Spin determination from the angular distributions of identical colliding nuclei

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(Received 22 February 1988)

We are concerned here with collisions involving two identical nuclei either in the entrance or in the exit channels (or in both) for which the differential cross section reads $d\sigma/d\Omega \propto |f(\theta)|^2 + |f(\pi-\theta)|^2 + 2\alpha\,\mathrm{Re}[f(\theta)f^*(\pi-\theta)]$. We propose a simple graphical method for obtaining the parameter α directly from the measured angular distribution.

Consider a collision involving two identical particles either in the entrance or in the exit channels (or in both) for which the angular dependence of the differential cross section reads

$$\frac{d\sigma}{d\Omega} \propto |f(\theta)|^2 + |f(\pi-\theta)|^2 + 2\alpha|f(\theta)|\,|f(\pi-\theta)|\cos\varphi(\theta) \; . \tag{1}$$

In (1), $\varphi(\theta)$ is the phase difference between the forward and the backward reaction amplitudes, and α is a parameter determined from the symmetrization procedure, according to the Pauli relation between spin and statistics.

Equation (1) applies to various collision processes:

(i) elastic scattering of identical spinless particles, or of nonzero spin particles if spin-dependent forces are neglected;

$$\alpha = (-)^{2s}/(2s+1) = 1, -\tfrac{1}{2}, \tfrac{1}{3}, \ldots , \tag{2}$$

(ii) inelastic and transfer reactions involving identical nuclei under particular conditions. The parameter α is then related to the angular momentum transferred (see the following).

The aim of the present Brief Report is to propose a simple graphical method for obtaining the parameter α directly from the measured angular distributions, i.e., in a model-independent way.

To achieve this we shall use the "envelope method,"[1] which has proved accurate in decomposing oscillatory angular distributions into interfering components.

Let us consider a collision experiment involving two identical particles and giving rise to an angular distribution with well-defined oscillations resulting from the interference between the forward and the backward amplitudes (see Fig. 1). Equation (1) tells us that $d\sigma/d\Omega$ oscillates between two limit curves or "envelopes" $E_\pm$, defined by

$$E_\pm(\theta) = |f(\theta)|^2 + |f(\pi-\theta)|^2 \pm 2|\alpha|\,|f(\theta)|\,|f(\pi-\theta)| \; . \tag{3}$$

Solving (3) for $|f(\theta)|^2$ and $|f(\pi-\theta)|^2$ one readily gets

$$\frac{E_+(\theta)-E_-(\theta)}{E_+(\theta)+E_-(\theta)} \begin{cases} = |\alpha| \;\text{ if }\; |f(\theta)|^2 = |f(\pi-\theta)|^2 \, , & \text{(4a)} \\ < |\alpha| \;\text{ if }\; |f(\theta)|^2 \neq |f(\pi-\theta)|^2 \, . & \text{(4b)} \end{cases}$$

Furthermore, $|\alpha| \le 1$.

To apply this result to the determination of $|\alpha|$, one should be able to draw the limit curves $E_\pm$ from the experimental data. There is no unique and rigorous way to do so. However, a reasonable criterion for drawing such curves is to assume that $|f(\theta)|^2$ is smooth enough so that the stationary points of $d\sigma/d\Omega$ lie very close to the maxima and minima of $\cos\varphi(\theta)$. Then the "envelope" E_+ (E_-) should be the "simplest" smooth curve passing through (or very close to) the maxima (minima) of the experimental angular distribution. The smoothness condition for $|f(\theta)|^2$ is well satisfied in nucleus-nucleus collisions (at least in a broad angular range around $\theta = 90°$) when the Sommerfeld parameter is large, $\eta = (Z_1Z_2e^2/\hbar v) \gg 1$.

To illustrate the applications of the method, one first considers the elastic scattering of ^{24}Mg on ^{24}Mg ($E_c = 17.37$ MeV).[2] As α is well known ($\alpha = 1$), this example also provides a test of the method.

Figure 1 shows the experimental data as well as the interpolated "envelopes" $E_\pm$. To obtain $|\alpha|$ from Eq. (4a), one should determine E_+ and E_- for $\theta = 90°$ where the two interfering amplitudes become identical. However, as seen in the diagram the lower "envelope" cannot be

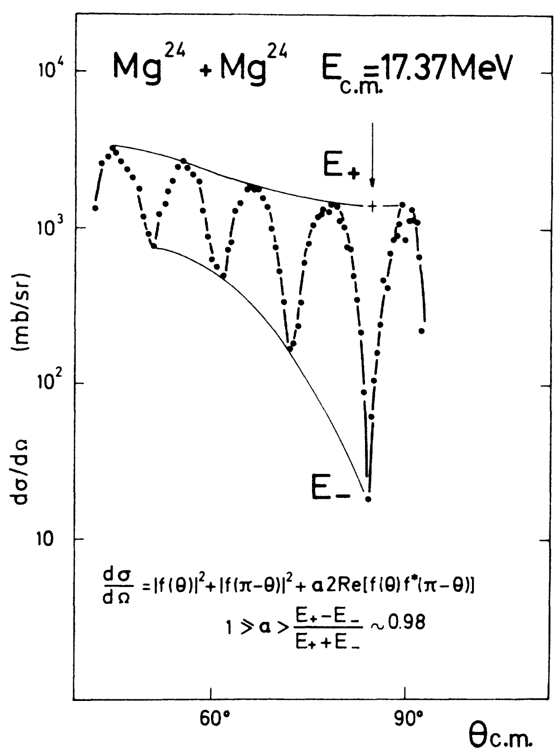

FIG. 1. The points are the experimental data of the elastic scattering of ^{24}Mg on ^{24}Mg, $E_c = 17.37$ MeV (from Ref. 2). The curve through the points is to guide the eye. Also shown the "envelopes" of the oscillatory pattern, $E_\pm$.

drawn for $\theta = 90°$. In such cases one should take the $E_\pm(\theta)$ values for the angle nearest to 90° for which both curves E_+ and E_- can be accurately drawn. In fact as long as θ remains close to 90°, one has $|f(\theta)|^2 \sim |f(\pi-\theta)|^2$ and (4a) holds approximately. So, for $\theta \sim 90°$ and taking into account (4b), one has

$$|\alpha| \gtrsim \frac{E_+(\theta)-E_-(\theta)}{E_+(\theta)+E_-(\theta)} . \tag{5}$$

With this criterion in mind one takes the values of E_+ and E_- for the angle corresponding to the minimum indicated by the arrow in the diagram.

One obtains $(E_+ - E_-)/(E_+ + E_-) \simeq 0.98$, and (5) gives

$$|\alpha| \gtrsim 0.98 . \tag{6}$$

From (2), the only possible value of α compatible with (6) is $\alpha = 1$, i.e., $s = 0$.

Consider now the reaction ^{14}N(^{14}N,^{13}N)^{15}N ($E_c = 9$ MeV) (Ref. 3) in which a neutron is transferred from a $p_{1/2}$ ($l_1 = 1, j_1 = \frac{1}{2}$) state in ^{14}N to a $p_{1/2}$ ($l_2 = 1, j_2 = \frac{1}{2}$) state in ^{15}N. The angular momentum transferred is restricted by the usual triangular rules,

$$|l_1 - l_2| \le l \le l_1 + l_2, \quad |j_1 - j_2| \le l \le j_1 + j_2 ,$$

which allow two values $l = 0,1$. However, if recoil effects are ignored, i.e., if the ratios of the neutron mass to that of the ^{14}N and ^{15}N nuclei are neglected,[4] an additional rule emerges: $l_1 + l_2 + l = \text{even}$. In that case, the angular dependence of the cross section is given, within the DWBA, by (see Ref. 4 for details):

$$\frac{d\sigma}{d\Omega} \propto \sum_{l\lambda} (j_1 \tfrac{1}{2}, l0 \,|\, j_2 \tfrac{1}{2})^2 \left[|T_{l\lambda}(\theta)|^2 + |T_{l\lambda}(\pi-\theta)|^2 \pm \hat{j}_1^2 \hat{j}_2^2 \begin{Bmatrix} l & j_1 & j_2 \\ j_1 & c_1 & c_2 \\ j_2 & a_1 & a_2 \end{Bmatrix} 2\,\mathrm{Re}\{T_{l\lambda}(\theta) T^*_{l\lambda}(\pi-\theta)\} \right] , \tag{7}$$

where ($a_1 \equiv a_2$), c_1, and c_2 are the spin of the ^{14}N, ^{13}N and ^{15}N nuclei, respectively. So, for nonrecoil, only $l = 0$ is allowed and (7) reduces to (1) with $\alpha = -\frac{1}{3}$.

There are many heavy ion transfer reactions involving identical nuclei for which Eq. (1) holds. This occurs either because $l = 0$ is the only possible value, or because other allowed values of l are strongly inhibited. Examples of these are as follows:[5]

$$^{30}\mathrm{Si}(^{28}\mathrm{Si},^{29}\mathrm{Si})^{29}\mathrm{Si}, \quad s_{1/2}(l_1 = 0, j_1 = \tfrac{1}{2}) \to s_{1/2}(l_2 = 0, j_2 = \tfrac{1}{2}), \quad l = 0 .$$

Equation (1) holds with $\alpha = +1$.

^{15}C(^{16}O,^{15}N)^{15}N and ^{13}C(^{13}C,^{12}C)^{14}C

(Refs. 6 and 7),

$$p_{1/2}(l_1 = 1, j_1 = \tfrac{1}{2}) \to p_{1/2}(l_2 = 1, j_2 = \tfrac{1}{2}), \quad l = 0,1 .$$

If $l = 1$ is inhibited, Eq. (1) holds with $\alpha = +1$.

Now, what can be learned from the angular distribution of Fig. 2, using the relations (4)?

We note first that, as in the preceding example, there is one "envelope" (E_+) which cannot be accurately extrapolated for $\theta = 90°$. We shall again take the angle nearest to 90° for which both curves E_+ and E_- can be drawn.

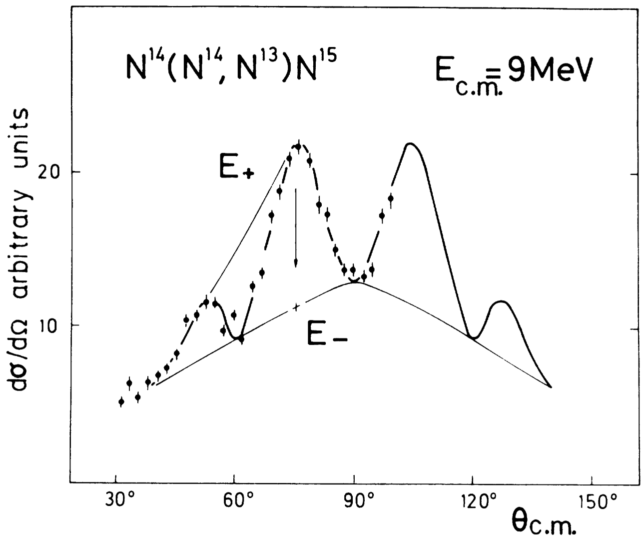

FIG. 2. Same as for Fig. 1 for the one-neutron transfer reaction $^{14}N(^{14}N,^{13}N)^{15}N$, $E_c=9$ MeV (from Ref. 3).

This angle is indicated by the arrow in the diagram. One obtains $(E_+-E_-)/(E_++E_-)\simeq 0.33$ and (5) gives

$$|\alpha| \gtrsim 0.33 .$$

Furthermore, since $d\sigma/d\Omega$ is minimum for $\theta=90°$, α must be negative.

One may wonder whether the results obtained by the "envelope method" are reproducible. To answer this question six people were asked to draw (for different angular distributions) the limit curves, and to read, for several angles, the corresponding $E\pm(\theta)$ values. The mean deviation found in this "draw and read experiment" was less than about 6%.

This makes us confident in the results obtained. Furthermore, in the present case the value $|\alpha| \lesssim 0.33$ is a clear indication that the $l=1$ component, although allowed, is strongly inhibited.

Concerning the practical applications of the method, it should be pointed out that in general for $\theta=90°$ one cannot accurately interpolate both curves E_+ and E_-. Therefore since the angle which is used to determine the ratio $(E_+-E_-)/(E_++E_-)$ should be as close as possible to 90°, the method works better if the period $\Delta\theta$ of the oscillatory pattern is small, particularly near 90°. In the case of nuclear-nucleus collisions under the condition $\eta \gg 1$, this period is roughly given[8] by $\Delta\theta \sim \pi/\eta$, for $\theta \sim 90°$.

It is a pleasure to acknowledge fruitful discussions with Dr. S. Klarsfeld.

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