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Model analysis of the effect of elementary degrees of freedom in the $^3S_1$ nucleon-nucleon system

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Using a modified Lee model for the nucleon-nucleon interaction we show the equivalence of this model to an interaction based on the quark compound bag model recently proposed by Bhasin and Gupta. It is found that their choice of the sign of the parameter coupling the hadronic to quark degrees of freedom leads to unphysical results. By investigating the model with the correct sign and more general form factors, we observe that elementary degrees of freedom in terms of the elementary particle (e.g., six-quark or dibaryon) eigenstate components of the two-nucleon system, will not be resolved by studying the on-shell behavior of the T matrix, but may be seen by studying effects that depend on the detailed structure of the two-nucleon wave function.

I. INTRODUCTION

The correct theory of strong interaction is now believed to be quantum chromodynamics (QCD). Although it is difficult to derive (low-energy) nuclear physics from QCD, it ought to be consistent with the fundamental assumptions and results of QCD. Since at higher energy the properties of the nucleons can be understood in terms of their quark-gluon structure, it is expected that the nucleon-nucleon interaction also depends on quark degrees of freedom. A correct derivation of the nuclear force would therefore include the quark and gluon constituents of the nucleons. It is not clear however to what extent the quark degrees of freedom influence low-energy scattering (and low-energy nuclear physics) and the extent to which the low-energy phenomena can be explained solely in terms of the hadronic properties of the nucleons.

It is well known that the long-range part of the nucleon-nucleon potential is described quantitatively by the one-pion-exchange potential (OPEP) and there should be some way that this aspect of the nuclear force is related to QCD. It has been found that in the limit of infinitely many colors, QCD is equivalent to a meson theory in which baryons are topological solitons. Thus for low energy at which QCD cannot be treated perturbatively, one has in recent years attempted to derive the nucleon-nucleon interaction using the Skyrme model. It is nevertheless still an open question whether quark degrees of freedom are more directly involved in the short-range part of the interaction. In nonrelativistic potential models,[1] the repulsion seen at higher energy in $^3S_1$ scattering is obtained by including a short-range, soft or hard, repulsive core. It is of interest therefore to study the relationship between such short-range behavior of the interaction and the quark degrees of freedom which are also expected to be effective at higher energy.

Recently a number of authors[2–5] have attempted to explain the inner region of the nucleon-nucleon interaction by means of general properties of the quark structure of the system when the two nucleons are a small distance apart. Although such discussions do not involve the precise dynamics of the six-quark system, they make use of the fact that six-quark eigenstates dominate the scattering process at short distances and hence lead to strong short-range repulsion.

We wish to point out that these results can also be obtained using a model derived from the Lee model and applied to the two-nucleon system. In this model the nucleons couple to elementary baryon states with the same quantum numbers as those of the two-nucleon system. In order to compare this and previously derived models to the experimental data, one needs to take care in the choice of parameters for the effective nucleon-nucleon interaction. In particular, the interaction derived by Bhasin and Gupta[3] displays some unphysical properties because of their choice of the strength parameter associated with the energy-dependent part of the interaction. In this paper it is not our intention to provide an accurate fit to the data, but rather to discuss some properties of the interaction when elementary particle states are incorporated. To focus on these properties we employ the modified Lee model which leads to analytic solutions but does not incorporate complicating features such as the boson exchange potential tail and the tensor force. Nor are processes such as $N\Delta, \Delta\Delta$, etc., scattering and meson production included. All these processes need to be considered when a detailed comparison with experimental data is made.

A similar model has been applied to the two-nucleon system to discuss the elementary particle component of the deuteron.[6] In contrast to earlier results we now find that the elastic scattering data can be adequately described by an interaction that gives a large elementary particle component in the deuteron. The model is described in Sec. II; its application to the $^3S_1$ state of the deuteron and the ensuing consequences are discussed in Sec. III, which is followed by a brief concluding section.

II. MODEL INTERACTION

The model which we employ is a modified Lee model which has been used in a number of contexts.[7–10] We will generalize an earlier derivation[6] which was also applied to the two-nucleon system.

In this model the two nucleons, besides interacting with each other directly, couple to one or more elementary particle states. The latter simulate the six-quark or dibaryon states which lie at considerable energy above threshold.[11] The Hamiltonian describing the system is

$$H = H_0 + H_I \tag{1}$$

with

$$H_0 = \sum_{i=1}^{n} m_i D_i^\dagger D_i + \int d^3k \epsilon_k \eta_k^\dagger \eta_k \tag{2}$$

and

$$H_I = \sum_{i=1}^{n} g_i \int d^3k u_i(k)[D_i \eta_k^\dagger + \eta_k D_i^\dagger]$$

$$-G \int \int d^3k d^3k' u_0(k) u_0(k') \eta_k^\dagger \eta_{k'} . \tag{3}$$

The operator D_i annihilates the ith elementary particle state of which we have assumed that there are n, and η_k annihilates the two-nucleon state with relative momentum $\mathbf{k}$ in the center-of-mass frame and energy $\epsilon_k = 2(m_N^2 + \mathbf{k}^2)^{1/2}$, where m_N stands for the nucleon mass. The noncommuting field operators are

$$[D_i, D_j^\dagger] = \delta_{ij}, \quad [\eta_k, \eta_{k'}^\dagger] = \delta(\mathbf{k} - \mathbf{k}') ; \tag{4}$$

all other field operators commute. Since the operator

$$Q_1 = \sum_{i=1}^{n} D_i^\dagger D_i + \int d^3k \eta_k^\dagger \eta_k \tag{5}$$

commutes with H, it is a constant of motion. For the two-nucleon system Q_1 is 1. As is the case with the Lee model, this Hamiltonian allows one to obtain exact algebraic bound and scattering state solutions.

The states created by $\eta_k^\dagger$ and $D_i^\dagger$ will have baryon number equal to 2. Because of baryon number conservation no other combination of nucleons and elementary particle states will be possible provided we stay with the two-nucleon sector and do not include particle-antiparticle pairs. The form factor u_0 controls the direct interaction between nucleons; the term of the Hamiltonian including this form factor simulates the hadronic interaction between the nucleons. In a realistic calculation this term would include the boson-exchange part of the potential. The term involving the form factors $u_i(k)$, $i = 1, \ldots, n$, couples the hadronic states to the elementary (e.g., six-quark) states.

We define $|0\rangle$ to be the state for which $\eta_k |0\rangle = D_i |0\rangle = 0$; since $H_0 |0\rangle = H_i |0\rangle = 0$, $|0\rangle$ represents the physical as well as the bare vacuum. We normalize the physical eigenstates so that $\langle 0|0\rangle = \langle \psi | \psi \rangle = 1$ for bound states and $\langle \psi_k | \psi_{k'} \rangle = \delta(\mathbf{k} - \mathbf{k}')$ for scattering states. For notational convenience we label the bare particle states as $|\mathbf{k}\rangle = \eta_k^\dagger |0\rangle$ and $|D_i\rangle = D_i |0\rangle$.

In order to study the scattering in the two-nucleon sector we introduce the T matrix which satisfies the operator equation

$$T(z) = H_I + H_I \frac{1}{z - H_0} T(z) . \tag{6}$$

In our representation, upon insertion of a complete set of states, the matrix elements of T are

$$\langle \mathbf{k} | T(z) | \mathbf{k}' \rangle = \langle \mathbf{k} | H_I | \mathbf{k}' \rangle + \int d^3k'' \frac{\langle \mathbf{k} | H_I | \mathbf{k}'' \rangle \langle \mathbf{k}'' | T(z) | \mathbf{k}' \rangle}{z - \epsilon_k''} + \sum_{i=1}^{n} \frac{\langle \mathbf{k} | H_I | D_i \rangle \langle D_i | T(z) | \mathbf{k}' \rangle}{z - m_i} \tag{7}$$

and

$$\langle D_i | T(z) | \mathbf{k}' \rangle = \langle D_i | H_I | \mathbf{k}' \rangle + \int d^3\mathbf{k}'' \frac{\langle D_i | H_I | \mathbf{k}'' \rangle \langle \mathbf{k}'' | T(z) | \mathbf{k}' \rangle}{z - \epsilon_{k''}} . \tag{8}$$

We have used the fact that $\langle D_i | H_I | D_j \rangle = 0$. The effective interaction potential in the nucleon-nucleon channel is

$$\langle \mathbf{k} | V_{\text{eff}}(z) | \mathbf{k}' \rangle = \langle \mathbf{k} | H_I | \mathbf{k}' \rangle + \sum_{i=1}^{n} \frac{\langle \mathbf{k} | H_I | D_i \rangle \langle D_i | H_I | \mathbf{k}' \rangle}{z - m_i} . \tag{9}$$

By inserting $\langle D_i | T(z) | \mathbf{k}' \rangle$ of Eq. (8) in Eq. (7), we obtain an expression for the two-nucleon T matrix. It has the form

$$\langle \mathbf{k} | T(z) | \mathbf{k}' \rangle = \sum_{i=0}^{n} a_i(z) u_i(k) B_i(z, \mathbf{k}') , \tag{10}$$

where

$$a_0 = -G, \quad a_i(z) = \frac{g_i^2}{z - m_i}, \quad \text{for } i = 1, \ldots, n \tag{11}$$

and

$$B_i(z, \mathbf{k}') = u_i(k') + \int d^3k'' \frac{u_i(k'') \langle \mathbf{k}'' | T(z) | \mathbf{k}' \rangle}{z - \epsilon_{k''}} . \tag{12}$$

Substitution of the expression for the T-matrix element of Eq. (10) in Eq. (12) yields a set of linear algebraic equations for the B_i's. Formally we write

$$B_i(z,k')=\sum_{j=0}^{n}[A^{-1}(z)]_{ij}u_j(k') , \tag{13}$$

where the matrix $A(z)$ is defined by its entries,

$$A_{ij}=\delta_{ij}-a_j(z)J_{ij}(z) \tag{14}$$

with

$$J_{ij}(z)=\int d^3k\frac{u_i(k)u_j(k)}{z-\epsilon_k} . \tag{15}$$

The determination of the (in general, off-shell) T matrix is equivalent to the inversion of an $(n+1)\times(n+1)$ matrix.

If the form factors depend only on the magnitude of the vector $\mathbf{k}$, the interactions will occur in the S state only. The S-state scattering phase shifts, $\delta(k)$, are determined from the on-shell T matrix since

$$\langle \mathbf{k}' \mid T(\epsilon_k+i\eta) \mid \mathbf{k}\rangle_{|\mathbf{k}'|=|\mathbf{k}|}=-\frac{1}{\pi^2\epsilon_k k}e^{i\delta(k)}\sin\delta(k) . \tag{16}$$

The scattering wave function is

$$\mid z_{\mathbf{k}}\rangle=\mid \mathbf{k}\rangle+\int d^3k'\frac{1}{\epsilon_k+i\eta-\epsilon_{k'}}\mid \mathbf{k}'\rangle \times\langle \mathbf{k}' \mid T(\epsilon_k+i\eta)\mid \mathbf{k}\rangle . \tag{17}$$

In order to solve for the bound states, we define the state vector

$$\mid \psi_B\rangle=\sum_{i=1}^{n}c_iD_i^\dagger\mid 0\rangle+\int d^3k\chi_B(\mathbf{k})\eta_k^\dagger\mid 0\rangle , \tag{18}$$

so that $\mid \psi_B\rangle$ satisfies the equation

$$H\mid \psi_B\rangle=\epsilon_B\mid \psi_B\rangle . \tag{19}$$

Taking the scalar product of Eq. (19) with $\langle 0 \mid D_j$, we obtain

$$c_j=\frac{g_j}{\epsilon_B-m_j}\int d^3k u_j(k)\chi_B(\mathbf{k}) . \tag{20}$$

Similarly we form the scalar product of Eq. (19) with $\langle 0 \mid \eta_k$ to obtain

$$\chi_B(\mathbf{k})=\frac{1}{\epsilon_B-\epsilon_k}\sum_{i=0}^{n}a_i(\epsilon_B)u_i(k)B_i , \tag{21}$$

where

$$B_i=\int d^3k'u_i(k')\chi_B(\mathbf{k}') . \tag{22}$$

By inserting the expression of $\chi_B(\mathbf{k})$ of Eq. (21) into Eq. (22), we get a homogeneous system of linear equations for the B_i's, i.e.,

$$\sum_{j=0}^{n}A_{ij}(\epsilon_B)B_j=0,\quad i=0,\ldots,n . \tag{23}$$

Bound states exist when $\det A(\epsilon_B)=0$. For each such ϵ_B, there exists an eigenvector $\mathbf{B}$ so that the corresponding $\chi_B(\mathbf{k})$ and c_i's can be determined.

Apart from a volume part of the optical potential which varies linearly as the energy, this model is equivalent to calculations by Simonov,[12] who employs the quark compound bag model. This becomes evident when it is observed that the scattering amplitude (for a single elementary state) is identical to that obtained by Bhasin and Gupta[3] who derived the scattering amplitude from the quark compound bag model. The term of the Hamiltonian with coupling constant G corresponds to the hadronic interaction, whereas the other terms are derived from the quark-quark interaction by means of the resonating-group method. Even though the quark-hadron coupling, leading to the nucleon-elementary-state coupling can, in principle, be obtained from the quark-quark interaction, Simonov[2,12] and Bhasin and Gupta[3] have parametrized the interaction assuming that the surface part is nonzero only for a single value of the separation distance of the nucleons. This simulates the surface interaction of two quark bags that approach one another.

Simonov[2] showed that the quark compound bag model is a natural basis for the P-matrix analysis of Jaffe and Low.[13] By assuming a surface interaction in coordinate space at $r=b$ and a cutoff for the hadronic interaction so that it is zero for $r<b$, the poles of the P matrix evaluated at b correspond to the elementary-state energies. It can be shown, however, that when the quark-hadronic interaction is smeared over a range of separation distances peaked at b, the P-matrix poles shift and are no longer exactly at the elementary-state energies. The poles of the T matrix occur at the NN bound-state energy and at scattering resonances. Although the introduction of an elementary state introduces a resonance, the energy at which it occurs is not necessarily the same as the energy of the elementary-particle state. The results described in Sec. III bear this out.

III. CALCULATIONS AND RESULTS

The form factors are chosen so that the nucleons couple to a single elementary-particle state by means of a δ-function interaction in coordinate space at separation distance $r=b$. For the nucleon-to-nucleon coupling we choose a separable Yukawa-type interaction for $r>b$. The form factors in momentum space are

$$u_0(k)=\frac{e^{-\beta b}[\cos(kb)+\beta\sin(kb)/k]}{\beta^2+k^2} , \tag{24}$$

$$u_1(k)=\frac{1}{\pi\sqrt{2}k}\sin(kb) . \tag{25}$$

For this interaction $n=1$ and β is the range of the separable Yukawa potential. This interaction is the same as that of Bhasin and Gupta.[3] However, these authors have used a negative value for the parameter corresponding to our g^2. This means that the fundamental Hamiltonian [Eq. (1)] is non-Hermitian.

Although an underlying non-Hermitian Hamiltonian is unacceptable, one might consider the effective energy-dependent interaction, which is derived from the theory and which is to be inserted in the Schrödinger equation, as one in which the parameters are to be adjusted to obtain a fit to the data. From this point of view and without recourse to the underlying Hamiltonian, are there unphysical features when a negative value for g^2 is used?

A modified form of Levinson's theorem[9,14] states that $\delta(0)-\delta(\infty)=(m-n)\pi$, where m is the number of bound states of the total Hamiltonian H and n is the number of discrete eigenstates of H_0. In Fig. 1 the $^3S_1$ phase shifts calculated with the parameters of Bhasin and Gupta[3] are plotted as a function of the logarithm of the laboratory energy. These phase shifts give a reasonable fit to experiment up to 400 MeV (see Fig. 2 of Ref. 3), but beyond 400 MeV the phase shifts decrease very quickly and asymptotically approach $-\pi$. According to Levinson's theorem this implies that there are three bound states, whereas a careful search gives only one at the deuteron energy. Since Levinson's theorem is a consequence of the poles and zeros of the logarithmic derivative of the Fredholm determinant in the physical sheet, cut along the positive real axis, we searched for other than bound-state zeros in the physical energy plane. In the upper half k plane, onto which the physical energy plane maps, we find besides the bound-state zero at $k=45.71i$ MeV, two zeros at $k=\pm 613.32+36.01i$ MeV. The fact that the Fredholm determinant has a zero in the first quadrant of the k plane, makes the S matrix nonanalytic there, which implies a violation of causality.[15] In connection with this we examined the Wigner condition[16] for the rate of decrease of the S-wave scattering phase shifts, viz.

$$\frac{d\delta}{dk} \geq -R + \frac{\sin 2(kR+\delta)}{2k}, \tag{26}$$

where R is the range of the interaction. The minimum range of interaction consistent with the rate of decrease of the phase shifts shown in Fig. 1 is 4.7 fm. This clearly exceeds the range of the interaction actually used, indicating a violation of the Wigner condition or equivalently a violation of causality.

Källén and Pauli[17] have investigated a Lee model which upon renormalization yielded an imaginary coupling parameter. This led to an additional "ghost" bound state with a negative probability. The model that we (and Bhasin and Gupta[3]) are studying is similar to the Lee model except for the additional interaction term. One might conjecture the existence of the ghost state in this model with an imaginary g. However, the additional zeros of the Fredholm determinant were not found at real negative energies, but rather at complex energies.

With the same form factors but now with a positive value of g^2, it is possible to fit the $^3S_1$ phase shifts and the deuteron binding energy. A reasonably good description of the scattering can then be obtained up to a laboratory energy of 500 MeV (see Fig. 2). However, as shown in Fig. 3, at larger energies the phase shift shows a sudden rise resulting in a resonance which is not found experimentally.[18] In this case the elementary-particle component of the deuteron, i.e., $|c_1|^2$, is high at 34%. This feature can be ascribed to the fact that the hadronic part of the interaction is repulsive, and the binding is due to the coupling of the NN system to the elementary-particle state.

The value of b obtained from P-matrix calculations and from fits with realistic interactions[4,5] ranges from 1 to 1.5 fm. Our value is lower; it is like that of Ref. 3. Jaffe and Low[13] related b to the radius of the quark bag. Since there are different versions of the bag model whose radii are not all the same, b is not unambiguously deter-

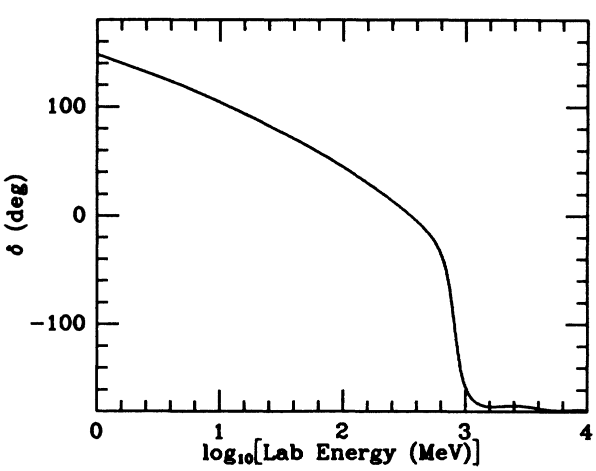

FIG. 1. $^3S_1$ phase shifts as a function of the logarithm of the laboratory energy obtained by using the parameters of Bhasin and Gupta (Ref. 3), i.e., $b=0.647$ fm, $\beta=457$ MeV, $m_1-2m_N=411$ MeV, $G=82\,720$ MeV^2, and $g^2=-44.06$ MeV^3.

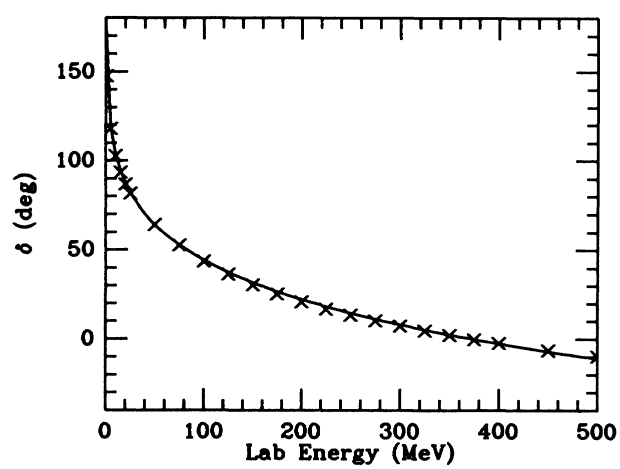

FIG. 2. $^3S_1$ phase shifts calculated with parameters $b=0.800$ fm, $\beta=244$ MeV, $m_1-2m_N=205$ MeV, $G=-7.289\times10^4$ MeV^2, and $g^2=153.5$ MeV^3. The crosses represent the experimental phases from Ref. 18. The scattering length is 5.30 fm and the effective range is 1.63 fm. The bound-state energy is 2.225 MeV with an elementary-particle component probability of 34%.

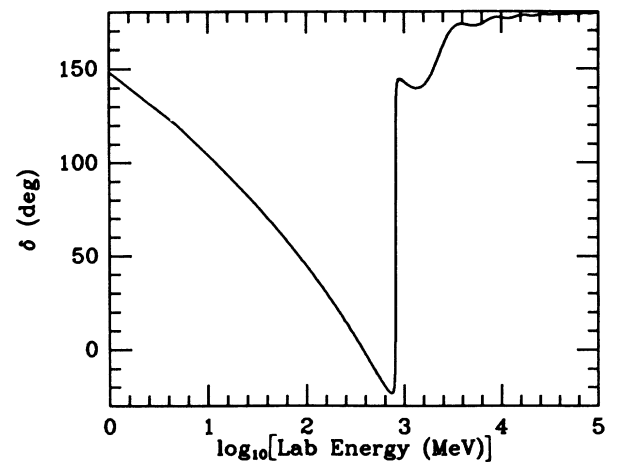

FIG. 3. $^3S_1$ phase shifts as a function of the logarithm of the laboratory energy with the same parameters of the interaction as found in the caption of Fig. 2.

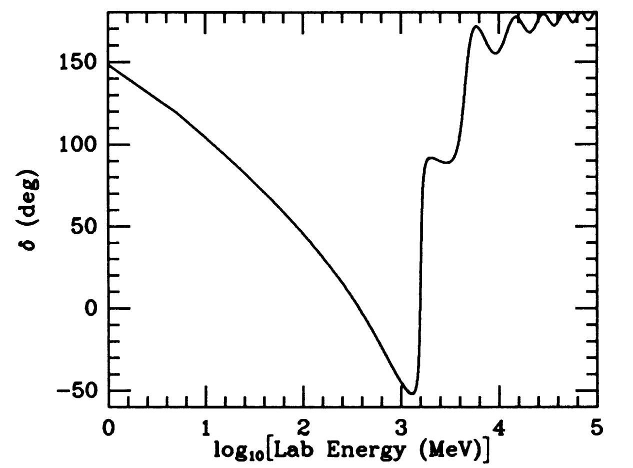

FIG. 4. $^3S_1$ phase shifts for the interaction with a square barrier calculated with parameters $b=0.600$ fm, $a=0.100$ fm, $\beta=316$ MeV, $m_1-2m_N=300$ MeV, $G=-3.236\times10^5$ MeV^2, and $g^2=362.4$ MeV^3. In this case the scattering length is 5.30 fm and the effective range is 1.63 fm. The bound-state energy is 2.228 MeV with an elementary-particle component probability of 25%.

mined.[19] In all our calculations the results are rather insensitive to the precise value of m_1.

In order to increase the flexibility of the interaction to describe the two-nucleon system and to avoid a singular surface interaction, we choose instead of the δ-function interaction a narrow square barrier of width $2a$. Then the form factor corresponding to a transition from hadronic to elementary-particle state in coordinate space is,

$$\tilde{u}_1(r)=\frac{1}{2a\sqrt{\pi}r}\theta(b+a-r)\theta(r-b+a) , \tag{27}$$

where $\theta(x)$ is the usual theta function. In momentum space the form factor is

$$u_1(k)=\frac{1}{2\pi\sqrt{2}ak^2}[\cos k(b-a)-\cos k(b+a)] . \tag{28}$$

The integrals can still be evaluated analytically. This form factor represents a smearing of the surface interaction. The surface width parameter a introduces greater freedom in describing the scattering and consequently it is possible to push the resonance to higher energies. The set of parameters given in the caption of Fig. 4 yield phase shifts that describe qualitatively the triplet S-wave elastic scattering up to 1000 MeV laboratory energy, but eventually they become positive again and approach 180°; they pass through 90° at around 1500 MeV as shown in Fig. 4. The wiggles seen at high energy in Fig. 4 are due to the sharp boundaries of the surface smearing function. The elementary-particle component of the deuteron given by this interaction is 25%. In the numerical calculations the scattering length and effective range fix the two parameters specifying the strengths of the interaction. The value of a needs to be nonzero in order not to have a resonance below 1000 MeV, leaving parameters b, m_1, and β to be adjusted so that the phase shift is zero at the appropriate energy. The last three parameters relate to the range of the surface interaction, the energy of the elementary particle, and the range of the hadronic force. Because of the simplifying assumptions made in our model we cannot interpret the values obtained strictly in terms of fundamental physical processes. Nevertheless the values of b and m_1 are consistent with those obtained in different calculations.[3,11]

In the calculation on the singlet S-wave scattering by Fasano and Lee,[5] a similar energy dependence of the phase shifts was observed when the hadron-elementary-particle coupling was reduced in the absence of an interaction term depending linearly on the energy. In their case the unobserved "dibaryon resonance" was eliminated by introducing an energy dependent term in the interaction.

In a previous calculation[6] we demonstrated that, contrary to an assertion of Weinberg,[20] one cannot conclude from the values of the low-energy scattering parameters and the smallness of the deuteron binding energy that the deuteron is a composite system or even nearly so. However, in that calculation when the phase shifts were constrained to change sign at some energy between 300 and 400 MeV, the elementary-particle state probability became small again. Our present work shows that by choosing a (smeared) surface-type nucleon-elementary-particle–type interaction this reduction of the elementary-particle-state probability need not occur. The elementary-particle component of the deuteron can be high for interactions giving a reasonable $^3S_1$ elastic phase shift up to 1000 MeV. It is not meaningful to consider much higher energies since then the nonrelativistic nuclear potential model breaks down and inelastic processes and relativistic effects begin to grow in importance. It is well known that the scattering and bound-state data of the $^3S_1$ wave can be described by interactions that do not include any elementary particle component. Such potentials can be obtained by fitting the data to parametrized forms of the interaction or by using procedures of the inverse scattering problem. This means that the elementary-particle component of the deuteron cannot be determined from the on-shell behavior of the nuclear T matrix at low energy[20] or at any energy of relevance in nonrelativistic nuclear potential models. It is a property that depends on the off-shell rather than on-shell characteristics of the T matrix in this energy range.

A recent analysis of the deuteron root-mean-square radius and its value predicted by nonrelativistic potential models[21] shows that such potential models are unable to explain the experimental rms radius simultaneously with the low-energy scattering parameters of the neutron-proton data. In fact, for the correct scattering length, the predicted rms radius turns out to be too large. Calculation of the rms radius of the deuteron with an elementary component will involve knowing the precise nature of this component, e.g., confinement radius, quark wave functions. However, since six-quark eigenstates are expected to have a radius that is smaller than the deuteron radius, the deuteron rms radius would be reduced when such a state is included as a component of the deuteron wave function. Kondratyuk *et al.*[22] claim that the deuteron radius cannot be modified noticeably by a substantial admixture of a six-quark state. They have considered however a case in which the nucleon-nucleon wave function is highly suppressed at small distances, whereas Fasano and Lee[5] show that the hadronic wave function need not be suppressed at energies far from the elementary-state energy.

IV. CONCLUSION

Using the modified Lee model one can derive simple nucleon-nucleon interactions such as proposed by Bhasin and Gupta[3] for which there are analytic solutions. However there are restrictions on the choice of parameters in the effective potential which is used in the Schrödinger equation. Parameters which lead to a non-Hermitian underlying Hamiltonian result in unphysical behavior in terms of causality violation.

From the examples with acceptable parameters it is clear that quark degrees of freedom in the form of six-quark or other multiquark eigenstates as simulated by the elementary particle states, will not be discerned by considering on-shell properties of the T matrix. This aspect of the nuclear force can be investigated only by using properties that depend on the details of the wave functions rather than their asymptotic form.

It has not been our intention to give a detailed fit to the scattering data but rather to use this analytically solvable model to discuss certain features of the nucleon-nucleon interaction. The interaction used here acts over the short and intermediate range of the nuclear force. The complexity of the potential can be increased by using a higher rank separable potential and including a greater number of elementary particle states or by adding an OPEP tail and a tensor component to describe the two-nucleon system accurately. One loses then the simplicity of algebraic solutions of the Schrödinger equation. Such modifications will not alter the conclusions of this paper. Of interest for future work is to study such a "realistic" nucleon-nucleon interaction with a specified form of the six-quark state in order to investigate the effect on the calculated rms radius of the deuteron, as well as on the magnetic moment and magnetic form factor of the deuteron which appear to be sensitive to the elementary-particle admixture in the deuteron.[4,22]

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