Damping of quadrupole motion in time-dependent density-matrix theory

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The time-dependent density-matrix theory which incorporates two-body collision effects into the mean-field theory is applied to the isoscalar quadrupole motion of ^{16}O. The collision term in the theory includes higher-order terms as well as the Born term. It is found that higher-order correlations are essential for the damping of the motion.

We have recently proposed the time-dependent density-matrix (TDDM) theory.[1,2] This is a straightforward extension of the time-dependent Hartree-Fock (TDHF) theory to include the effects of two-body collisions. We made the first application of TDDM to fusion reactions of $^{16}O+^{16}O$ (Ref. 2) and found that an additional dissipation due to two-body collisions resolved the fusion window anomaly in TDHF.[3] Although TDDM is constructed to describe large amplitude collective motions, we also applied it to small amplitude motions of ^{16}O (Ref. 4) to test the theory. We found that TDDM did not bring about the damping of the isoscalar quadrupole motion. In this paper we show that higher-order correlations, which are neglected in TDDM, drastically change the damping rate.

First we present TDDM which is generalized to include higher-order terms. The new TDDM consists of three coupled equations. The first equation determines the single-particle (SP) representation. The most convenient SP basis ψ_λ may be the solution of a TDHF-like equation

$$i\hbar\frac{\partial}{\partial t}\psi_\lambda(\mathbf{r},t)=h\psi_\lambda(\mathbf{r},t)=\left[\frac{-\hbar^2}{2M}\nabla^2+U(\rho)\right]\psi_\lambda(\mathbf{r},t)\ , \qquad (1)$$

where U is the self-consistent mean field and is a functional of the one-body density matrix

$$\rho(\mathbf{r},\mathbf{r}';t)=\sum_{\lambda\lambda'}n_{\lambda\lambda'}(t)\psi_\lambda(\mathbf{r},t)\psi^*_{\lambda'}(\mathbf{r}',t)\ . \qquad (2)$$

The second equation is for the occupation matrix $n_{\lambda\lambda'}$, of which equation of motion is directly related to the correlated part of the two-body Green's function in the equal-time limit[5,6] or the two-body density matrix.[7] In the SP representation given by Eq. (1), the equation of motion for $n_{\lambda\lambda'}$ becomes[5,7]

$$\frac{d}{dt}n_{\lambda\lambda'}(t)=\frac{i}{\hbar}\sum_{\alpha\beta\gamma}[\langle\lambda\gamma\,|\,v\,|\,\alpha\beta\rangle C_{\alpha\beta\lambda'\gamma}(t)-\langle\alpha\beta\,|\,v\,|\,\lambda'\gamma\rangle C_{\lambda\gamma\alpha\beta}(t)]\ , \qquad (3)$$

where v is the residual interaction and $C_{\alpha\beta\gamma\delta}$ is the correlated part of the two-body density matrix. The last equation of motion in TDDM gives the time evolution of $C_{\alpha\beta\gamma\delta}$. The equation of motion for the two-body density matrix, in general, contains a three-body density matrix as well as the one-body density matrix. To close the equation of motion the three-body density matrix is usually replaced by a product of the two-body density matrix and the one-body matrix. A full presentation of the equation for the two-body density matrix in coordinate space may be seen in Ref. 7. The equation motion for the two-body density matrix thus obtained contains several terms which represent various two-particle correlations. We take the correlations of two types, i.e., the first-order particle-particle correlation and the higher-order particle-hole (p-h) correlation. The former is included in the original TDDM and the latter, which is responsible for redistribution of 2p-2h level density, is the effect which we investigate in this paper. The equation of motion for the two-body density matrix is written as

$$\begin{aligned}i\hbar\frac{d}{dt}C_{\alpha\beta\gamma\delta}(t)=&\sum_{\alpha'\beta'\gamma'\delta'}\langle\alpha'\beta'\,|\,v\,|\,\gamma'\delta'\rangle\{[\delta_{\alpha\alpha'}-n_{\alpha\alpha'}(t)][\delta_{\beta\beta'}-n_{\beta\beta'}(t)]n_{\gamma'\gamma}(t)n_{\delta'\delta}(t)\\&-n_{\alpha\alpha'}(t)n_{\beta\beta'}(t)[\delta_{\gamma\gamma'}-n_{\gamma'\gamma}(t)][\delta_{\delta\delta'}-n_{\delta'\delta}(t)]\}\\&+\sum_{\alpha'\beta'\gamma'}(\langle\alpha\alpha'\,|\,v\,|\,\beta'\gamma'\rangle n_{\beta'\gamma}-\langle\beta'\alpha'\,|\,v\,|\,\gamma\gamma'\rangle n_{\alpha\beta'})C_{\beta\gamma'\delta\alpha'}\\&+\sum_{\alpha'\beta'\gamma'}(\langle\beta\alpha'\,|\,v\,|\,\beta'\gamma'\rangle n_{\beta'\delta}-\langle\beta'\alpha'\,|\,v\,|\,\delta\gamma'\rangle n_{\beta\beta'})C_{\alpha\gamma'\gamma\alpha'}\ .\end{aligned} \qquad (4)$$

For simplicity we neglect exchange terms. The first term on the right side is the Born term and Eqs. (1), (3), and (4) with this term are equivalent to the original TDDM. The second and third terms on the right-hand side of Eq. (4) represent higher-order *p-h* correlations among 2p-2h configurations. The TDDM equations (1), (3), and (4) conserve the total number of particles and the total energy consisting of the Hartree-Fock (HF) energy and the correlation energy:[2] the correlation energy $E_{\rm cor}$ is given in terms of the two-body density matrix as

$$E_{\rm cor}=-\frac{i}{2\hbar}\sum_{\alpha\beta\gamma\delta}\langle\alpha\beta|v|\gamma\delta\rangle C_{\gamma\delta\alpha\beta}(t)\ . \tag{5}$$

We study the isoscalar quadrupole motion of ^{16}O. It is excited by squeezing initially ($t=0$) the Hartree-Fock solution ϕ_λ with the quadrupole field as

$$\psi_\lambda(\mathbf{r},t=0)=e^{i\alpha r^2Y_{20}(\theta)}\phi_\lambda(\mathbf{r})\ , \tag{6}$$

where α is a parameter determining the amplitude of the oscillation. We use the TDHF code with axial symmetry.[8] The mean potential is calculated with the Bonche-Koonin-Negele force[9] and the SP states are taken up to the 2*s*-1*d* shell. The 1*s* and 1*p* states are assumed to be initially completely occupied and other states totally empty. The two-body density matrix $C_{\alpha\beta\gamma\delta}$ is assumed to be zero at $t=0$. We use a residual interaction of the δ function form $v=v_0\delta^3(\mathbf{r}-\mathbf{r}')$ with $v_0=-300$ MeV fm^3. The residual interaction gives the *NN* cross section of about 40 mb in the Born approximation. The strength α in Eq. (6) is adjusted to give the mean excitation energy of 22 MeV, which is close to the empirical excitation energy of the giant quadrupole resonance.[10] The amplitude of the motion depends on α but its time dependence is independent of α unless α is very large.

The time evolution of the quadrupole moments is shown in Fig. 1. We compare three different calculations. The dashed curve denotes the TDHF result. The quadrupole moment in TDHF oscillates with a frequency corresponding to the excitation energy of about 20 MeV and the amplitude of the oscillation slightly decays, probably due to particle emission. The dotted-dashed curve (referred to as TDDM') shows the TDDM calculation only with the Born term in Eq. (4). This calculation was done with SP states up to the 2*p*-1*f* shell. The frequency in TDDM' is lowered due to the residual interaction, and that the amplitude of the oscillation in TDDM' is slightly larger than that in TDHF is due to an increase in the HF energy associated with the decrease in the correlation energy.[2] As can be seen in Fig. 1, TDDM' brings about no significant damping of the motion, contrary to an expectation from experiment[10] suggesting the lifetime of about 10^{-22} sec. The result of the full TDDM calculation is shown in Fig. 1 by a solid curve. The inclusion of the *p-h* correlations drastically increases the damping rate of the motion, in qualitative agreement with the work of Bertsch, Bortignon, and Broglia.[11] The relaxation time in TDDM extracted from the figure is about 3×10^{-22} sec. This time corresponds to $\Gamma\approx4$ MeV. The correlations play an important role in redistributing the level density of 2p-2h configurations as was the case in shell-model calculations.[12] As a result of these correlations some of the 2p-2h configurations are shifted to the low-frequency region, increasing the level density around the frequency of the isoscalar motion.

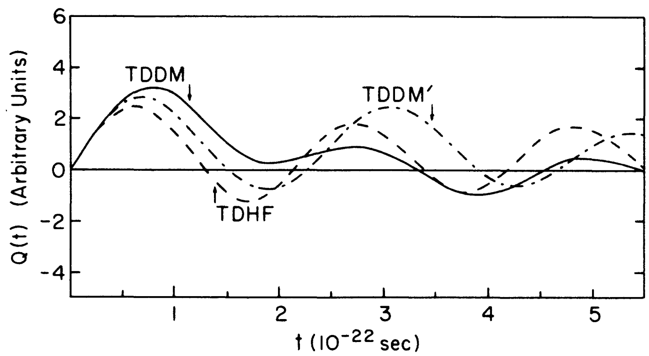

FIG. 1. Time evolution of the isoscalar quadrupole moments calculated in TDHF (dashed curve), in TDDM (solid curve), and in TDDM' (dotted-dashed curve), which includes only the Born term.

In summary, we studied the damping of the isoscalar quadrupole motion of ^{16}O in TDDM which incorporates particle-collision effects into the mean-field theory. It is found that the correlations among 2p-2h configurations are important to describe the damping of the motion. It is interesting to study the effects of these higher-order correlations on dissipations and fluctuations in heavy-ion collisions.

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