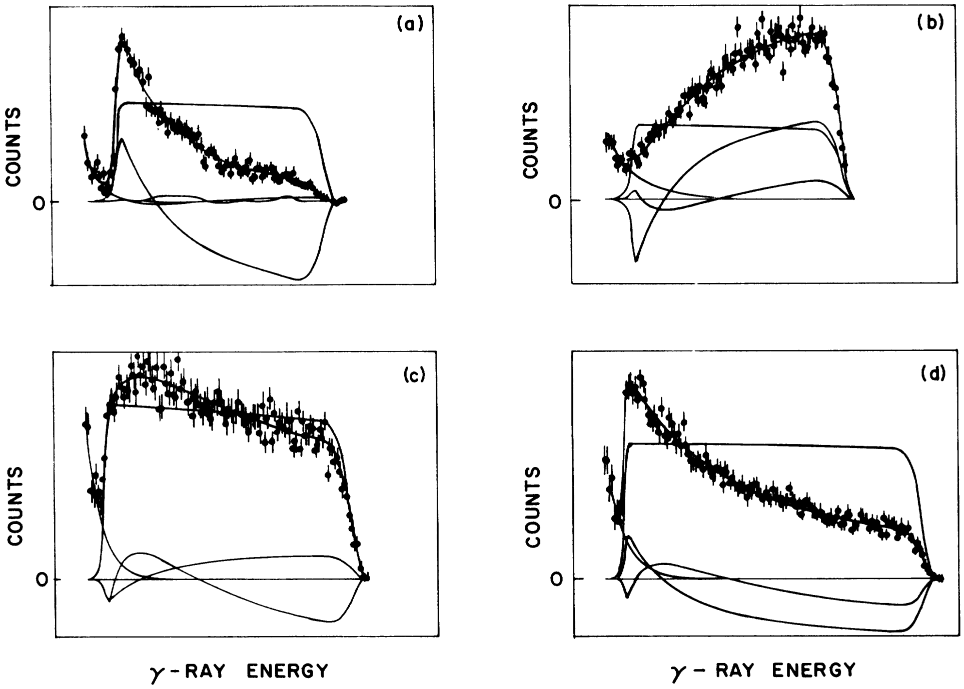

FIG. 2. The charge exchange γ-ray spectra and the fits at several different energies and angles. The contribution of different Legendre polynomial terms are also shown. The data were taken at (a) 45.6 MeV and 45°; (b) 62.2 MeV and 141°; (c) 76.4 MeV and 90°; and (d) 91.7 MeV and 90°.

$$I(k,\alpha)=\frac{N_p\Omega\epsilon F}{\beta_0\gamma_0 k'\gamma(1-\beta\cos\alpha)}\sum_{l=0}^{m}A_lP_l\left[\frac{\cos\alpha-\beta}{1-\beta\cos\alpha}\right]P_l\left\{\frac{1}{\beta_0}\left[1-\frac{k'}{\gamma_0\gamma k(1-\beta\cos\alpha)}\right]\right\}, \qquad (10)$$

where N_p is the number of protons in the target, Ω is the solid angle of the detector, ϵ is the total efficiency of the detector, and F is the π^- flux. The kinematic factors are as follows: β_0 and γ_0 are the relativistic parameters of the π^0 in the c.m. system for the reaction, k' is the γ-ray energy in the rest frame of the π^0 (viz., 67.48 MeV), β and γ are the relativistic parameters of the c.m. system for the reaction, and α is the laboratory angle of the γ ray. The most important feature of this equation is that the separation of the π^0 angular distribution amplitudes A_0, A_1, A_2, is retained in the final expression, so that it is easy to carry out the fitting procedure for the measured energy spectrum.

The expression for $I(k,\alpha)$ was folded with the detector resolution function $R(E_\gamma, A, B, C, D)$ and this was done for each Legendre polynomial and then stored. A look-up procedure was then used to obtain the best fit for the experimental spectra. Since the low-energy scattering is totally dominated by s and p waves, we set $m=2$, which gave us only three amplitudes to fit (d-waves contribute less than 1%, even at the highest energy that we measured).

TABLE II. Variations of parameters A_0, A_1, and A_2 with angle at fixed energy ($T_\pi=91.7$ MeV) for the charge exchange reaction. Since the errors assigned to these parameters from the fitting program were unreasonably small, we have chosen the weighted standard derivations of these parameters for statistical errors.

Angle	A_0	A_1	A_2
45°	1.27±0.01	−1.43±0.05	0.54±0.30
60°	1.22±0.01	−1.38±0.08	0.92±0.07
75°	1.22±0.02	−1.51±0.34	0.79±0.06
90°	1.27±0.02	−0.98±0.20	0.88±0.05
105°	1.39±0.03	−1.19±0.09	1.03±0.11
120°	1.28±0.02	−1.44±0.10	0.30±0.67
142°	1.46±0.03	−1.28±0.08	0.81±0.08
Average	1.26±0.03	−1.37±0.10	0.83±0.10

TABLE III. Experimental results for the ($\pi^- p \to \pi^0 n$) reaction. The differential cross section is expressed in terms of Legendre polynomials with coefficients A_l. The error quoted for these coefficients does not include an overall normalizing error of 3.1%. However, this error is included for the total cross section.

	$T_\pi=45.6$ MeV	$T_\pi=62.2$ MeV	$T_\pi=76.4$ MeV	$T_\pi=91.7$ MeV	$T_\pi=106.8$ MeV	$T_\pi=121.9$ MeV
A_0 (mb/sr)	0.516±0.010	0.654±0.020	0.879±0.020	1.26±0.03	1.68±0.08	2.28±0.04
A_1 (mb/sr)	−0.709±0.020	−0.90 ±0.03	−1.16 ±0.03	−1.37±0.10	−1.49±0.10	−1.72±0.15
A_2 (mb/sr)	0.20 ±0.04	0.36 ±0.04	0.50 ±0.06	0.83±0.10	1.27±0.10	1.71±0.15
σ_{total} (mb)	6.48 ±0.24	8.2 ±0.4	11.1 ±0.4	15.8 ±0.6	21.1 ±1.2	28.7 ±1.0

In addition to the π^0 γ rays, we observe a significant background which we attribute to neutrons. The size of the background is very sensitive to the cuts so, for instance, a slight timing shift between target in and target out can make a substantial difference to the number left in the subtracted spectrum. (The π^0 γ rays are much less sensitive to such effects.) We were apprehensive about the shape of the background in the energy region of the π^0 γ rays. Various tests were therefore conducted by finding the energy spectrum of events close to the timing cuts in the TINA time-of-flight spectrum. All the evidence supported our believe that the background was monotonic and well behaved. We therefore chose to describe the background by a simple exponential.

A further background was observed at the low energies used in our previous experiment.[1] This comes from a scattered π^- which stops in the target and is captured at rest. This produces a clear signature of a monochromatic γ ray at 129 MeV and a π^0 box between 55 and 83 MeV. At 26.4 and 39.3 MeV this feature was quite obvious, but at the lowest energy of the present experiment (45.6 MeV) there was no clear evidence for the effect, but the fit was slightly better for including it. However for $T_\pi \geq 76.4$ MeV, we set this parameter equal to zero. The stopped π^- spectrum was taken from a special run where we added absorbers to deliberately stop the beam in the target. Again we used a table look-up method in the fitting procedure. Thus, the final fitting function was

$$F(x)=A_{BG}\exp(-bx)+A_{\text{stop}}S+\sum_{l=0}^{2}G_l(A_lP_l) \qquad (11)$$

where G_l is the convoluted form of P_l in $I(k,\alpha)$ of Eq. (10), and S in the spectrum for stopped π^-.

IV. RESULTS AND DISCUSSION

Some typical fits to the π^0 γ rays are shown in Fig. 2. Note that at 45.6 MeV, the effect of the stopped π^- spectrum is just discernible to the trained eye. At higher energies it cannot be seen. The various components are given to illustrate their very different shapes; the $l=0$ is a square box, $l=1$ crosses the axis once while $l=2$ crosses the axis twice. The angles which we have chosen to illustrate are those which have the greatest sensitivity to all three components. The least sensitive to $l=1$ and $l=2$ is

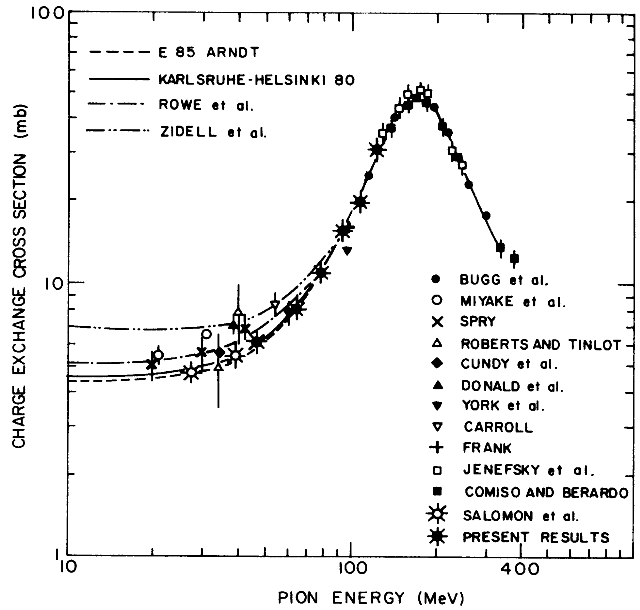

FIG. 3. The total cross sections for the charge exchange reaction $\pi^- p \to \pi^0 n$, including previous experimental results (Refs. 59–70) and the recent phase-shift analyses of Arndt *et al.* (Ref. 12), Koch and Pietarinen (Karlsruhe Helsinki) (Ref. 13), Rowe *et al.* (Ref. 72), and Zidell *et al.* (Ref. 71).

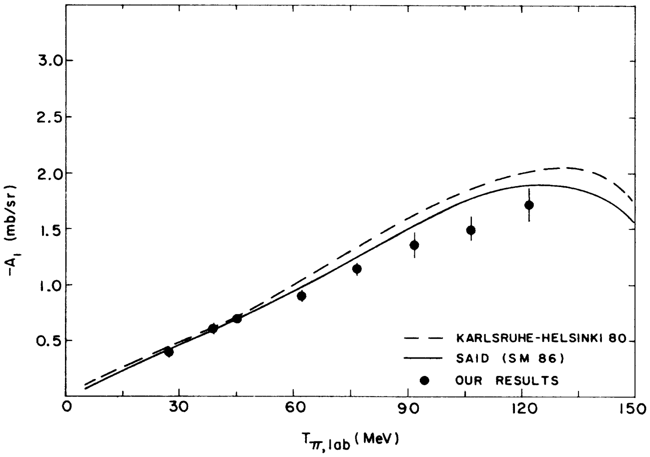

FIG. 4. The variation of the Legendre polynomial coefficient A_1 for the differential cross section of the charge exchange reaction $\pi^- p \to \pi^0 n$. Also illustrated are the phase-shift analysis SAID (SM86) and the Karlsruhe-Helsinki analysis (Ref. 13).

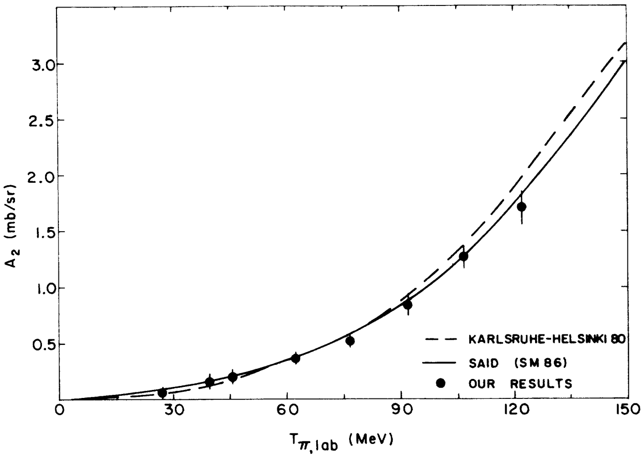

FIG. 5. The variation of the Legendre polynomial coefficient A_2 for the differential cross section of the charge exchange reaction $\pi^- p \rightarrow \pi^0 n$. Also illustrated are the phase-shift analysis SAID (SM86) and the Karlsruhe-Helsinki analysis (Ref. 13).

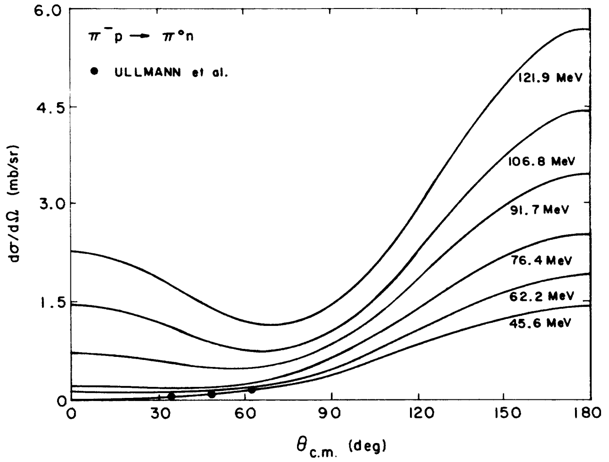

FIG. 6. The differential cross sections for the charge exchange reaction at different energies, as obtained from our best fits. The data points are from Ullmann *et al.* (Ref. 19) at 48.9 MeV.

at 90° in the c.m., i.e., at a laboratory angle of 78°; several angles are insensitive to $l=2$.

To illustrate the variation in the Legendre coefficients, we present the full results for $T_\pi = 91.7$ MeV in Table II for all laboratory angles of the detectors. The errors quoted for each angle are the fitting error as given by the MINUIT fitting program. It is clear that these errors are too small, so we have chosen to use a weighted standard deviation of these parameters as our final error. These fitting fluctuations contribute the major error for the experiment. They represent to some extent the correlations between the parameters together with a better accounting of background fluctuations. Even though we use somewhat conservative errors, they are relatively small because the number of counts in each spectrum was very large.

The final results for all the energies are given in Table III. As noted in the caption, we consider that there is an overall normalization error of 3.1% which takes account of systematic effects such as uncertainty concerning the beam composition, π^- decay, hydrogen target density, detector solid angle, and resolution function. This factor is included in the total cross-section error.

The total cross section is illustrated in Fig. 3 and compared with previous data[59–70] and with various phase-shift analyses. We see that our new results agree with the trend of our *M*13 results at low energy, and all are considerably lower than previous experiments. We believe that many of the early experiments would have had a large contribution from stopped negative pions, but they would have been unaware of this effect because of the poor resolution of their γ-ray detectors (the effect is small for us at 45.6 and 62.2 MeV because of our relatively thin hydrogen target). At our higher energies there is excellent agreement with previous data, in particular with the very precise experiment of Bugg *et al.*[60] which used a totally different technique. This gives us confidence that the overall method is very reliable.

The agreement with the phase shifts is adequate. The old VPI version of Zidell *et al.*[71] is certainly excluded by all our measurements. The more recent versions include our low-energy points but fit the present data equally well. The older analysis of Rowe *et al.*[72] is a little too high, reflecting the reliance on poor quality data.

The Legendre polynomial coefficients are shown in Figs. 4 and 5 and compared with the Karlsruhe-Helsinki and SAID (SM86) phase shift analyses. The latter fits slightly better, but not perfectly.

In Fig. 6 we illustrate the π^0 angular distributions

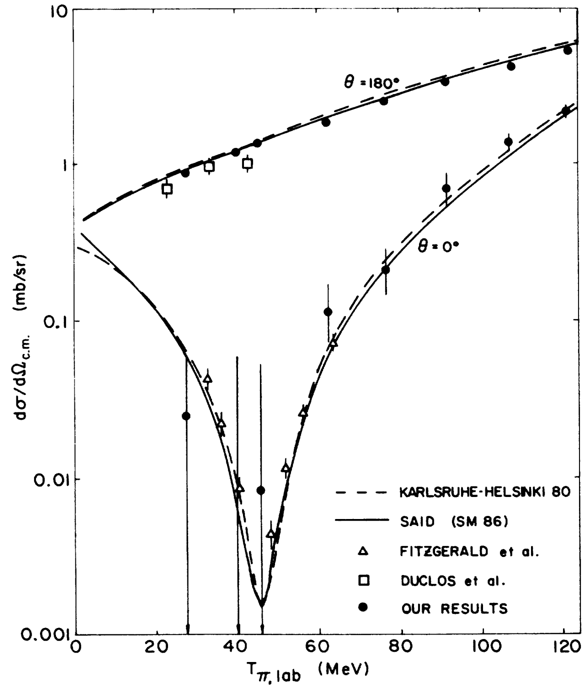

FIG. 7. The differential cross sections at 0° and 180° for the ($\pi^- p \rightarrow \pi^0 n$) reaction including the Saclay data of Duclos *et al.* (Ref. 73) and the recent Los Alamos data of Fitzgerald *et al.* (Ref. 6).

TABLE IV. Differential cross sections at 0° and 180° for the ($\pi^- p \rightarrow \pi^0 n$) reaction, obtained from our best fits.

T_π (MeV)	$\frac{d\sigma}{d\Omega}(0°)$ (mb/sr)	Relative error (mb/sr)	Absolute error (mb/sr)	$\frac{d\sigma}{d\Omega}(180°)$ (mb/sr)	Relative error (mb/sr)	Absolute error (mb/sr)
27.4[a]	0.03	0.04	0.04	0.87	0.04	0.05
39.3[a]	−0.02	0.07	0.07	1.22	0.07	0.08
45.6	0.01	0.05	0.05	1.43	0.05	0.07
62.2	0.12	0.05	0.05	1.91	0.05	0.08
76.4	0.21	0.07	0.07	2.54	0.07	0.11
91.7	0.73	0.15	0.15	3.46	0.15	0.18
106.8	1.46	0.16	0.17	4.44	0.16	0.21
121.9	2.27	0.22	0.23	5.70	0.22	0.28

[a] Salomon *et al.* (Ref. 1).

which result from our fits; note the well-known forward minimum at low energies. The experimental points come from the experiment of Ullmann *et al.*[19]

An interesting special case is the cross section at 0° and 180°, and we present our results in Fig. 7 and Table IV. Also included in Fig. 7 are the old results of Duclos *et al.*[73] and the very recent results of Fitzgerald *et al.*[6]. All the measurements are compatible, although those of Duclos *et al.* fall a little low. Note that the results of Duclos *et al.* do not constitute a direct determination of the *s*-wave scattering length as is so often stated. Even at 30 or 40 MeV, *p*-wave scattering makes the dominant contribution to the cross section (the 0° and 180° cross section would be the same in the limit of no *p* wave). It is therefore essential to do a full phase-shift analysis.

Our results at 0° are not as accurate as those of Fitzgerald *et al.* because of the very small size of the cross section at this particular angle. The absolute value of our errors is relatively independent of angle, so it looks good when the cross-section is large as at 180°, but depressingly large on a log plot when the cross section dips to a low value. Clearly, for 0° the better technique is to measure that cross section directly. However, our results are compatible with the dip near 45 MeV.

TABLE V. The *S*-wave scattering length ($a_1 - a_3$) obtained from this experiment and other references (in natural units $\hbar/m_\pi c$).

		($a_1 - a_3$)
This experiment	$T_\pi = 45.6$ MeV	0.267±0.013
	$T_\pi = 62.2$ MeV	0.253±0.017
	$T_\pi = 76.4$ MeV	0.270±0.015
	$T_\pi = 91.7$ MeV	0.273±0.015
	$T_\pi = 106.8$ MeV	0.263±0.015
	$T_\pi = 121.9$ MeV	0.270±0.017
Salomon *et al.* (Ref. 1)	$T_\pi = 27.4$ MeV	0.260±0.012
Salomon *et al.* (Ref. 1)	$T_\pi = 39.4$ MeV	0.265±0.012
Duclos *et al.* (Ref. 73)		0.270±0.014
Spuller *et al.* (Ref. 75)		0.263±0.005
Zideell *et al.* (Ref. 71)		0.302±0.006
Rowe *et al.* (Ref. 72)		0.283±0.008
Koch *et al.* (Ref. 13)		0.274±0.005
Koch (Ref. 14)		0.275
Bugg *et al.* (Ref. 76)		0.262±0.004
Arndt *et al.* (Ref. 12)		0.258±0.006

We applied the electromagnetic corrections of Tromborg *et al.*[74] and obtained the *s*-wave scattering lengths. We have extrapolated to zero energy using the relation

$$a_1 - a_3 = \frac{(\tan\delta_1 - \tan\delta_3)}{q} + cq^2 + \text{resonant term} .$$

The correction terms are relatively small and were obtained from energy dependence phase-shift analyses. There is no obvious trend of the values for the scattering length as a function of the pion energy, which justifies the standard assumptions. Our results are compared with other determinations in Table V. Some of these previous calculations have used quite sophisticated extrapolation techniques, but the data available was often of relatively poor quality.

V. CONCLUSION

For the first time, a technique, which was proposed many years ago, has finally been successfully used to make a comprehensive series of measurements on pion charge-exchange scattering at low energies. This has been possible because of the excellent properties of the TINA detector coupled with the well matched characteristics of the TRIUMF pion beams which have low electron contamination and 100% macroscopic duty cycle. The analysis is sophisticated but includes no major uncertainties, so the derived parameters are stable against any reasonable change in the analysis procedure.

The results are far more reliable than earlier experiments because the excellent energy resolution of TINA made it possible to avoid background problems which were almost certainly present in earlier experiments but went unobserved and unheeded. The existing phase shifts describe these new data fairly well, but by no means perfectly. A reoptimization of the parameters is recommended and will soon be available for the VPI solutions. It would also be useful to have a reanalysis using the Karlsruhe-Helsinki technique. The revised analyses should then be used as a basis for the determination of the Σ term.

There have been several suggestions that *s*-wave pion-nucleon scattering cannot be described by a normal effective-range expansion, i.e., the effective range is anomalously large or the scattering length is "energy dependent." There is no obvious evidence for this in our results since the accepted approach applied to all our energies gives consistent values for the scattering length. However, the errors are large and some small effects cannot be excluded. If a complete analysis is made, including the latest results on $\pi^{\pm}p$ elastic scattering, it should be possible to make a more definitive statement.

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