Cluster-orbital shell model and its application to the He isotopes

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A model is proposed in which we take full advantage of both the shell and cluster models in order to describe a system of valence nucleons weakly coupled to a core. This is realized by introducing radius vectors of the valence nucleons relative to the core, new coordinates of independent particle nature. The model is applied to the neutron-rich He isotopes, $^{6-8}$He, with reasonable success.

Recently, the interaction cross section has been measured by making use of exotic isotope beams produced in the projectile fragmentation of high-energy heavy ion collisions.[1,2] A Glauber-type analysis[1,2] of the interaction cross section has led to a determination of the root-mean-square radii of unstable nuclei and demonstrated that some of the neutron-rich nuclei in the vicinity of the neutron drip line have anomalously large rms radii. These observations have raised a number of questions challenging the knowledge of nuclear structure which has been established on the basis of the properties of nuclei of normal density: These include the possible existence of a very weakly bound neutron halo,[3] the significance of the magic numbers in extremely neutron-rich nuclei, e.g., a "doubly magic" nucleus ^{10}He, and the density dependence of the nuclear interaction.

An attempt at understanding the structure of neutron-rich nuclei has been undertaken in shell model[4] and Hartree-Fock like[5] approaches. The purpose of this paper is to propose a cluster-orbital shell model which takes full advantage of both the shell and cluster models and to apply it to a structure study of the He isotopes. The model we propose does not require the heavy computational effort of the no core shell model,[4] and we believe that it gives a clear insight into the physics involved.

In neutron-rich nuclei like the He isotopes, $^{6-8}$He, we assume that weakly bound neutrons move around a stable core cluster. Let n and f be the number of valence neutrons and the mass number of the fragment core cluster, respectively, and $\mathbf{r}_1,\ldots,\mathbf{r}_n,\mathbf{r}_{n+1},\ldots,\mathbf{r}_{n+f}$ be the position vectors of the nucleons. The normalized center-of-mass coordinate of the core cluster, $\mathbf{R}_C$, is defined by

$$\mathbf{R}_C=(\mathbf{r}_{n+1}\cdots+\mathbf{r}_{n+f})/\sqrt{f}\ .$$

We introduce a system of coordinates which suits the cluster-orbital shell model:

$$\begin{aligned}\boldsymbol{\xi}_i&=\left[\frac{f}{f+1}\right]^{1/2}\left[\mathbf{r}_i-\frac{\mathbf{R}_C}{\sqrt{f}}\right]\quad(i=1,\ldots,n)\ ,\\ \mathbf{R}&=(\mathbf{r}_1+\cdots+\mathbf{r}_{n+f})/\sqrt{n+f}\\ &=(\mathbf{r}_1+\cdots+\mathbf{r}_n)/\sqrt{n+f}+\left[\frac{f}{n+f}\right]^{1/2}\mathbf{R}_C\ ,\end{aligned}\tag{1}$$

where $\boldsymbol{\xi}_i$ is the normalized radius vector from the center-of-mass of the core to the ith valence neutron and $\mathbf{R}$ the total center-of-mass coordinate. Using these coordinates enables us to express the total Hamiltonian as

$$H=H_C+\sum_{i=1}^{n}\left[\frac{1}{2m}\mathbf{p}_i^2+U_i\right]+\sum_{i<j}^{n}\left[v_{ij}+\frac{1}{f+1}\frac{1}{m}\mathbf{p}_i\cdot\mathbf{p}_j\right]\ ,\tag{2}$$

where H_C is the core Hamiltonian and $\mathbf{p}_j=-i\hbar\partial/\partial\boldsymbol{\xi}_j$ the momentum conjugate to $\boldsymbol{\xi}_j$. The interaction potential v_{ij} acts between the valence neutrons and its radial dependence is given by

$$\mathbf{r}_i-\mathbf{r}_j=\left[\frac{f+1}{f}\right]^{1/2}(\boldsymbol{\xi}_i-\boldsymbol{\xi}_j)\ .$$

The U_i denotes the interaction potential between the valence neutron and the core. It would in general be nonlocal and energy dependent. Although it is possible to use a nonlocal potential in our model, we assume a local, energy-independent potential U_i depending on $\boldsymbol{\xi}_i$. It is easy to show that the total angular momentum of the system, $\mathbf{J}$, is expressed in terms of the new coordinates as

$$\mathbf{J}=\mathbf{J}_C+\sum_{i=1}^{n}(\boldsymbol{l}_i+\mathbf{s}_i)\ ,\tag{3}$$

where $\mathbf{J}_C$ is the angular momentum of the core and $\boldsymbol{l}_i=\boldsymbol{\xi}_i\times\mathbf{p}_i$ the orbital angular momentum of the ith valence neutron. The difference of our model from the usual shell model approach is rather conceptual: The latter assumes the self-consistent Hartree-Fock field $U_{\rm HF}$ generated from all the nucleons and treats residual interactions perturbatively. In our model the single-particle Hamiltonian, $h=1/2m\,\mathbf{p}^2+U$, is relevant to the relative motion of the valence particle around the core and can be used to define cluster orbitals for valence particles, and the interaction potential v is not a residual interaction but considered a realistic effective interaction acting between the valence particles. One of the advantages of our model over the shell model is that any excitation of the valence neutrons does not lead to spurious center-of-mass excitations. The terms $\mathbf{p}_i\cdot\mathbf{p}_j$ of Eq. (2) appear because the coordinate transformation from $\mathbf{r}_i$, $\mathbf{R}_C$

to ξ_i, $\mathbf{R}$ is not orthogonal. This nonorthogonality imposes the correct normalization of wave functions as

$$\int |\psi(\xi_1,\ldots,\xi_n)|^2 d\xi_1 \ldots d\xi_n = \left| \frac{\partial(\mathbf{r}_1,\ldots,\mathbf{r}_n,\mathbf{R}_C)}{\partial(\xi_1,\ldots,\xi_n,\mathbf{R})} \right| = \left[\frac{n+f}{f} \right]^{3/2} \left[\frac{f}{f+1} \right]^{3n/2} . \quad (4)$$

We assume ^{4}He as a core in applying the cluster-orbital shell model to the He isotopes. The neutron- (and proton-) ^{4}He interaction was extensively studied by the cluster model. In particular, Kanada *et al.*[6] nicely reproduced the s- and p-wave phase shifts at low energies by a resonating group method (RGM) calculation with the use of the Hasegawa-Nagata (HN) potential.[7] The HN potential is a realistic effective potential determined from a reaction matrix calculation for ^{4}He, ^{6}Li and nuclear matter and includes the spin-orbit and tensor components as well as the central part. (See Table I of Ref. 6 for details). Based on their RGM calculation, Kanada *et al.* further determined a parity-dependent, local nucleon-^{4}He potential consisting of central and spin-orbit parts [Eq. (10) of Ref. 6]. This potential can be regarded as including the exchange effects between the nucleon and the nucleons in the ^{4}He cluster. We employ this potential as U_i. For v_{ij} we use the same HN potential consistent with the calculation of Ref. 6. Since the valence neutron is not allowed to take the same $s_{1/2}$ orbit as that in the ^{4}He cluster and thus, the potential U becomes most attractive in the $p_{3/2}$ orbit ($p_{3/2}$=the quantum number of $\mathbf{j}=\mathbf{l}+\mathbf{s}$), we assume the simplest configuration $(j=p_{3/2})^n$ for $^{4+n}$He ($n=2,3,4$). Assuming the resultant angular momentum to be 0, $\frac{3}{2}$, and 0 for $n=2$, 3, and 4, use of coefficients of fractional parentage[8] leads to the energies of the He isotopes relative to the core, $\Delta E(n)=E(^{4+n}\text{He})-E(^4\text{He})$, as

$$\Delta E(2)=2\varepsilon_{3/2}+V_0, \quad \Delta E(3)=3\varepsilon_{3/2}+\tfrac{1}{2}V_0+\tfrac{5}{2}V_2 , \quad \Delta E(4)=4\varepsilon_{3/2}+V_0+5V_2 , \quad (5)$$

where $\varepsilon_j=\langle jM \mid h \mid jM\rangle$ is the single-particle energy of the cluster-orbital j, and $V_J=\langle j^2JM \mid v \mid j^2JM\rangle$ the two-particle interaction matrix elements. Note that the $\mathbf{p}_i\cdot\mathbf{p}_j$ terms make no contribution to the energy, provided that we take into account orbitals of the same parity only. If the $p_{3/2}$ orbits of the He isotopes are all identical, the observed values of $\Delta E(n)$, listed in Table I, can simultaneously be reproduced by the following:

$$\varepsilon_{3/2}=1.02 \text{ MeV}, \quad V_0=-3.02 \text{ MeV} , \quad V_2=-0.84 \text{ MeV} . \quad (6)$$

Figure 1 displays the dependence of the three quantities on radial form of the $p_{3/2}$ orbit. Here the orbit is assumed to be given by

$$\phi_a=\left[\frac{64a^5}{9\pi b^6}\right]^{1/4}\frac{r}{b}\exp\left[-\frac{a}{2}\left[\frac{r}{b}\right]^2\right], \quad (r=\sqrt{5/4}\,\xi) , \quad (7)$$

TABLE I. Energies and root-mean-square radii of the He isotopes. Results obtained with the approximate $p_{3/2}$ resonance wave function are shown in parentheses.

	Energy (MeV) $\Delta E(n)$		rms radii (fm) R^M_{rms}			R^P_{rms}			R^N_{rms}	
	present	exp.	present	HF[a]	exp.[b]	present	HF[a]	exp.[b]	present	HF[a]	exp.[b]
^{6}He($n=2$)	0.47 (0.72)	−0.975	2.40 (2.65)	2.49	2.48±0.03	1.81 (1.88)	1.80	2.21±0.03	2.65 (2.96)	2.77	2.61±0.03
^{7}He($n=3$)	1.56 (1.77)	−0.535	2.69 (2.92)			1.85 (1.91)			2.96 (3.23)		
^{8}He($n=4$)	0.52 (1.17)	−3.112	2.73 (3.11)	2.59	2.52±0.03	1.83 (1.92)	1.83	2.15±0.02	2.97 (3.42)	2.80	2.64±0.03

[a] Reference 5.
[b] Reference 2.

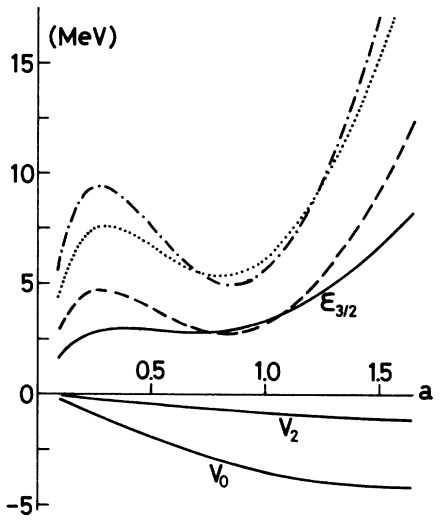

FIG. 1. Dependence of the single-particle energy and the two-body interaction matrix elements on the parameter a of the $p_{3/2}$ single-particle orbit. Also shown are the energies of the He isotopes given in Eq. (5): Dashed curve; ^{6}He, dotted curve; ^{7}He, and dash-dotted curve; ^{8}He.

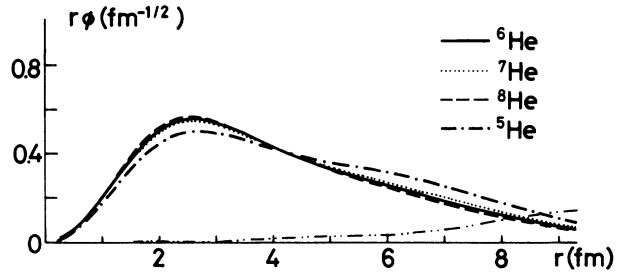

FIG. 2. The $p_{3/2}$ single-particle orbits in the He isotopes. The dash-dot-dot curve denotes the difference between the exact and approximate solutions of the $p_{3/2}$ resonance wave function of ^{5}He.

where r is the distance between the valence nucleon and the core, b, set to 1.8 fm, is introduced to scale the length and a is a parameter determining the radial extension of the orbit. The interaction matrix elements V_0 and V_2 at $a\sim 1$ are close to those required in Eq. (6) but $\varepsilon_{3/2}$ is too large. The $p_{3/2}$ orbit actually extends over a wide region because the single-particle Hamiltonian, h, does not have a bound state but has a $p_{3/2}$ resonance at 0.9 MeV. To simulate the $p_{3/2}$ resonance wave function we take a linear combination of ϕ_a's, $\phi=\sum_a C_a\phi_a$, and choose five points of a at $a=0.3$, 0.54, 0.97, 1.75, 3.15. An approximate $p_{3/2}$ orbit obtained by the best choice of C_a's is shown in Fig. 2 together with the difference from the exact resonance wave function which is obtained under the correct resonance boundary condition and normalized at the peak. The approximate $p_{3/2}$ orbit yields $\varepsilon_{3/2}=1.19$ MeV, $V_0=-1.66$ MeV, $V_2=-0.39$ MeV, and the corresponding values of $\Delta E(n)$ are listed in parentheses in Table I. Although $\varepsilon_{3/2}$ is now close to the value of Eq. (6), the interaction matrix elements are about half of the needed values.

Now we extend our calculation by choosing an optimal $p_{3/2}$ cluster orbital, ϕ, so as to minimize $\Delta E(n)$ for each n. The same five points of a are chosen and coefficients C_a are varied to reach energy minimum. Figure 2 shows the curves of ϕ obtained for the He isotopes. They are drawn inward more strongly than the resonance orbit and do not differ very much from one another. The resulting values of $\Delta E(n)$ are listed in Table I. They are underbound by 1.4 MeV, 2.1 MeV, and 3.6 MeV for $n=2$, 3, and 4, respectively. This corresponds to 30–40 % lack of the strength of the needed two-body interaction matrix elements.

The energy gain due to lifting the $p_{3/2}$ neutrons to the $p_{1/2}$ orbit is easily evaluated. For ^{6}He the admixture of the $(p_{1/2})^2$ configuration is small, i.e., we have gotten $0.991\psi((p_{3/2})^2)+0.136\psi((p_{1/2})^2)$ for ^{6}He, and the value of $\Delta E(2)$ becomes 0.36 MeV resulting in an energy gain of only 0.1 MeV. We conclude that the effect of the $p_{1/2}$ orbit and probably the $s_{1/2}$ orbit can be neglected in the He isotopes, $^{6-8}$He.

We estimate the rms radii of proton, neutron, and nucleon, $R^P_{\rm rms}$, $R^N_{\rm rms}$, and $R^M_{\rm rms}$. It is easy to derive the following equations for the He isotopes ($f=4$):

$$[R^P_{\rm rms}(^{4+n}\mathrm{He})]^2=[R^P_{\rm rms}(^4\mathrm{He})]^2+\frac{1}{(n+f)^2}\frac{f+1}{f}\left\langle\left[\sum_{i=1}^{n}\xi_i\right]^2\right\rangle, \tag{8}$$

$$[R^N_{\rm rms}(^{4+n}\mathrm{He})]^2=\frac{2}{n+2}[R^N_{\rm rms}(^4\mathrm{He})]^2+\frac{1}{n+2}\frac{f+1}{f}\left\langle\sum_{i=1}^{n}\xi_i^2\right\rangle-\frac{1}{n+f}\left[\frac{2}{n+2}-\frac{1}{n+f}\right]\frac{f+1}{f}\left\langle\left[\sum_{i=1}^{n}\xi_i\right]^2\right\rangle, \tag{9}$$

where the expectation values are calculated with the wave function of the valence neutrons. It is noteworthy that the proton radius $R^P_{\rm rms}$ includes the second term of Eq. (8) which is determined by the extent of the neutron orbits. Note that the matter radius $R^M_{\rm rms}$ is given by

$$[R^M_{\rm rms}(^{4+n}\mathrm{He})]^2=\frac{2}{n+4}[R^P_{\rm rms}(^{4+n}\mathrm{He})]^2+\frac{n+2}{n+4}[R^N_{\rm rms}(^{4+n}\mathrm{He})]^2. \tag{10}$$

Table I lists the rms radii calculated by using $R^P_{\rm rms}(^4\mathrm{He})=R^N_{\rm rms}(^4\mathrm{He})=1.57$ fm.[2] All of the rms values are in reasonable agreement with those of a Hartree-Fock-type calculation.[5] Although the $R^P_{\rm rms}$ values are smaller compared with experiment,[2] we think that it does not indicate a serious drawback of our model: According to the authors of Ref. 2, the $R^M_{\rm rms}$ values are reliably determined from their experiment but the deduction of the $R^P_{\rm rms}$ values is subject to a questionable assumption of the proton distribution.

In summary, we have proposed a model which is aimed to apply to a system of valence particles weakly coupled to a core cluster. We have introduced the radius vectors of the valence particles measured from the core. This enables us to utilize results of a recent microscopic study of nucleon-nucleus potentials, including a derivation of optical potentials, and the effects of the Pauli principle. Our model is also very convenient to take account of possible Pauli-forbidden states between the valence particle and the core. Diagonalizing the single-particle Hamiltonian, h, in an appropriate basis provides us with orthonormal cluster orbitals. Distributing the valence particles to the cluster orbits, we can make best use of shell-model techniques to calculate the matrix elements of two-body operators. Note also that no spurious center-of-mass excitation appears in our model. We have applied our model to the He isotopes. The input needed for the present calculation has been taken entirely from the literature, and no adjustable parameters have been included in the calculation. We have calculated the energies and root-mean-square radii, assuming $(p_{3/2})^n$ configurations. Though the binding energies are a little short, the result is rather satisfactory considering the simplicity of the calculation. There are some effects to be examined in order to improve the agreement between theory and experiment. One of these is the state and density dependence of the effective interaction to be used in neutron-rich nuclei as emphasized in Ref. 9. Another is the possibility of the core changing with the number of the valence particles as studied in Ref. 10. These points will be investigated elsewhere.

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