Global interpretation of low-energy octupole states in spherical and weakly deformed $Z>28$ nuclei

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We present a parameter that provides a unified interpretation of the 3_1^- states of a large number of spherical and weakly deformed $Z>28$ nuclei. This parameter suggests criteria for identifying nuclei having anomalous octupole behavior.

In an atomic nucleus, a collective octupole vibrational state can be understood as the coherent sum of a number of one-particle-one-hole (1p-1h) or two-quasiparticle (2qp) excitations which can couple to an angular momentum of $3\hbar$. As pointed out by Bohr and Mottelson,[1] excitations between orbits differing in orbital angular momentum l by $3\hbar$ and having the same intrinsic spins (i.e., the change in total angular momentum $\Delta j=\Delta l$) dominate the systematic behavior of low-energy octupole states because of geometric properties of angular momentum coupling. In heavy nuclei (Z and $N>28$), the proton and neutron valence shells each contain such a pair of orbits, which we shall call a $\Delta l=3$ pair. The filling of the orbits in the $\Delta l=3$ pairs (both proton and neutron) determines the variation of the energy of the low-lying octupole states with changing N and Z in spherical and weakly deformed nuclei.[1] This behavior lends itself to a highly schematic but useful description. As particles are added to the lower-energy orbit of a $\Delta l=3$ pair and additional 1p-1h excitations contribute, the excitation energy of the low-energy octupole state, which we denote by $E(3_1^-)$, is driven downward. Likewise, as the higher-energy orbit of the pair is filled and 1p-1h excitations are blocked, $E(3_1^-)$ increases.

Recently, a simple parametrization[2] for $E(3_1^-)$ based on this qualitative description has been developed and used to interpret the systematic behavior of low-energy octupole states in each of four regions of the periodic table where spherical and weakly deformed nuclei are found. In the present paper, we formulate a new parameter which provides a unified description of the nuclei of these four regions which are "well-behaved" with respect to the above description. In particular, when the energies of the first 3^- states from all four mass regions are plotted on a single graph using this new parametrization, they fall nearly on a single line. When all the known 3_1^- states of nuclei not falling in the two large regions of well-deformed nuclei—that $N=88-106$ rare-earth isotopes and the heavy actinide isotopes—are added to the plot, it is possible to immediately determine those nuclei which cannot be described by the simple picture of octupole behavior presented here.

The parameter B_n+B_p of Ref. 2 used for individual regions is calculated by using the energies of spherical single-particle orbits ϵ_v, a rough estimate for the ground-state pairing gap ($\Delta=12/A^{1/2}$, where A is set to an average value for the region of interest), the number of valence particles N, and the Bardeen-Cooper-Schrieffer (BCS) equations

$$2\sum_v V_v^2=N \ ,$$

$$V_v^2=(1/2)[1-(\epsilon_v-\lambda)/E_v] \ ,$$

$$E_v=[(\epsilon_v-\lambda)^2+\Delta^2]^{1/2} \ .$$

The occupation probabilities V_v^2, the Fermi energy λ, and the quasiparticle energies E_v are calculated iteratively. We then use these probabilities to calculate "occupation numbers" for the contributing 1p-1h states. If we let $j1$ and $j2$ be the lower- and higher-energy members of a $\Delta l=3$ pair, respectively, then the occupation number for a neutron 1p-1h state B_n is given by

$$B_n=\min[V_{j1}^2,1-V_{j2}^2] \ ,$$

and a corresponding proton quantity B_p is determined in the same way. As the lower orbit of the $\Delta l=3$ pair fills, "more" 1p-1h states become available and B_n (or B_p) increases. Once the higher-energy orbit begins to fill, 1p-1h states are blocked and B_n (or B_p) decreases. We would expect that $E(3_1^-)$ would reach a maximum where $B_n+B_p=0$, and a minimum where this parameter is a maximum.

At first, the relationship of the above description of the systematic behavior of 3_1^- states to the accepted methods for calculating $E(3_1^-)$, such as the quasiparticle random-phase approximation (QRPA), may not be clear. However, the present parameter, motivated by the Brown-Bolsterli schematic model for the giant dipole resonance,[3] is grounded in a simple but useful picture for nuclear vibrations and well reproduces systematic trends of octupole states in a number of regions of the periodic table. Furthermore, we can draw a mathematical parallel between the present parameter and the QRPA. In the QRPA, the occupation number dependence of $E(3_1^-)$ is given by the term[4]

$$(V_{j1}U_{j2}+U_{j1}V_{j2})^2 \ .$$

When orbit $j1$ is nearly full and $j2$ is nearly empty, we have

$$(V_{j1}U_{j2}+U_{j1}V_{j2})^2 \simeq (V_{j1}U_{j2})^2 = V_{j1}^2(1-V_{j2}^2) .$$

The behavior of the final expression is quite similar to that of the above expression for $B_{n(p)}$. Even though the present parameter is based on a highly simplified physical picture, it is therefore qualitatively consistent with the formal QRPA approach.

Figures 1 and 2 display the energy of the first 3^- state,[5–10] $E(3_1^-)$ vs B_n+B_p for the well-behaved members of the four regions mentioned above. We will refer to these regions as the Zn region ($Z=30$–34, $N=30$–38), the Kr region ($Z=32$–38, $N=40$–50), the Ru region ($Z=40$–46, $N=50$–66), and the Te region ($Z=50$–60, $N=60$–82, with the exception of ^{132}Sn, which we will discuss later). It is clear that the parametrization we have just described provides a reasonable description of the relationship of the 3_1^- states of nuclei within each region.

To motivate the formulation of a parameter which can relate the systematic trends observed in these four regions to one another, we recall the Brown-Bolsterli schematic model for giant dipole resonances.[3] For this schematic model, the secular equation for a large number of 1p-1h states was given a particularly simple form,

$$0=\begin{vmatrix} \epsilon_1+D_{11}-\lambda & D_{12} & D_{13} & \cdots \\ D_{12} & \epsilon_2+D_{22}-\lambda & D_{23} & \cdots \\ D_{13} & D_{23} & \epsilon_3+D_{33}-\lambda & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{vmatrix} ,$$

where ϵ_i is the energy of the ith 1p-1h state and D_{ij} is the interaction between the ith and jth 1p-1h states.

In order to formulate an expression for $E(3_1^-)$ within

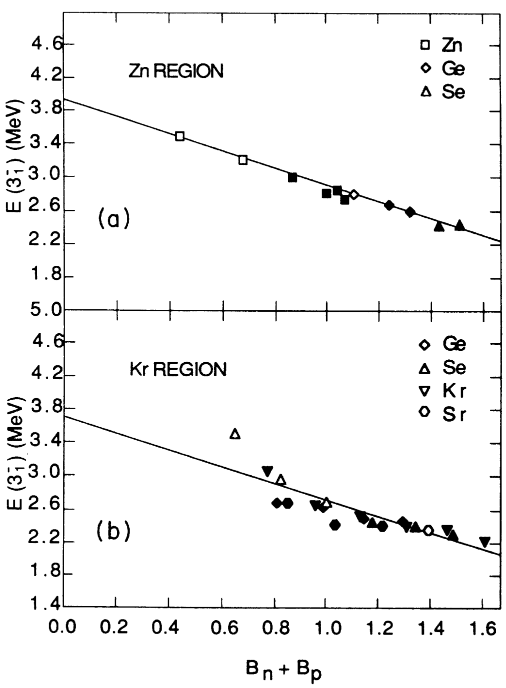

FIG. 1. Plots of $E(3_1^-)$ vs B_n+B_p for the "well-behaved" regions near (a) Zn and (b) Kr. The lines are fitted to the points shown by linear regression. Data taken from Ref. 5.

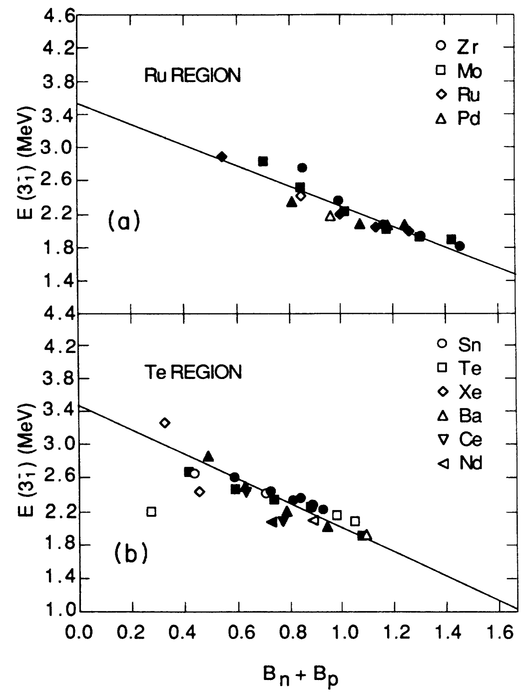

FIG. 2. Plots of $E(3_1^-)$ vs B_n+B_p for the "well-behaved" regions near (a) Ru and (b) Te. The lines are fitted to the points shown by linear regression. Data are taken from Refs. 5–10.

the present schematic framework, we will assume that there are n_p (n_n) proton (neutron) 1p-1h states contributing, all of which are excitations from the lower to the higher members of the $\Delta l=3$ orbits we have been discussing. In doing this, we neglect the contributions of particle excitations from one major shell to another, which are relatively important near closed shells and, of course, are the only configurations available for doubly magic nuclei. We also neglect the contributions of other intrashell 1p-1h (or 2qp) excitations; as we noted earlier, these contributions are suppressed by angular momentum coupling. Finally, we set D_{ij} to a constant D.

The resulting eigenvalues are

$$\lambda=[\epsilon_p+\epsilon_n+D(n_p+n_n)\pm X]/2 ,$$

where

$$X=[(\epsilon_p+\epsilon_n)^2+D^2(n_p+n_n)^2 +2D(\epsilon_p-\epsilon_n)(n_p-n_n)]^{1/2} .$$

The systematic behavior of low-energy octupole states in heavy nuclei demonstrates the importance of the $D(n_p+n_n)$ term for the excitation energy. Therefore we approximate X to be $D(n_p+n_n)$, so that

$$\lambda=(\epsilon_p+\epsilon_n)/2$$

or

$$\lambda=(\epsilon_p+\epsilon_n)/2+D(n_p+n_n) . \quad (1)$$

For the low-energy octupole mode,[3] $D<0$; consequently, the second solution, the collective one, possesses the well-known dependence on the number of contributing

1p-1h states.

We can write n_p+n_n in an approximate fashion in terms of B_n+B_p. For a given $\Delta l=3$ pair, the number of available 1p-1h states is $2l_{\text{low}}+2$, where l_{low} is the orbital angular momentum of the lower spin member of the pair. This statement, arising as it does from our schematic framework, reflects the fact that more nucleons are available to participate in octupole excitations of heavier nuclei. Therefore, if we take B_n to be the "occupation number" for the 1p-1h excitations, we have

$$n_n=B_n(2l_{\text{low}}+2)$$

for neutrons, and likewise for protons. The sum n_p+n_n can be written

$$n_p+n_n=2(B_p+B_n)(l_{\text{ave}}+1)+2B_p(l_{\text{low}\,p}-l_{\text{ave}})$$
$$+2B_n(l_{\text{low}\,n}-l_{\text{ave}}) ,$$

where

$$l_{\text{ave}}=(l_{\text{low}\,p}+l_{\text{low}\,n})/2 .$$

If we neglect the latter two terms (because $l_{\text{low}\,p}$ and $l_{\text{low}\,n}$ never differ by more than 1), then Eq. (1) becomes

$$\lambda=(\epsilon_p+\epsilon_n)/2+2D(l_{\text{ave}}+1)(B_n+B_p) .$$

If we use linear regression to fit straight lines to the data of the well-behaved regions shown in Figs. 1 and 2, then we would expect that the extracted slopes and intercepts can be interpreted in terms of Eq. (1). The values extracted by such a fit are listed in Table I. A global interpretation is strongly suggested by the observation that the values of D found for all four regions fall in a narrow range, -0.25 MeV to -0.26 MeV. At present, there is no reason to expect that D should be constant over such a wide range of masses; however, we shall exploit this observation here. If we set D to -0.25 MeV and y_0 to be the appropriate y intercept for each nucleus, then $y_0-E(3_1^-)$ should be approximately equal to $D(n_p+n_n)$ for nuclei in all four regions. Therefore, if we plot the values of $y_0-E(3_1^-)$ against $D(n_p+n_n)$, the points should fall nearly on a single straight line with slope 1 and intercept 0. This plot is shown in Fig. 3. All of the points (with the exception of the tentatively identified 3_1^- states in ^{84}Se and ^{132}Te) fall within 0.35 MeV of the $y=x$ line. Consequently, the band extending 0.35 MeV on either side of the $y=x$ line can be regarded as the region of "normal" behavior for $E(3_1^-)$ for all heavy ($A>60$) nuclei which are not well deformed. The scattering of points within the band arises, in part, from the contributions of 2qp configurations other than corresponding to the $\Delta l=3$ pairs.

TABLE I. Values for slope, D, and y intercept for lines fitted to "well-behaved" regions.

Region	Slope (MeV)	D (MeV)	y intercept (MeV)
Zn	−0.997	−0.25	3.89
Kr	−0.989	−0.25	3.74
Ru	−1.286	−0.26	3.57
Te	−1.480	−0.25	3.52

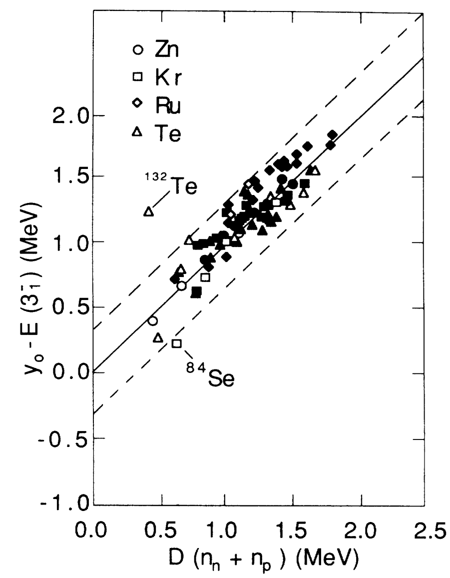

FIG. 3. Plot of $y_0-E(3_1^-)$ vs $D(n_p+n_n)$ for the four "well-behaved" regions. The solid line is the $y=x$ line, and the dotted lines fall 0.35 MeV above and below the $y=x$ line. Data are taken from Refs. 5–10.

In Fig. 4, three additional regions of the periodic table are included: the Sm region ($Z=54$–66, $N=82$–86), the Pt region ($Z=78$–82, $N=108$–126), and the Rn region ($Z=82$–90, $N=126$–134). Plots of $E(3_1^-)$ vs B_n+B_p for these regions[5,6,11–19] are shown in Fig. 5. Single-particle binding energies used to calculate B_n+B_p for these regions are taken from Refs. 20 (for Sm), 21 (for Pt), and 22 (for Rn). For the Pt and Rn regions, linear fits were

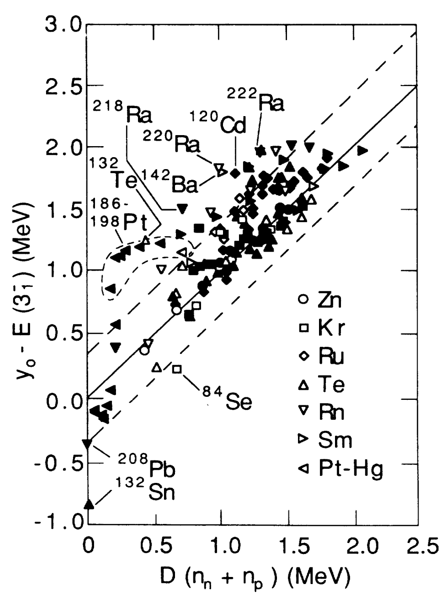

FIG. 4. Plot of $y_0-E(3_1^-)$ vs $D(n_p+n_n)$ for all known 3_1^- states of nuclei not in the deformed rare-earth ($Z=54$–76, $N=88$–116) or heavy actinide ($Z\geq 88$, $N\geq 136$) regions. Data are taken from Refs. 5–19.

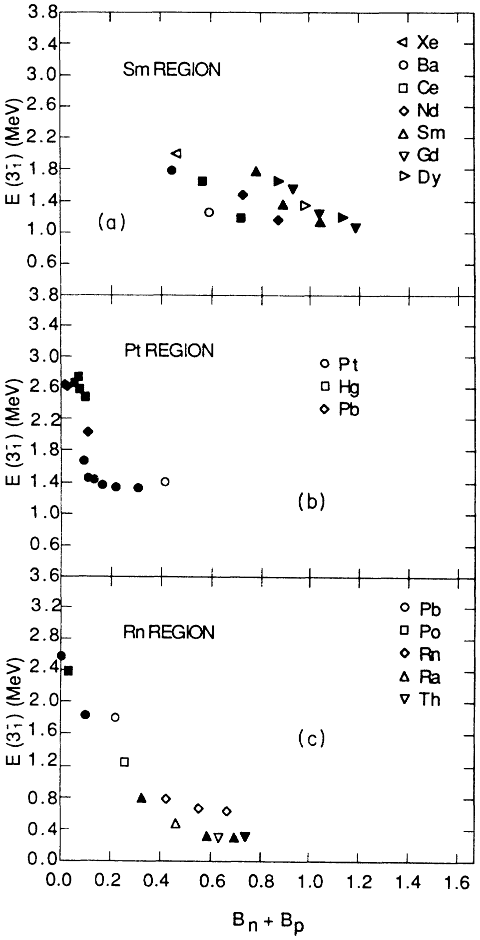

FIG. 5. Plots of $E(3_1^-)$ vs B_n+B_p for (a) Sm, (b) Pt, and (c) Rn regions. Data are taken from Refs. 5, 6, and 11–19.

made to all nuclei in the regions; only $N=82$–86, Sm, Gd, and Dy isotopes were used for the Sm region fit. Extracted y intercepts used for the calculation of $E(3_1^-)-y_0$ for these regions are listed in Table II. In addition, other nuclei[5–10] are included in the plot so that it displays the known 3_1^- states of all nuclei which are not in the deformed rare-earth ($Z=54$–76, $N=88$–116) or heavy actinide ($Z\geq 88$, $N\geq 136$) regions. These isotopes are denoted by symbols for the four regions discussed previously (y_0 values used for these nuclei are those extracted for the same regions). The nuclei ^{86,88}Zr and ^{90}Mo are included with the Kr region; ^{88}Kr, ^{94}Sr, and $^{106-120}$Cd are denoted by the symbol for the Ru region; and ^{132}Sn, ^{142}Sm, and ^{144}Gd are included with the Te region.

TABLE II. Slopes and y intercepts for Sm, Pt, and Rn regions.

Region	Slope (MeV)	y intercept (MeV)
Sm[a]	−1.693	3.05
Pt	−3.873	2.53
Rn	−3.008	2.25

[a]Fit to Sm, Gd, and Dy isotopes only.

Among the nuclei clearly deviating from the band of "normal" behavior are the doubly magic isotopes ^{132}Sn and ^{208}Pb. These two nuclei are doubly magic with respect to major shells (in contrast with ^{146}Gd and ^{90}Zr, which are spherical but fall in the centers of their respective proton major shells). Consequently, their 3_1^- states are composed entirely of cross-shell 1p-1h states, and we would not expect them to conform to the description represented by the present parametrization.

The Pt region is clearly anomalous as well. There is a discontinuity of approximately 1 MeV in $E(3_1^-)$ between the Pt isotopes ($Z=78$) and Hg ($Z=80$) which is not seen in any other region of the periodic table above mass 20. Several investigators have suggested that the 3_1^- states of the Pt isotopes are not of collective octupole nature;[23] however, a recent analysis[24] of 35 MeV proton inelastic scattering data[25] finds $B(E3;0^+_{\text{g.s.}}\rightarrow 3_1^-)$ matrix elements of between 6 and 9 W.u. in ^{194,196,198}Pt. These results are not very different from the value[26] measured for ^{204}Hg (14 W.u.). Another way of investigating this problem is to search for octupole states in light isotopes of Hg and Pb. Recently, Van Duppen *et al.*[19] have found the 3_1^- state of ^{196}Pb via the decay of ^{196}Bi at an energy of 1992 keV, significantly below the energies of 3_1^- states in heavier Hg and Pb isotopes, but still above those of Pt isotopes. Further studies of light Pb and Hg isotopes are needed to better understand octupole behavior.

A number of members of the Rn region lie significantly above the "normal" band in Fig. 4; we may account for this apparent deviation in several ways. One explanation involves the possible onset of static octupole deformation. Such a deformation has been predicted[27,28] for several nuclei in this region, and may alter the systematic behavior of the 3_1^- states. Alternatively, the presence of some other nuclear mechanism, such as alpha particle clustering,[29] may play a role in the 3_1^- states of several of these nuclei.

Static octupole deformation may also occur[30] in the vicinity of ^{144}Ba, a nucleus which is not well behaved in the sense of the present work. While this deformation may contribute to the deviation, it is also possible that changes in shell structure, specifically the spacing between the $d_{5/2}$ and $h_{11/2}$ proton orbits, may change between $Z=56$ and 62. These orbits form the proton $\Delta l=3$ pair for the nuclei in the Sm region. Any change in the spacing of these orbits, which is reflected in the variable ϵ_p in Eq. (1), would directly influence the behavior of $E(3_1^-)$. Such changes certainly occur[31] between the $N=82$ and 90 isotopes of Gd.

Finally, we discuss the behavior of 3_1^- states of heavy Cd isotopes, including ^{120}Cd. A β-decay study[32] of ^{118}Cd recently showed that the compiled[5] assignment of the 1935-keV level as the 3_1^- state, which was deduced from a study of the ^{122}Sn(d, ^{6}Li)^{118}Cd reaction, was incorrect. Data from a (d,^{6}Li) reaction were also the basis[5] for the 3_1^- assignment to the 1920-keV state in ^{120}Cd. This assignment, which is clearly anomalous in Fig. 4, may also be incorrect.

In summary, we have presented a global interpretation of the excitation energies of 3_1^- states of $Z>28$ spherical and weakly deformed nuclei in terms of a parameter which represents the effects of the strongest coherent 1p-1h (or 2qp) contributions to these states. By using this parameter, we have been able to identify nuclei in which the behavior of the 3_1^- state is unusual. Finally, we have briefly surveyed possible explanations for this behavior.

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