Angular correlation function as a detector for two-step processes

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The reaction $^{12}C(^6Li,^6Li')^{12}C^*(^6Li,d)^{16}O^*(\alpha)^{12}C$ proceeding via the quasibound 4_2^+ ($E^*=11.09$ MeV) state of ^{16}O has been suggested to proceed via a two-step process. We reinvestigate this case. Computing the angular correlation function $W(\Omega_d,\Omega_\alpha)$ it is confirmed that the previous suggestions are indeed correct. Furthermore, our results indicate that the comparison of calculated $W(\Omega_d,\Omega_\alpha)$ with experimental phase shifts may be used as a detector for two-step processes.

I. INTRODUCTION

The (^{6}Li,d) reaction on ^{12}C with an α transfer into the quasibound 4_2^+ ($E^*=11.09$ MeV) state of ^{16}O as studied in Ref. 1 could be due to a two-step process of the type $^{12}C(^6Li,^6Li')^{12}C^*(^6Li,d)^{16}O^*(\alpha)^{12}C$. We would like to discuss this case in some detail to present evidence showing that this interpretation is indeed correct.

Let us first recall that the reaction $^{12}C(^6Li,d)^{16}O\rightarrow{}^{12}C+\alpha$ has been shown[2] to have (for $\theta_d=0°$) such a geometry that a related angular correlation function W of the form of a Legendre polynomial of second order implies that the (^{6}Li,d) reaction proceeds via a *direct* process. In the case under consideration the measured[1] angular correlation function W as a function of angle θ does have the form of a $P_l^2(\cos\theta)$ polynomial so that the (^{6}Li,d) reaction mechanism is indeed a direct one. However, it has been noted[1] that there is a large shift of $\Delta\theta\simeq 15°$ between the respective polynomials $W(\theta_\alpha)\sim P_4^2(\cos\theta)$ of the 4_1^+ (10.35 MeV) and 4_2^+ (11.09 MeV) states. Since the energy difference between the two levels is rather small (i.e., $\sim$750 keV) such a large shift $\Delta\theta$ can neither be understood in terms of the DWBA (since the differences in the kinematics are negligible), nor can it be explained by differences in the structures of the two levels.

Microscopic considerations are expected to lead to a correct interpretation of the observed discrepancies. To start with, let us recall that the resonating group method (RGM) with coupled channels has[3,4] been applied to a discussion of the $T=0$ states in ^{16}O. The formalism used allows us to account for the collective properties exhibited by the core ^{12}C. Its first excited 2^+ state at 4.43 MeV is easily activated in inelastic scattering processes. From the point of view of Ref. 4 the 4_2^+ (11.09 MeV) state under consideration appears as the ground state of an α particle orbiting in the field of the excited core ^{12}C (2^+). In terms of the complete set of wave functions u_{nl} with the weights $\gamma^2_{I,l,n}$, the spectroscopic structure of this 4_2^+ state is given by the following numbers: $\gamma^2_{0,4,1}=0.095$; $\gamma^2_{2,2,2}=0.575$; $\gamma^2_{2,4,2}=0.016$; $\gamma^2_{2,6,2}=0.001$.[4] It is obvious that the dominant contribution corresponds to an α particle orbiting around the excited core ^{12}C (2^+), i.e., to $\gamma^2_{2,2,2}=0.575$. In this connection the ratio of the measured reduced widths is also of interest: The one of the 4_2^+ under consideration is very small, $\sim 10^{-3}$, indicating that it relates to a clean α-cluster state.[5]

The traditional DWBA cannot properly account for the excitation of the core in transfer reactions. Hence, it is suggestive to resort to the method of (strongly) coupled channels within the DWBA, which was originally developed in the context of (d,p) reactions.[6] It leads to a system of coupled differential equations implying that the corresponding computations will be rather laborious.

In a somewhat different approach to the problem, two approximations of the adiabatic type[7,8] have been proposed to account for the effect of the excited core in (d,p) reactions.[9] The discussion of these approximations is not just to help us in the qualitative determination of the nature of the process under consideration, but also to provide us with a pictorial view of the reaction mechanism. In Sec. II we will discuss the gist of the method to derive the formulas required for the computations to be discussed in Sec. III. The final section is devoted to a short summary.

Before going on to the details, let us recall that weakly bound states may well be interpreted within the finite range DWBA; however, it does not allow for an adequate treatment of quasibound or resonance states with such high excitation energies as of the order of 10 MeV. Hence, one has to resort to the zero-range DWBA which will also be applied in here. Since we do know that the zero-range DWBA does provide a good approximation to this particular reaction,[10,11] there are no serious objections against its use in exploring the role of two-step processes in such reactions.

II. FORMAL CONSIDERATIONS AND APPROXIMATIONS

The first approximation put forward in Ref. 7 assumes that the scattering in the reaction channel involves a "frozen" target. Formally that implies the factorization of the total channel wave function $\Psi^{(+)}(\mathbf{r},\xi)$ depending on the internal variables ξ and the relative coordinate $\mathbf{r}$:

$$\Psi^{(+)}(\mathbf{r},\xi)=\psi^{(+)}(\mathbf{r},\xi)\Phi(\xi)\ . \tag{1}$$

$\Phi(\xi)$ represents the internal wave function of the respec-

tive channel and $\psi^{(+)}(\mathbf{r},\xi)$ the distorted wave. However, this $\psi^{(+)}$ does not just depend on $\mathbf{r}$ as within the usual DWBA, but also on the collective variables. Approximation (1) becomes more exact, the higher the energy of relative motion E in the reaction channel. This is readily appreciated by the aid of simple physical considerations: Let E^* be the energy of the collective state excited in the scattering; $t \sim 2R/v$ is then the approximate collision time and $T \sim \hbar/E^*$ the period of the internal motion. The condition

$$t \gg T \text{ or } E \gg (KR)E^* \tag{2}$$

is then sufficient for (1) to be justified. Practice shows that condition (2) is more restrictive than necessary, i.e., even at lower energies of relative motion (1) turns out to work nicely.

The second approximation[8] amounts to assuming that the angular part of the wave function of relative motion $\psi^{(+)}(\mathbf{r},\xi)$ depends only weakly on the internal variables ξ. Consequently it is possible to represent it in the form of the traditional partial wave expansion for spherically symmetric fields. Generally speaking, this approximation is sensible if the main contribution to the cross section stems from the partial waves with $l \sim l_0 = KR_0 \gg L_0$, where L_0 is the characteristic change of momentum during the excitation. More rigorous estimates[8] show that for the scattering amplitude such a picture is justified up to a precision of order $O(I/(KR)^{1/3})$.

We are going to consider only such collective excitations of the nucleus A which involve motions of nuclear surfaces of the type

$$R(\hat{r}) = R[1+\Delta(\hat{r})] \text{ with } \Delta(\hat{r}) = \sum_{\lambda\mu} \xi_{\lambda\mu} Y^*_{\lambda\mu}(\hat{r}) . \tag{3}$$

The coordinates $\xi_{\lambda\mu}$ are then the corresponding collective variables. Under these approximations the wave functions of relative motion in entrance and exit reaction channels may be written (ignoring spin-orbit interactions and using j for ^{6}Li and d for the deuteron) as

$$\psi^{(+)}(\mathbf{K}_j\cdot\mathbf{r}) = \frac{4\pi}{r} \sum_{l_j m_j} i^{l_j} \chi_{l_j}(r,R(\hat{r})) \times Y_{l_j m_j}(\Omega_r) Y^*_{l_j m_j}(\Omega_{K_j}) , \tag{4}$$

$$\psi^{(-)}(\mathbf{K}_d,\mathbf{r}) = \frac{4\pi}{r} \sum_{l_d m_d} i^{l_d} \chi_{l_d}\left(\frac{m_A}{m_B} r, R(\hat{r})\right) \times Y_{l_d m_d}(\Omega_r) Y^*_{l_d m_d}(\Omega_{K_d}) . \tag{5}$$

Now the radial functions in (4) and (5) are expanded in terms of a Tailor series which may be cast into the compact form

$$\chi_l(r,R(\hat{r})) = \exp\left[\Delta(\hat{r}) R \frac{\partial}{\partial R}\right] \chi_l(r,R) . \tag{6}$$

We shall ignore all terms except for the first two, i.e., we are going to work in the framework of perturbation theory in respect to the deformations:

$$\chi_l(r,R(\hat{r})) = \left[1+\Delta(\hat{r}) R \frac{\partial}{\partial R}\right] \chi_l(r,R) . \tag{7}$$

It is self-evident that the contributions to direct processes are generated by the first term in (7). Two-step processes arise if the second term is different from zero.

Let us draw attention to some points of note for further consideration: The expression for $\Delta(\hat{r})$ appears as an operator acting on the collective variables, whence the functions $\psi^{(\pm)}(r,R(\hat{r}))$ also become operators. Therefore it is important in the formula for the amplitude in which sequence they appear relative to the other functions of the collective variables. That is, the latter should appear at the "outer parts" of the matrix elements sandwiching the $\psi^{(\pm)}$ standing inbetween.

For the sake of simplicity we do not discuss the superposition between the amplitudes of one- and two-step processes (as would be required within a more general treatment). Such a simplification is justified if the states under consideration are (at least almost) clean states. In the present case this is guaranteed (see above) because the ratio of the reduced widths for the decay of the 4_2^+ state in ^{16}O into the ^{12}C (g.s.) state relative to the one of a decay of the 4_1^+ state in ^{16}O into the ^{12}C (g.s.) is about 1000.

Since we are interested in those states which decay via α particles, i.e., into a single reaction channel, the angular correlation function is determined by

$$W(\theta_d,\theta_\alpha) = \frac{d^3\sigma}{dE_d d\Omega_\alpha d\Omega_d} \Big/ \frac{d^2\sigma}{dE_\alpha d\Omega_d} , \tag{8}$$

with the normalization

$$\int W(\Omega_d,\Omega_\alpha) d\Omega_\alpha = 1 \tag{9}$$

for the registration of an α particle in the solid angle 4π. Relation (9) is the formal statement for the fact that we have only a single decay (exit) channel. For the threefold differential cross section we have[9]

$$\frac{d^3\sigma}{dE_\alpha d\Omega_\alpha d\Omega_d} = \frac{\mu_{\mathrm{Li},A}\mu_{\alpha,A}\mu_{d,B}}{(2\pi)^5\hbar^6} \frac{K_\alpha K_d}{K_{\mathrm{Li}}} \sum |T|^2 , \tag{10}$$

where T is the amplitude of the process $A + {}^6\mathrm{Li} \rightarrow A + \alpha + d$. The other symbols have their conventional meanings. The summation implies an averaging over the angular momentum projections of the initial state and a summation over the projections of the final states.

For the evaluation of W we follow the notion put forward in Ref. 12. It suggests that it is sensible to represent the amplitude of the process $A({}^6\mathrm{Li},d)B^*(\alpha)A$ in a factorized form, i.e.,

$$T^{m_i m_A}_{m_i' m_A' m_\alpha}(\Omega_d,\Omega_\alpha) \equiv T(\Omega_d,\Omega_\alpha) = T(\Omega_d)\cdot T(\Omega_\alpha) \equiv \sum_m T^{m_i m_A}_{m_i' m}(\Omega_d) T^{m}_{m_A' m_\alpha}(\Omega_\alpha) . \tag{11}$$

The schematic representation of this reaction displayed in Fig. 1 helps to specify the notation used. The physical picture implied by (11) is rather simple. There are two independent processes occurring one after the other: *First* the α particle is transferred from ^{6}Li into a resonance (quasibound) state of the nucleus B, and *then* the nucleus B decays into the target nucleus A plus an α particle, both being in their respective ground states. Bearing in mind that the spin of the α particle is zero, we have in the case of a spin-zero target ($I_A = I'_A = 0$) the relation

$$T^{m_i}_{m'_i}(\Omega_d,\Omega_\alpha) = \sum_m T^{m_i}_{m'_i m}(\Omega_d) T^m(\Omega_\alpha) \ . \tag{12}$$

On the other hand, condition (9) implies that we do not have to worry about normalization factors in the amplitude $T^m(\Omega_\alpha)$. Hence, we may use the proportionality $T^m(\Omega_\alpha) \sim Y^m_L(\Omega_\alpha)$ (where L is the angular momentum of the respective state in B, e.g., here $L=4$) to proceed towards the evaluation of W.

Taking, within the zero-range approximation, the excitation of the core into account, the amplitude of the first of the two partial processes for the transfer of an α particle from the projectile ^{6}Li into the nucleus B is represented by

$$T^{m_i}_{m'_i m}(\Omega_d) = D_0 \langle \Psi^{(-)}_{d,B}(\mathbf{r},\xi) \mid \Psi^{(+)}_{A,\mathrm{Li}}(\mathbf{r},\xi) \rangle \tag{13}$$

(Ref. 13). D_0 is due to the zero-range approximation for the α-d interaction, and $\Psi^{(+)}_{\mathrm{Li},A}$ and $\Psi^{(-)}_{d,B}$ are the complete channel functions. The use of the zero-range approximation for the interaction $V_{\alpha d}(r_{\alpha d})$ is in the case under consideration justified (see also Ref. 10). This is due to the process being a peripheral one with $\theta_d = 10°$ (lab) and the reaction amplitude containing the tail of the wave function of relative motion between α particle and deuteron. The difference in the respective tails of the 1s state of the deuteron and of the 2s state of ^{6}Li are not important.

Some standard manipulations[9] lead us now to the correlation function for a pure two-step process, namely

$$W(\theta_d,\theta_\alpha) = N^{-1} \left| \sum_m G^m_{Ll}(\theta_d) Y^m_L(\theta_\alpha) \right|^2 , \tag{14}$$

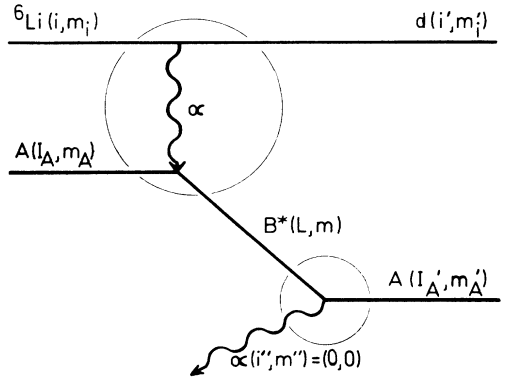

FIG. 1. To specify the notation for the momentum algebra a schematic representation is given of the reaction $A(^6\mathrm{Li},d)B^*(\alpha)A$.

where N is the normalization constant with

$$N = \sum_m | G^m_{Ll}(\theta_d) |^2 ,$$

$$G^m_{Ll}(\theta_d) = \left[R_{\mathrm{Li}} \frac{\partial}{\partial R_{\mathrm{Li}}} + R_d \frac{\partial}{\partial R_d} \right] \tilde{G}_{Ll}(\theta_d) \ , \tag{15}$$

$$\tilde{G}_{Ll}(\theta_d) = \sum_{l_i l_d} \Gamma^{Lm}_{l_i l_d} J_{l_i l_d l} P^m_{l_d}(\cos\theta_d) \ ,$$

and

$$J_{l_i l_d l} = \int dr \, \chi_{l_d} \left[\frac{m_A}{m_B} K_d r \right] u_l(r) \chi_{l_i}(K_i r) \ ,$$

$$\Gamma^{Lm}_{l_i l_d} = (2l_d+1) i^{l_i - l_d - l} \left[\frac{(l_d - m)!}{(l_d + m)!} \right]^{1/2} \tag{16}$$

$$\times (l_d 0 l 0 \mid l_i 0)(l_d m L - m \mid l_i 0) \ .$$

The differentiations in (15) have to be performed in respect to the radii of the real parts of the optical potential in the ^{6}Li and d channels and one has to include the contribution (of at least one) of the states of the orbiting α particle, $u_{nl}(r_\alpha)$. In principle one has to sum over all the u_{nl} in respect to l. However, as already discussed, we are here dealing with a clean state so that there is no need for such a summation.

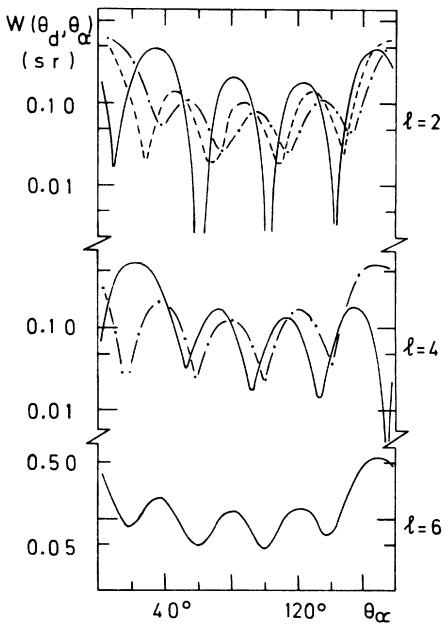

FIG. 2. For different quantum numbers l and n, the angular correlation function for the reaction $^{12}\mathrm{C}(^6\mathrm{Li},d)^{16}\mathrm{O}^*(\alpha)^{12}\mathrm{C}$ (via the 4^+_2 state in ^{16}O at 11.09 MeV; $E_{\mathrm{Li}} = 34$ MeV; $\theta_d = 10°$ lab) is plotted versus the scattering angle of the emitted α particles. From top to bottom the angular momentum l increases from 2 to 6; full curves correspond to $n=1$, dotted ones to $n=2$, and the broken one to $n=3$.

TABLE I. The optical model parameters are given. The ones for the elastic scattering of ^{6}Li off ^{12}C stem from Ref. 14 and the ones in the deuteron channel from Ref. 15; in all cases $r_{Cl}=1.3$ fm.

	V_0 (MeV)	r_v (fm)	a_v (fm)	W_v (MeV)	W_d (MeV)	r_W (fm)	a_W (fm)
^{6}Li + ^{12}C	245.0	1.20	0.800	12.50		1.75	1.000
d + ^{16}O	101.4	1.00	0.717		8.75	1.58	0.625

III. DISCUSSION

Crudely speaking, (14)–(16) demand that we evaluate four versions of the conventional breakup. Since we would then also be obliged to perform the summations over all n and l, this would lead to a rapid increase in the required computer time. To circumvent this problem, we try to explore the characteristics of the "one-channel" angular correlation functions with different n and l in u_{nl} to thus arrive at a qualitative picture of the physics involved.

An example of such a qualitative analysis is given in Fig. 2. It displays the functions $W(\Omega_d,\Omega_\alpha)$ for the 4_2^+ states of ^{16}O with different values of the quantum numbers n and l of the $u_{nl}(r_\alpha)$ of the weakly bound (~ -500 keV) α particle in the field of the excited core ^{12}C ($I^\pi=2^+$). As usual, u_{nl} is evaluated by adjusting the depth of a Saxon-Woods potential ($R=2.98$ fm; $a=0.5$ fm) so as to provide the appropriate binding energy. The related optical model parameters are displayed in Table I.

In the case of transfers involving quasibound states it is in general necessary to regularize the respective radial integrals. However, in the given case of the reaction ^{12}C(^{6}Li,^{6}Li')^{12}C*(^{6}Li,d)^{16}O*(α)^{12}C involving the 4_2^+ (11.09 MeV) in ^{16}O there is no need to do so. Due to the binding of the α particle in the field of the excited core this is automatically done by the two-step process.

Figure 2 nicely demonstrates the strong dependence of $W(\Omega_d,\Omega_\alpha)$ on the quantum numbers of the wave function of the captured α particle; compare in particular the dot-dash curve (two-step) for $l=2$ with the solid curve (one-step) for $l=4$. Seemingly this is related to the specifics of the given case explicitly involving the bound state of the α particle. As already discussed, the state with $u_{nl}(r_\alpha)=u_{22}(r_\alpha)$ is the only one of note for the comparison to experiment. Indeed, the calculated $W(\Omega_d,\Omega_\alpha)$ for the two-step process agree rather nicely with the experimental data;[1] see the full curve in Fig. 3. The dash-dot curve contrasts this result for the two-step process ^{12}C(^{6}Li,^{6}Li')^{12}C*(^{6}Li,d)^{16}O*(α)^{12}C with the characteristics of the one-step process (involving the 4_1^+ state at 10.35 MeV; the details of the curves depend very little on smaller variations in the energy, say from 10.35 MeV to 11.09 MeV). Thus we have a clear indication that the reaction mechanism is governed by a comparatively clean two-step process.

The small shift in the theoretical function $W(\Omega_d,\Omega_\alpha)$ in respect to experiment with $\Delta\theta\sim 3°$ may be attributed to the following:

(1) The suppression of effects of second, third, etc., order in $\Delta(\hat{r})$; see Eqs. (6) and (7);

(2) The somewhat incorrect transfer form factor of $u_{nl}(r_\alpha)$. Strictly speaking it is necessary to evaluate the form factor within the coupled channel RGM.[4] The one we relied upon does not sufficiently accurately account for the microscopic structure of the respective state and for the effects of antisymmetrization [both of which *are* of importance when the details are of interest (see Refs. 10 and 11) and references therein]. However, varying the parameters of the Saxon-Woods potential for $u_{nl}(r_\alpha)=u_{22}$ also gives rise to a similar shift of the order of $\Delta\theta\sim 2°-3°$.

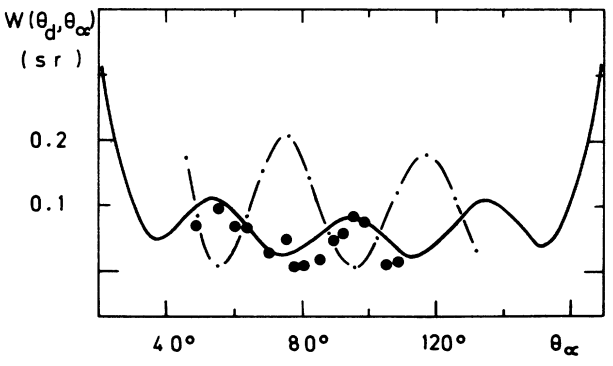

FIG. 3. The angular correlation function for the same reaction as in Fig. 2 is plotted versus the scattering angle of the emitted α particles. The experimental points have been taken from Ref. 1. The calculations represented by the full curve correspond to a pure two-step process [i.e., to an $u_{nl}(r)$ with $l=2$ and $n=2$]. The dash-dot curve illustrates the effect of a one-step process involving the 4_1^+ state (10.35 MeV; $l=4$, $n=1$).

IV. SUMMARY

Our results unambiguously confirm the notion[1] that the reaction ^{12}C(^{6}Li,d)^{16}O*(α)^{12}C proceeds via a two-step process of the type ^{12}C(^{6}Li,^{6}Li')^{12}C*(^{6}Li,d)^{16}O*(α)^{12}C. But it is even more important to recognize that in general the angular correlation function is so sensitive to the details of the reaction process that it does not just tell us that the process is a direct one, but even more explicitly what type of a direct reaction. For the experimentalists it is of note to realize that phase-shift measurements in liaison with computed angular correlation functions thus help to determine the particular type of a reaction process.

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