Bag-model nuclear equation of state in the Wigner-Seitz approximation

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The nuclear equation of state is calculated using the soliton bag model and the Wigner-Seitz approximation for baglike states on a lattice. Since the Wigner-Seitz treatment averages over lattice structure, it is more realistic for a fluid like nuclear matter than is a periodic crystal model. A crucial feature of the model presented here is the manner of filling the Bloch band of independent particle states: We argue for a dilute filling of the entire band. Quark wave functions sigma field configurations, and energies are calculated as a function of baryon density. The energies are only qualitatively similar to semiempirical nuclear energies. Within the context of this crude model, we find a first-order phase transition corresponding to a nuclear matter phase which sets in at about six times normal density and a plasma phase at roughly 12 times normal nuclear density.

I. INTRODUCTION

The description of individual nucleons as three-quark bags or clusters has had remarkable success in reproducing elementary properties of hadrons. The treatment of interactions between such structures is more complex, and the calculation of N-N scattering has been studied by a number of research groups. Nuclear matter is still more complicated, particularly if one wishes to go beyond the independent pair approximation. Even the analog of the Fermi gas is already a very complicated problem.

An alternate, albeit simplistic, approach is to consider nuclear matter as a collection of bags, like the holes in Swiss cheese, and to study the structure of the system under compression. We anticipate that the interstices between the bags—the physical vacuum—should disappear as the density is increased leading ultimately to a quark plasma. Is the transition continuous or discontinuous, i.e., what is the order of the "phase" transition? Models which place nucleon bags on a regular lattice have already received considerable attention.

Achtzehnter, Scheid, and Wilets[1] calculated the equation of state for a periodic lattice in the nontopological model. The scalar sigma field was taken to have the symmetry of a cubic crystal and the quarks are then Dirac-Bloch waves. In the mean-field approximation (MFA) which was used, the valence quarks occupy one-fourth of the lowest Brillouin band, since each site can accommodate 12 quarks (two spin, two flavor, three color) and here each site is identified as a baryon consisting of three quarks. Zhang, Derreth, Schäfer, and Greiner[2] solved the problem using the MIT model on a crystalline lattice. This was a remarkable feat, since the quarks are then subject to boundaries which are hard with sharp corners. Banerjee, Glendenning, and Soni[3] employed the hybrid topological soliton model utilizing the Wigner-Seitz approximation. Reinhardt, Dang, and Schulz also used the Wigner-Seitz approximation, but with a nontopological soliton bag model.[4] Goldman and Stephenson[5] have studied a periodic quark model in order to understand quark tunneling in nuclei.

In the present work, we use the soliton bag model in the Wigner-Seitz approximation. However, inasmuch as nuclear matter behaves more like a fluid than a crystal, we believe that the Wigner-Seitz approximation is more physical than the crystal model since it represents an angular average over the location of neighboring sites and carries no reference to particular crystalline symmetry. This approach has been used very successfully in condensed matter physics to calculate the equation of state of liquids. The original arguments were that the approximation should be good for a close-packed crystal since the Brillouin zone surface is close to spherical.

An important consideration in our calculations is the manner of filling of the Bloch states within the lowest band. The filling is clear in the pure MFA, or independent-particle model (IPM), since then the lowest states would be occupied with unit amplitude up to some Fermi energy determined by the density. In fact, however, quarks are strongly correlated by color electric and magnetic forces. Hence, levels within a band are mixed. This mixing leads to an effective probability of occupation within a band, and has (as we shall see) a crucial effect on the equation of state and the stability of the system.

II. THE SOLITON BAG MODEL

We utilize here the soliton bag model of Friedberg and Lee.[6–8] The model is *qualitatively* similar to the MIT model, but has several important factors not present in boundary-condition models.

The first is that for each dynamical coordinate which appears in the Lagrangian, the time derivate of that coordinate also appears. Thus, a Hamiltonian can be constructed and dynamics calculated utilizing techniques familiar from nuclear physics. For the calculations presented here, however, dynamics are not yet involved. The second feature—more important in the present context—is that the model leads to solutions with a smooth, finite bag surface. This is important computationally and, we believe, physically as well.

In the soliton model, the (effective) Lagrangian density is

$$\mathcal{L}=\mathcal{L}_q+\mathcal{L}_\sigma+\mathcal{L}_{q,\sigma}+\mathcal{L}_G \ , \tag{2.1}$$

where the individual terms have the following interpretations:

$\mathcal{L}_q=\sum_f \bar{\psi}_f(\gamma\cdot p-m_f)\psi_f$ describes the quarks as Dirac particles of mass m_f, where f is the flavor. We take $m_u=m_d=0$.

$\mathcal{L}_\sigma=\frac{1}{2}(\partial\sigma)^2-U(\sigma)$ describes the scalar soliton field σ, which represents the complex structure of the vacuum, arising from virtual gluons and quark-pairs interacting among themselves. The momentum operator conjugate to σ is $\pi=\dot{\sigma}$, and the two satisfy the canonical equal-time commutation relations. The nonlinearity of the soliton field enters through the self-interaction function

$$U(\sigma)=\frac{a}{2}\sigma^2+\frac{b}{3!}\sigma^3+\frac{c}{4!}\sigma^4+B \ . \tag{2.2}$$

The polynomial terminates in fourth order to ensure renormalizability. $U(0)=B$ is to be identified with the "bag constant" or volume energy density of a cavity. With suitable adjustment of the constants, the function has two minima, one at $\sigma=0$, and another, lower minimum, at $\sigma=\sigma_{\text{vac}}$. The physical vacuum corresponds to the lower of the two minima, and the constant B is chosen so that $U(\sigma_{\text{vac}})=0$.

$\mathcal{L}_{q,\sigma}=-g\bar{\psi}\sigma\psi$ gives the interaction of the quarks with the soliton field. In the presence of valence quarks, the sum $U(\sigma)+g\bar{\psi}\sigma\psi$ may have a minimum (depending on the parameters) near $\sigma=0$ (the perturbative vacuum). This leads to a cavity in the σ field, referred to as the "bag."

$\mathcal{L}_G$: Color gluon fields are introduced as in QCD, except that they interact with the soliton field through a dielectric function $\kappa(\sigma)$, chosen such that $\kappa(0)=1$ and $\kappa(\sigma_{\text{vac}})=0$. The magnetic susceptibility is $\mu=\kappa^{-1}$. The dielectric function is not uniquely prescribed in the model, and a choice must be made as to its functional form. This and other matters relating to gluonic effects in the model have been discussed elsewhere.[9] However, we note that the general requirements on κ do yield absolute color confinement.

III. THE LATTICE MODEL

As an introduction to the Wigner-Seitz approximation, we first consider the replacement of the moving, fluctuating bags by a regular, periodic face-centered cubic (fcc) lattice of bags. These are characterized by the lattice displacement vectors $\mathbf{a}_n$. We work in the mean-field approximation, with σ a c number. We take $\sigma(\mathbf{r})$ to be periodic in the crystal translation vectors,

$$\sigma(\mathbf{r})=\sigma(\mathbf{r}+\mathbf{a}_n) \ , \tag{3.1}$$

and to contain the reflectional and discrete rotational symmetries of the fcc lattice.

In the absence of OGE interactions, the quark functions then satisfy Bloch's theorem

$$\psi_{\mathbf{k}}(\mathbf{r})=e^{i\mathbf{k}\cdot\mathbf{r}}\phi_{\mathbf{k}}(\mathbf{r}) \ , \tag{3.2}$$

where $\mathbf{k}$ is a continuous vector and $\phi_{\mathbf{k}}(\mathbf{r})$ is periodic in the $\mathbf{a}_n$

$$\phi_{\mathbf{k}}(\mathbf{r})=\phi_{\mathbf{k}}(\mathbf{r}+\mathbf{a}_n) \ , \tag{3.3}$$

although it need not possess the other symmetries of σ. The $\phi_{\mathbf{k}}$ satisfy the Dirac equation

$$[\boldsymbol{\alpha}\cdot(\mathbf{p}+\mathbf{k})+g\beta\sigma(\mathbf{r})]\phi_{\mathbf{k}}=\epsilon_{\mathbf{k}}\phi_{\mathbf{k}} \ , \tag{3.4}$$

where the eigenvalues $\epsilon_{\mathbf{k}}$ have the characteristic band spectra of the fcc crystal.

At very low density—well separated bags—the self-consistent solutions for σ and ψ are those of isolated bags and the low-lying energy spectrum becomes discrete. As the bags are moved closer together, the eigenvalues $\epsilon_{\mathbf{k}}$ spread out into bands.

IV. THE WIGNER-SEITZ CELL

Although the lattice calculation is feasible, as has been demonstrated by the work of Achtzehnter *et al.*,[1] we choose to use the Wigner-Seitz spherical cell approximation. As discussed in the introduction, we believe that it is more physical than the lattice model for nuclear matter, and use it here as a *model* in its own right, rather than as an approximation to the crystal model. However, certain boundary conditions associated with the model are conveniently determined by reference back to the crystal.

A single "bag" is enclosed in a Wigner-Seitz sphere of radius R such that its volume is the same as that ascribed to each bag in the crystal. (In solid state literature this is usually denoted by r_S.) Because of the assumed spherical symmetry, the lowest band assumes the form for s states; ψ_k can be represented by

$$\psi_k(\mathbf{r})=\begin{bmatrix} u_k(r) \\ i\boldsymbol{\sigma}\cdot\hat{\mathbf{r}}v_k(r)\end{bmatrix}\chi \tag{4.1}$$

so that

$$du_k/dr=(g\sigma+\epsilon_k)v_k \ , \tag{4.2a}$$

$$dv_k/dr+2v_k/r=(-g\sigma+\epsilon_k)u_k \ , \tag{4.2b}$$

and

$$-\nabla^2\sigma(r)+U'(\sigma)+\frac{9g}{\bar{k}^3}\int_0^{\bar{k}}k^2dk[u_k^2(r)-v_k^2(r)]=0 \ , \tag{4.2c}$$

where $\bar{k}$ is the highest k value in the band that is filled, and is determined below. The factor $9/\bar{k}^3$ assures three quarks per bag irrespective of $\bar{k}$. χ is the spin-flavor-color function. Here the quark functions are normalized to

$$4\pi\int_0^R r^2dr[u_k^2(r)+v_k^2(r)]=1 \ . \tag{4.3}$$

The boundary conditions on σ are

$$\sigma'(0)=\sigma'(R)=0 \ . \tag{4.4}$$

The lowest member of the quark band satisfies the boundary condition

$$u_b'(R)=0 \Longrightarrow v_b(R)=0 \ . \tag{4.5}$$

At the top of the band, we have

$$u_t(R)=0 \ . \tag{4.6}$$

Using these boundary conditions and a given $\sigma(r)$, we can solve for the corresponding ϵ_b and ϵ_t. The intermediate ϵ's lie in the continuum of the band and do not require the solution of an eigenvalue problem. Rather ϵ_k is specified and Eqs. (4.2a) and (4.2b) are integrated from $r=0$ to $r=R$ without an eigenvalue search. Let $s=k/k_{\rm top}$ ($k_{\rm top}$ is inversely proportional to the lattice spacing); $s\leq 1$. We make the reasonable ansatz that

$$\epsilon(s)=\epsilon_b+(\epsilon_s-\epsilon_b)\sin^2(\pi s/2) \ . \tag{4.7}$$

Using the reduced momentum label s instead of k, the inhomogeneous term in (4.2c) can now be written

$$\frac{9g}{\bar{s}^3}\int_0^{\bar{s}} ds(u_s^2-v_s^2) \ . \tag{4.8}$$

V. BAND FILLING

Each level is 2(spin)$\times$2(flavor)$\times$3(color)=12-fold degenerate. If we were to neglect gluonic interactions, the lowest band would be densely filled one-fourth of the way up, implying $\bar{s}=(\frac{1}{4})^{1/3}$. However, the nucleon and delta are each particular linear combinations of products of single-quark states, and we are interested in the nucleon states.

For isolated bags, there are

$$\begin{pmatrix}12\\3\end{pmatrix}=220$$

three-quark states which can be constructed from spacially identical orbitals. Of these, there are only 20 color-singlet states; the four nucleon and 16 delta states. The soliton model guarantees color confinement for isolated bags, although color percolation can occur for overlapping bags. The color-electric matrix elements (required for color confinement) are of the order of $\alpha_s/\langle r\rangle^{1/2}$, which is several hundred MeV. The color magnetic interaction is responsible for at least half of the N-Δ splitting of 293 MeV; the rest can be interpreted as pionic interaction (which must also be included in the following argument). In the case of band structure in crystals or the Wigner-Seitz model, these gluonic matrix elements lead to a mixing of the band members and a separation of the many-particle states into nucleon bands, delta bands, and bands of different color symmetry. In the results presented below, we find that the width of the lowest band varies from zero (for well-separated bags) to about 400 MeV at $R=0.8$ fm, which corresponds to about 3.4 times normal nuclear density. For large separations the mixing matrix elements are large compared to the band width; for small separations they may become comparable. We are thus, led to the reasonable assumption that band levels are fully mixed by the color electric and magnetic forces, and that all levels within a band have equal probability of occupation. This corresponds to setting $\bar{s}=1$.

Note that the next band, corresponding to the first excited s level, lies, in the MIT model, at 2.645 times the lowest s level, roughly 500 MeV above the lowest level. Hence, we ignore mixing to the higher bands.

The issue of the occupation probability of levels within a band could be clarified by diagonalizing the Hamiltonian matrix for a discretized band. The matrix elements are well defined within the soliton model. Combinatorics expand the size of the matrix very rapidly, but a two, three, or even four level approximation should be feasible. This exercise is left to the interested researcher. A similar approach has been used by Kerman and Dagdeviren[18] to produce three-quark correlations in the plasma phase. There the calculation of gluon-exchange energies is simplified by the fact that the Block waves are simply plane waves.

As will be seen below, the structure of the Wigner-Seitz cells appear to scale approximately under compression. In order to obtain a simple estimate for the color-magnetic interaction for each cell, we take one-half the N-Δ splitting and multiply it by the ratio of the rms radius of quarks in a proton to that of quarks in a cell of radius R.

VI. THE UNIFORM PLASMA

In the high-density limit we may assume that a uniform plasma is the preferred phase. In the MFA, the energy per unit volume of this uniform plasma is given by ($\bar{d}^3k=dk^3/(2\pi)^3$)

$$\begin{aligned}\frac{E}{V}&=12\int \bar{d}^3k[k^2+(g\sigma)]^{1/2}\theta(k_F-k)+U(\sigma)\\&=\frac{3}{4\pi^2}\left[2k_F\epsilon_F^3-(g\sigma)^2k_F\epsilon_F\right.\\&\qquad\left.-(g\sigma)^2\ln\left[\frac{k_F+\epsilon_F}{g\sigma}\right]\right]+U(\sigma)\ ,\end{aligned}\tag{6.1}$$

with $\epsilon_F=[k_F^2+(g\sigma)^2]^{1/2}$; 12 is the degeneracy factor. The baryon density, which is one-third of the quark density, is given by

$$\frac{N}{V}=(\tfrac{4}{3}\pi R^3)^{-1}=4\int \bar{d}^3k\,\theta(k_F-k)=\frac{2}{3\pi^2}k_F^3\ , \tag{6.2}$$

or

$$k_F=\tfrac{1}{2}(9\pi)^{1/3}R^{-1}\ . \tag{6.3}$$

The energy (6.1) must be minimized with respect to σ for fixed volume. Since $U(\sigma)$ has a local minimum at $\sigma=0$ and the integral has an absolute minimum there, we see that $\sigma=0$ is always a local minimum (and usually the lowest minimum) of the full energy density. For those cases we see immediately that

$$\frac{E}{V}=\frac{3}{2\pi^2}k_F^2+B \tag{6.4}$$

or

$$\frac{E}{N}=\tfrac{9}{4}k_F+\frac{3\pi^2}{2k_F^3}B=3.4273R^{-1}+\tfrac{4}{3}\pi R^3B \ . \qquad (6.5)$$

Note that (6.5) depends only on the bag constant B and the cell radius R. It is of the same *form* as the energy expression in the MIT model, except that here the coefficient of R^{-1} is 3.4273 compared with the MIT value (for three quarks) of $3\times 2.0428=6.1284$. As should be well known, the uniform plasma in the MIT model has a lower energy per baryon than isolated bags when gluonic effects are ignored.

As we have seen, gluonic effects will be needed if the model is to provide more than a crude qualitative description of the hadronic phase. Similarly we should include gluonic contributions to the energy of the plasma. These have been calculated by Freeman and McLerran[10] and by Baluni.[11] To order $\alpha_S \ln\alpha_S$, the plasma energy is

$$\frac{E}{V}=\frac{3}{2\pi^2}k_F^4\left\{1+\frac{2\alpha_S}{3\pi}+\frac{\alpha_S^3}{3\pi^3}[2\ln(2\alpha_S/\pi)+6.79]\right\}+B\ , \qquad (6.6)$$

where α_S is the strong coupling constant appropriate to the Fermi momentum. The leading-logarithm expression for α_S is

$$\alpha_S(k_F)=\frac{12\pi}{29\ln(k_F^2/\Lambda^2)}\ , \qquad (6.7)$$

where we use $\Lambda=150$ MeV for the QCD scale parameter.

At sufficiently high densities, other flavors of quarks would also appear. Strange quarks, for example, could be created when $k_F(u,d)=m_S\sim 280$ MeV, which is comparable to normal nuclear matter density. We do not pursue that interesting possibility here.[12]

For some values of the volume (or R), there is another minimum of (6.5). These minima depend on the soliton model parameters a, b, c, and g. This branch plays no role here since it lies above either the $\sigma=0$ plasma curve or the nuclear matter curve; it was considered in Ref. 1.

VII. NUMERICAL RESULTS

We have performed calculations based on several sets of parameters, all belonging to the family $a=0$. The parameters b and g were taken from the compilation of Horn[13] which are to fit the mean baryon mass and the proton rms charge radius, including recoil effects. To study the dependence on the parameter c, parameter sets were chosen from that compilation for $c=10^3$, 10^4, and 10^5. The results do not differ appreciably even over this wide range of values of c, except that solutions are "lost" sooner (i.e., lower density) for the smaller values of c. For $c=10^5$, for example, solutions were obtained for all values of R; for $c=10^4$, the solution was lost somewhere in the range $0.8<R<0.9$ fm. For calculations reported, we choose the parameter set with $c=10^4$ because it gives a "reasonable" value for the glueball mass,[14] $[U(\sigma_{vac})]^{1/2}$, although we appreciate that the glueball has not been observed and that the lattice calculations remain uncertain. The full set used here is

$$a=0.0\ ,$$
$$b=-700.43\ \text{fm}^{-1}\ ,$$
$$c=10^4\ ,$$
$$g=10.98\ ,$$

from which it follows that $B=0.27\ \text{fm}^{-4}=53\ \text{MeV/fm}^3$, $g\sigma_{vac}=2.3\ \text{fm}^{-1}=454$ MeV, $m_{GB}=8.58\ \text{fm}^{-1}=171$ MeV.

The lowest s band ranges, bounded by ϵ_b and ϵ_t, are displayed in Fig. 1 as a function of the cell radius over the range $0.9\le R\le 2.0$ fm. We note that stable Wigner-Seitz solutions could not be obtained, even at the radius of normal nuclear density ($R_0=1.12$ fm), with fillings corresponding to $\bar{s}$ significantly less than unity.

The total energy per baryon is plotted in Fig. 2 for the parameter set previously listed; the other sets did not differ significantly, except where they terminated. For filling corresponding to $\bar{s}$ significantly less than unity, the energy was found to decrease with decreasing R. Since these calculations are based on the MFA, they do not contain the higher-order terms necessary to describe recoil corrections nor details of the N-N interaction, namely the effective attraction attributed to one and two pion (or "sigma meson") exchange. The energies are too high at low density inasmuch as the parameters have been fit to yield the mean nucleon-delta mass including recoil corrections. The magnetic energy, which splits the nucleon and the delta, has been accounted for phenomenologically. This has been done by subtracting a term equal to half the nucleon-delta splitting multiplied by the ratio of the quark rms radius for isolated bags to that calculated for the given cell radius. The curve should be shifted downward (at least at low densities) to agree with the free nucleon mass. On the same diagram, we have plotted the phenomenological equation of state for nuclear matter in the vicinity of equilibrium: Saturation at -16 MeV binding at a density corresponding to $R=R_0=1.12$ fm and a compressibility modulus

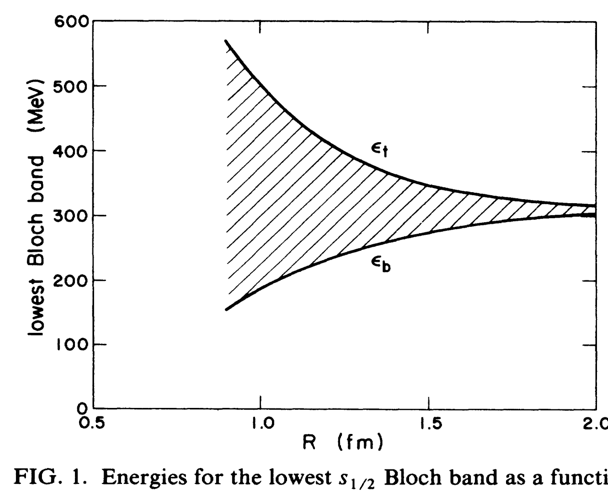

FIG. 1. Energies for the lowest $s_{1/2}$ Bloch band as a function of the cell radius. The standard text parameters were used.

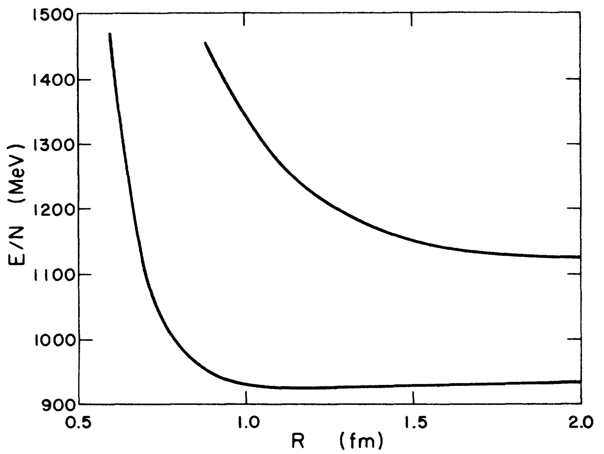

FIG. 2. The energy per baryon is plotted against the cell radius. The upper curve is calculated from the standard parameters in the text, which were fitted to yield the mean nucleon-delta mass, including recoil corrections. This curve has been corrected for the nucleon-delta color magnetic splitting, but not for recoil. The latter corrections would bring it down to 939 MeV at low density. The lower curve is from semiempirical nuclear energies.

$$K = R^2 \frac{\partial^2(E/N)}{\partial R^2}\bigg|_{R_0} = 9\rho^2 \frac{\partial^2(E/N)}{\partial \rho^2}\bigg|_{\rho_0} \approx 200 \text{ MeV} . \tag{7.1}$$

It is, in fact, the phenomenological form which we use below to discuss the phase transition. Even after shifting the W-S energies to agree at low density, the agreement between the two curves is at best qualitative. In addition to recoil corrections (which we only know how to do well at low density), one should consider the various meson exchange corrections and many body correlations.

The quark density (normalized to unity per cell) and the σ functions are displayed in Fig. 3. In these calculations, the functions shrink as R is reduced, but the value of σ at the cell boundary does not decrease as we might have expected. This means that the dielectric function essentially vanishes across the boundary and gluons are confined to their own cell. Individual quarks cannot percolate, but color-singlet clusters of three can.

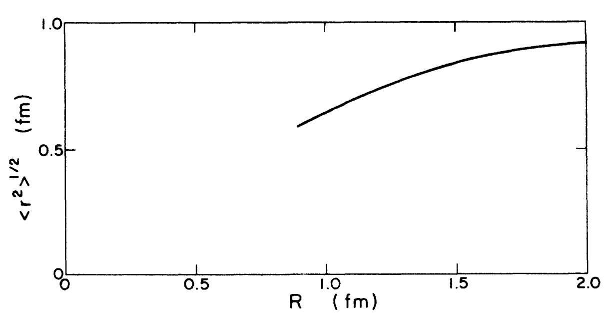

FIG. 3. The quark densities (normalized to one per cell, left-hand scale), and the sigma function (right-hand scale) are plotted against r for several cell sizes.

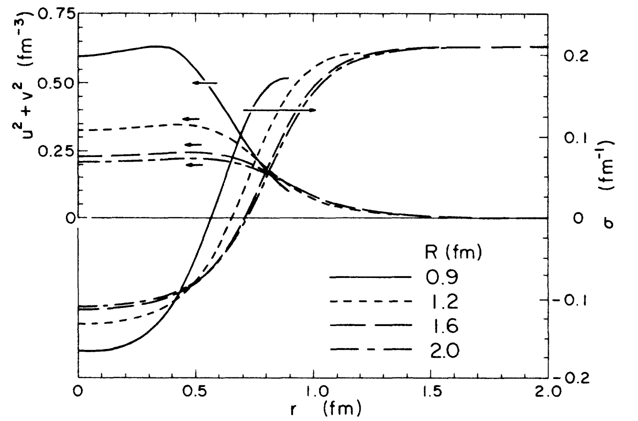

FIG. 4. The rms quark radius vs cell radius. For large R, the curve asymptotes to the free nucleon size, uncorrected for recoil. For small R it approaches zero linearly.

In Fig. 4 the quark rms radius is plotted as a function of the cell size. It asymptotes, for large R, to the free bag values (without recoil corrections). It eventually goes to zero linearly with R.

In Fig. 5 we have plotted E/N vs V/N for the two phases of nuclear matter. The negative derivative of these curves yields the pressure P. In a P-V diagram, the pressure would display a local minimum, an inflection point, and a local maximum. Equilibrium between the phases is determined by Maxwell construction which consists of drawing a horizontal (constant pressure) line which leaves equal area above and below the straight line segment. On the E/N vs V/N diagram, this is equivalent to connecting the two regions by a line which is tangent to the curves in the two regions. This yields the equation of states without requiring detailed knowledge of the intermediate, connecting region. Similar calculations of the phase transition to a quark plasma have been done by Engelbrecht and Brown,[15] using a theoretical model for the nuclear equation of state.

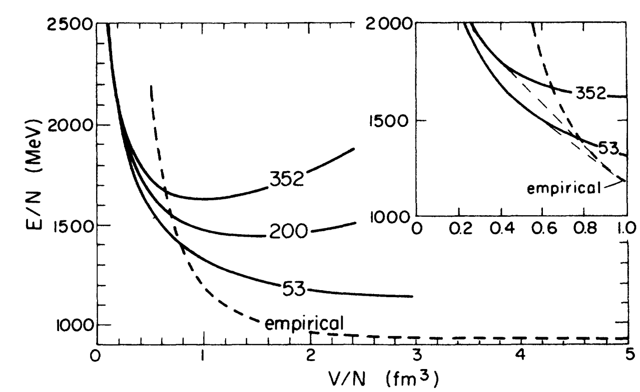

FIG. 5. The energy per cell as a function of cell volume. The solid curves are plasma energies for three different values of B, labeled in MeV/fm^3. The lowest value is close to the MIT value. The dashed curve is the semiempirical nuclear mass, given by Eq. (7.2). The Maxwell construction (dotted line) is shown in the insert for two values of B. Normal nuclear density corresponds to $V_0 = \frac{4}{3}\pi(1.12)^3 = 5.9$ fm^3.

The W-S energies shown in Fig. 5 have been shifted to give the correct nucleon mass at low densities. For comparison we plot also the "empirical" values based on the mass formula binding energy, the density of normal nuclear matter, and nuclear compressibility. In extending the energy to high density, there is an ambiguity whether to choose a quadratic form in the radius parameter or in the density, or some other functional form. This is actually not a crucial question, since the Maxwell line is tangent close to normal nuclear density. In Fig. 5, we use the density form,

$$\frac{E}{N} \approx \frac{K}{18}\left[\frac{\rho}{\rho_0}-1\right]^2 + 923 \text{ MeV} , \tag{7.2}$$

with $K = 200$ MeV;[16] the 923 MeV includes both the nucleon rest mass and the binding energy of nuclear matter. It is rather striking to note that nuclear matter is considered to be stiff under compression on a scale of nuclear energies but it is actually very soft on a scale of hadronic energies.

On the basis of the simple quark plasma model and empirical data we can draw conclusions which are, however, somewhat dependent on the bag constant and the QCD scale parameter. There is a first-order phase transition at zero temperature. The two phases in equilibrium correspond to a nuclear matter phase at roughly 6 times normal density and a plasma phase of roughly 12 times normal nuclear density. It should be pointed out that heavy ion collisions have already achieved densities of about 4 times normal nuclear matter without a signal of a phase transition. However, our W-S approximation to the equation of state for nuclear matter is only qualitatively similar to the semiempirical one. The latter leads to equilibrium between the two phases from roughly 6–12 times normal nuclear matter density.

VIII. SUMMARY

The soliton bag model is solved in the Wigner-Seitz cell approximation to obtain quark wave functions, confining sigma field configurations, and energies as a function of baryon density. The manner of filling the Bloch bands is seen to be critical. Combining this with a simple model of the plasma[10] (consistent with both the MIT and soliton bag models, plus gluon corrections) shows the existence of a phase transition between nuclear matter and a quark plasma. This is qualitatively similar to results found in other models.[15,17]

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