Dileptons as a probe of pion, kaon, nucleon, and antinucleon dynamics in nuclear matter

Charles Gale
Atomic Energy of Canada, Ltd., Theoretical Physics Branch, Chalk River Nuclear Laboratories, Chalk River, Ontario, Canada K0J-1J0

Joseph Kapusta
School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455
(Received 16 May 1988)

We continue our study of dilepton radiation from high-temperature nuclear matter by computing the rates for the annihilations $K^+K^-\rightarrow e^+e^-$ and $N\bar{N}\rightarrow e^+e^-$. In general this radiation would yield information on the abundances of kaons and antinucleons in hot and dense nuclear matter. More specifically, the former may yield information on the approach to kaon condensation, and the latter may yield information on the nucleon-antinucleon dispersion relations as predicted by relativistic field theories of nuclear matter. However, these yields are mostly overshadowed by the processes $np\rightarrow npe^+e^-$ and $\pi^+\pi^-\rightarrow e^+e^-$, so that dilepton production remains essentially a probe of pion dynamics in the nuclear medium.

I. INTRODUCTION

The main purpose of colliding heavy nuclei at high energy is to study the properties of nuclear matter in temperature and density regions far from equilibrium. We then hope to learn about the behavior of excited nuclear matter and perhaps to reveal some exotic new phenomena. The conjectured liquid-gas phase transition[1] and quantum chromodynamics phase transition[2] are often mentioned in this context. It is the purpose of this paper to continue our study of the emission spectrum of dileptons in high-energy heavy ion collisions.[3]

Dileptons may serve as an interesting probe of the relativistic many-body system. In general, electromagnetic signals offer obvious advantages over strongly interacting probes: (a) They do not suffer from strong final-state interactions, i.e., they travel relatively unscathed from the interaction zone. If the dileptons are produced mainly in incoherent processes at the single-particle level, they carry valuable information about the space-time region where those interactions are most frequent: the high temperature and density phase. (b) Their coupling to other particles is fairly well known. The main drawback of course is the extremely low counting rate, mainly because of the size of α. Note that the rate of dilepton emission is of order α^2, whereas photon emission goes as α.

In our first work[3] we concentrated on the thermal emission rates for $np\rightarrow npe^+e^-$, treated in the soft-photon approximation, and for $\pi^+\pi^-\rightarrow e^+e^-$. Generally the former is dominant for the invariant mass $M<400$ MeV and the latter is dominant for $M>400$ MeV. However, for a soft pion dispersion relation in nuclear matter the rate from $\pi^+\pi^-$ annihilation can be enhanced by even several orders of magnitude. We emphasized the importance of looking at back-to-back e^+e^- in the nuclear matter rest frame. With this configuration the relation between the mass spectrum and the pion dispersion relation is direct. In fact, if the pion group velocity $d\omega/dk$ vanishes at nonzero k_0 then there is a peak in the mass spectrum at $M_0=2\omega(k_0)$.

In this paper we look at some other processes which could yield interesting information about high-temperature nuclear matter. The first of these is $K^+K^-\rightarrow e^+e^-$. The most obvious information carried by the yield of this reaction is the abundance of kaons (in the nuclear matter while it is hot and dense as opposed to the number observed in the final state) since the rate is proportional to the square of the kaon number density. Furthermore, it has recently been conjectured[4,5] that the kaon mass is a decreasing function of nuclear density, and that kaon condensation may set in at a critical density $n_c\sim 4n_0$ (four times nuclear matter density). We estimate the consequence of this for the dilepton mass spectrum. In fact, it could be that dileptons are the only signal that kaon condensation happened in the transient nuclear system.

The second process we examine is $N\bar{N}\rightarrow e^+e^-$. Since antinucleons are probably strongly annihilated during the final expansion and cooling of the nuclear system the dilepton spectrum may be a unique probe of their abundance in the high temperature and density phase. Also, relativistic field theories of nuclear matter[6] make characteristic predictions for the dispersion relations of nucleons and antinucleons in nuclear matter. Perhaps these predictions could be tested by the dilepton mass spectrum.

In Sec. II we compute the rates for the aforementioned processes. The results are plotted and compared to one another and to previously computed processes in Sec. III. Conclusions are presented in Sec. IV. Our main conclusion will be that nucleon and especially pion dynamics dominate over kaon and antinucleon dynamics for invariant masses $M<1$ GeV, but that for $M>1$ GeV most annihilation processes are of comparable magnitude.

II. COMPUTATION OF RATES

A. $\pi^+\pi^- \to e^+e^-$

The rate for this annihilation reaction was computed by us previously.[3] The rate when the e^+e^- pair is emitted with zero total momentum ($\mathbf{q}=0$) in the rest frame of the nuclear matter is

$$\frac{d^4R_{\pi\pi}}{d^3q\,dM}(\mathbf{q}=0,M)=\frac{\alpha^2}{3(2\pi)^4}\frac{|F_\pi(M)|^2}{(e^{\omega/T}-1)^2}\times\sum_{\substack{k\\ \text{such that}\\ 2\omega(k)=M}}\left[\frac{k}{\omega}\right]^4\left|\frac{d\omega}{dk}\right|^{-1}. \quad (1)$$

Here $\omega=\omega(k)=M/2$ relates the pion dispersion relation to the invariant mass M of the e^+e^- pair. In Ref. 3 we used a parametrization of the form

$$\omega(k)=[(k-k_0)^2+m_0^2]^{1/2}-U\ , \quad (2)$$

where k_0, m_0, and U are all density dependent. The minimum of $\omega(k)$ occurs at k_0 corresponding to the p-wave attraction between nucleons and pions. In free space of course $k_0=U=0$ and $m_0=m_\pi$. The pion electromagnetic form factor $F_\pi(M)$ has a peak near the rho resonance. We use[3] a modified relativistic Breit-Wigner type fit to the Gounaris-Sakurai formula.[7]

The important point about Eq. (1) is that the rate is inversely proportional to the group velocity of the pion and to the fourth power of k/ω. The group velocity appears simply as the Jacobian of the transformation between energy and momentum.

B. $K^+K^- \to e^+e^-$

This reaction may be especially interesting in light of the recent enticing concept that kaon condensation may occur at moderate nuclear densities.[4,5] Furthermore, the dilepton spectrum will explore the kaon electromagnetic form factor which, in the vector dominance approximation, is due to the superposition of the ρ, ω, and ϕ vector meson resonances. The approach to kaon condensation may be modeled very simply by a vanishing mass at the critical density:

$$m_K^*=m_K(1-n/n_c)^{1/2}\ . \quad (3)$$

Here $m_K=495$ MeV is the vacuum mass, and we choose $n_c=4n_0$. The kaon dispersion relation is thus

$$\omega(k)=(k^2+m_K^{*2})^{1/2}\ . \quad (4)$$

The functional form is the same as in free space and differs from the pion one because the dominant interaction between kaons and nucleons is the s-wave attraction.

The thermal rate for kaon annihilation has the same form as Eq. (1). Using Eq. (4) we get

$$\frac{d^4R_{KK}}{d^3q\,dM}(\mathbf{q}=0,M)=\frac{\alpha^2}{3(2\pi)^4}|F_K(M)|^2\times f_{K^+}f_{K^-}(1-4m_K^{*2}/M^2)^{1/2}\ . \quad (5)$$

We take the kaon electromagnetic form factor to be[8]

$$F_K(M)=\frac{1}{2}\frac{m_\rho^2}{m_\rho^2-M^2-im_\rho\Gamma_\rho}+\frac{1}{6}\frac{m_\omega^2}{m_\omega^2-M^2-im_\omega\Gamma_\omega}+\frac{1}{3}\frac{m_\phi^2}{m_\phi^2-M^2-im_\phi\Gamma_\phi}\ . \quad (6)$$

The masses and widths are taken from the particle data table and the coefficients are the ideal SU(3) values. The phase-space occupation factors are $f_{K^+}=(e^{(\omega-\mu_K)/T}-1)^{-1}$ and $f_{K^-}=(e^{(\omega+\mu_K)/T}-1)^{-1}$. Since K^+ and K^- are antiparticles of each other their chemical potentials have opposite signs but equal magnitude. Generally quantum statistics are not significant in this context unless we are interested in very small M and then only if phase space allows a nonzero rate. In the following we neglect quantum statistics in Eq. (5) and so $f_{K^+}f_{K^-}=e^{-M/T}$. The chemical potentials cancel and the rate depends only on temperature.

C. $N\bar{N} \to e^+e^-$

It is interesting to contemplate the rate for antinucleon annihilation on nucleons into dileptons at high temperature and density. This process may yield information on the abundance of antinucleons at high temperature and density. It may also depend on the dispersion relation of nucleons and antinucleons.

For illustration let us consider relativistic mean-field models of hot and dense nuclear matter.[6] Nucleons and antinucleons move independently in mean scalar and vector fields which are generated self-consistently. The Dirac equation is

$$[i\partial\!\!\!/-g_V\bar{V}_0\gamma^0-(m_N-g_s\bar{\phi})]\psi(\mathbf{x},t)=0\ . \quad (7)$$

Here $\bar{\phi}$ is the mean scalar field and $\bar{V}_0$ is the mean zeroth component of the vector field. In simple models these take the form

$$\bar{\phi}=(g_s/m_s^2)n_s\ ,\qquad \bar{V}_0=(g_V/m_V^2)n\ , \quad (8)$$

where n_s and n are the scalar and baryon densities ($n_s\to n$ at low density). The Dirac wave function is

$$\psi(\mathbf{x},t)=\int\frac{d^3p}{(2\pi)^{3/2}}\left[\frac{m_N^*}{E^*}\right]^{1/2}\sum_{\pm s}(b(\mathbf{p},s)u(\mathbf{p},s)\exp\{-i[(E^*+g_V\bar{V}_0)t-\mathbf{p}\cdot\mathbf{x}]\}+d^*(\mathbf{p},s)v(\mathbf{p},s)\exp\{i[(E^*-g_V\bar{V}_0)t-\mathbf{p}\cdot\mathbf{x}]\})\ , \quad (9)$$

where

$$m_N^* = m_N - (g_s/m_s)^2 n_s \ ,$$
$$E^* = (\mathbf{p}^2 + m_N^{*2})^{1/2} \ , \tag{10}$$

and b, d, u, and v assume the usual free-space forms but with $E \to E^*$ and $m_N \to m_N^*$. The single-particle energies, or dispersion relations, are

$$E = (\mathbf{p}^2 + m_N^{*2})^{1/2} \pm (g_V/m_V)^2 n \ , \tag{11}$$

where the $\pm$ signs refer to nucleon/antinucleon states.

The cross section for $N\bar{N} \to e^+e^-$ then is easily computed in the usual way when the N and $\bar{N}$ have equal but opposite momenta in the nuclear matter rest frame. It is

$$\sigma_{N\bar{N}\to e^+e^-} = \frac{2\pi\alpha^2}{3\beta M^2}[2|G_M|^2 + (1-\beta^2)|G_E|^2] \ , \tag{12}$$

where β is the velocity of the N or $\bar{N}$, and G_E and G_M are the nucleon electric and magnetic form factors evaluated at $s = M^2$. Relying on relativistic kinetic theory the thermal rate is computed to be

$$\frac{d^4R_{N\bar{N}}}{d^3q\,dM}(\mathbf{q}=0,M) = \frac{4\alpha^2}{3(2\pi)^4}[|G_M|^2 + 2(m_N^*/M)^2|G_E|^2] \times f_N f_{\bar{N}}[1 - 4m_N^{*2}/M^2]^{1/2} \ . \tag{13}$$

The occupation probabilities are approximated by their Boltzmann limits so that the product is

$$f_N f_{\bar{N}} = e^{-M/T} \ . \tag{14}$$

Notice that the result, Eqs. (13)–(14), depends only on the effective mass m_N^* and not on the chemical potential or on the energy shifts $\pm(g_v^2/m_v^2)n$. For the timelike form factors we use the following simple dipole parametrization:

$$|G_E| = |G_M| = [1.4 + (M^2 - 4m_N^{*2})/1\ \mathrm{GeV}^2]^{-2} \ . \tag{15}$$

These reproduce the measured low-energy cross sections for $\bar{p}p \to e^+e^-$ (Ref. 9) (when we substitute for m_N^* the proton rest mass in vacuum). Equation (13) refers to the contribution from $N = n$ or p. We use Eq. (15) and multiply Eq. (13) by 2, although that may be a slight overestimate of the neutron contribution. We do not pursue here the density or temperature dependence of the form factors, but that certainly could be done.

There are other annihilation modes such as $\bar{N}N \to e^+e^-X$, where X is one or more mesons, which could lead to e^+e^- masses less than $2m_N^*$. For a given invariant mass M these reactions will be further suppressed by the Boltzmann factor $\exp(-E_X/T)$, where E_X is the energy carried by X, as compared to the pure annihilations $\bar{N}N \to e^+e^-$, $\pi^+\pi^- \to e^+e^-$, and $K^+K^- \to e^+e^-$. Therefore, we do not include them here.

D. $np \to npe^+e^-$

In Ref. 3 we estimated this rate based on the soft-photon approximation. This corresponds to Fig. 1. Virtual photons are radiated only from the external proton lines and energy conservation is neglected. This probably leads to an overestimate of dilepton emission at large M. The result is

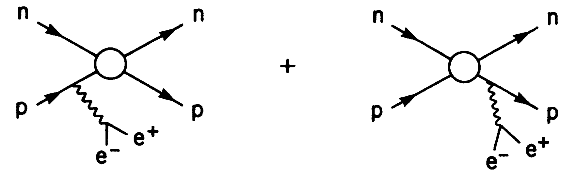

FIG. 1. Radiation from the external proton lines in np scattering.

$$\frac{d^4R_{np}^{\mathrm{soft}}}{d^3q\,dM}(\mathbf{q}=0,M) = \frac{\alpha^2}{6(2\pi)^3}\left[\frac{T}{m_N}\right]^6 \frac{n_p n_n}{[K_2(m_N/T)]^2}\frac{1}{M^4} \times \int_{z_{\min}}^{\infty} dz\, z^2(z^2 - 4m_N^2/T^2)^2 \times \sigma_{np}^{\mathrm{el}}(z)K_1(z) \ , \tag{16}$$

where $z = s^{1/2}/T$, $z_{\min} = (2m_N + M)/T$, and $\sigma_{np}^{\mathrm{el}}$ is the elastic np cross section. The rate is proportional to the product of the neutron and proton densities. Note that in Eq. (16) we use the free-space nucleon mass and the free-space cross section.

It was pointed out in Ref. 3 that the soft-photon approximation is probably inaccurate when $M > 100$–200 MeV. One would imagine a relativistic one-boson exchange model for NN scattering[10] at energies up to 1 or 2 GeV. Then one could compute dilepton radiation from this set of Feynman diagrams. This is a formidable task, which we have begun but will not report on here.

III. EVALUATION AND COMPARISON OF RATES

In this section we numerically evaluate the rates computed in Sec. II, plot them and compare the results. We must keep in mind that these are thermal rates which cannot be compared directly with experiment. In principle, these rates must be integrated over the space-time history of the nucleus-nucleus collision to obtain absolute numbers of pairs. A rough estimate would be to multiply by a space-time volume of 7 $A^{4/3}$ fm^4 for a central $A + A$ collision. More generally, if the momentum distributions of the hadrons are not thermal, then the occupation probabilities (the f's) must be replaced by the phase-space distributions appropriate to the dynamical model being used. This will be the case for an intranuclear cascade or Boltzmann-Uehling-Uhlenbeck simulation, for example.

The notation $d^4R/d^3q\,dM$ ($\mathbf{q}=0,M$) refers to the emission of a back-to-back e^+ and e^- integrated over all solid angle. If we require the e^- to be emitted in a particular direction, due to the placement of a detector for example, then we should divide the aforementioned rate by 4π. Note the relation

$$E_+E_-\frac{d^6\sigma}{d^3k_+d^3k_-} = 2\frac{d^6\sigma}{d^4q\,d\Omega_-^{\mathrm{c.m.}}} \ , \tag{17}$$

where $dq^0 = dM$ when $\mathbf{q} = 0$.

The first process to study is $N\bar{N}\to e^+e^-$. This rate is shown in Fig. 2 for $T=100$ MeV and $m_N^*=500$ MeV. Once the threshold at $M=2m_N^*$ is surpassed the rate does not depend on m_N^*. It depends on T but not on the density. As we see from the figure the rate for this process is comparable to the rate for $\pi^+\pi^-\to e^+e^-$ when the free-space pion dispersion relation is used and is smaller by an order of magnitude when the soft pion dispersion relation of Ref. 3 is used at $n=n_0$. The reason is not hard to understand. From Eqs. (1), (5), and (13) we see that for large M

$$\frac{d^4R^{\text{annihilation}}}{d^3q\,dM}(\mathbf{q}=0,M) \sim \frac{(\text{spin})^2\alpha^2}{3(2\pi)^4}|\text{form factor}|^2 e^{-M/T}\,. \quad (18)$$

Hence, large M annihilations are essentially a study of form factors. Since the nucleon is a rather large, composite object compared to the pion, its form factor is softer. This depresses the rate for $N\bar{N}$ annihilations compared to $\pi^+\pi^-$ annihilations, but in the mass range shown this is compensated by a spin-isospin factor of 8.

The next process to consider is $K^+K^-\to e^+e^-$. This rate is compared to the rate for $\pi^+\pi^-\to e^+e^-$ at $T=150$ MeV in Fig. 3, assuming free-space dispersion relations for both kaons and pions. The rate for kaons of course goes to zero at $2m_K=990$ MeV. The ϕ peak is clearly evident. It stands almost a factor of 100 above the pion rate. However, it is very narrow, and one would need a detector of excellent resolution to see it well. Above the ϕ resonance the rates due to kaons and to pions are comparable, a result of the fact that their form factors are comparable.

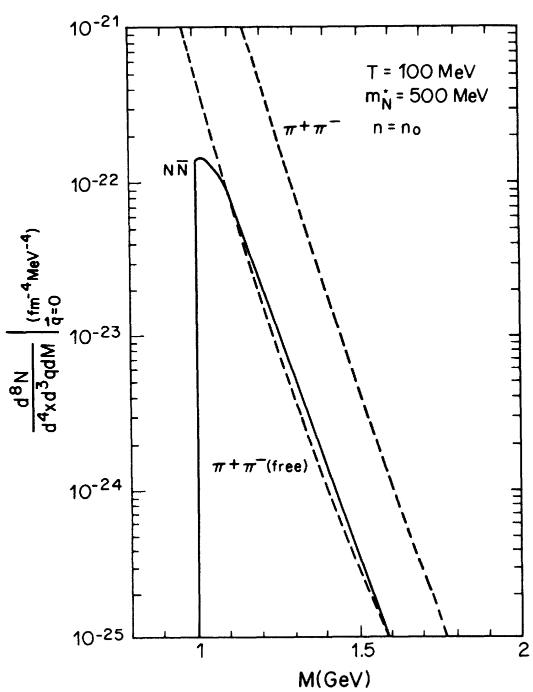

FIG. 2. Thermal dilepton production rates from $N\bar{N}\to e^+e^-$ and $\pi^+\pi^-\to e^+e^-$.

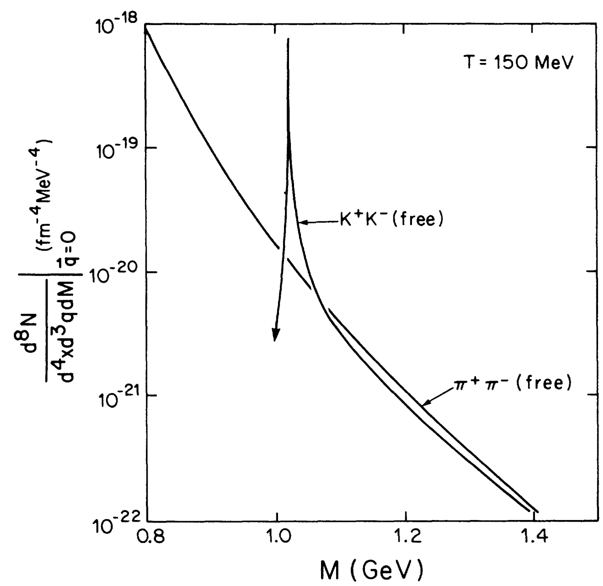

FIG. 3. Thermal dilepton production rates from $K^+K^-\to e^+e^-$ and $\pi^+\pi^-\to e^+e^-$ using free-space dispersion relations.

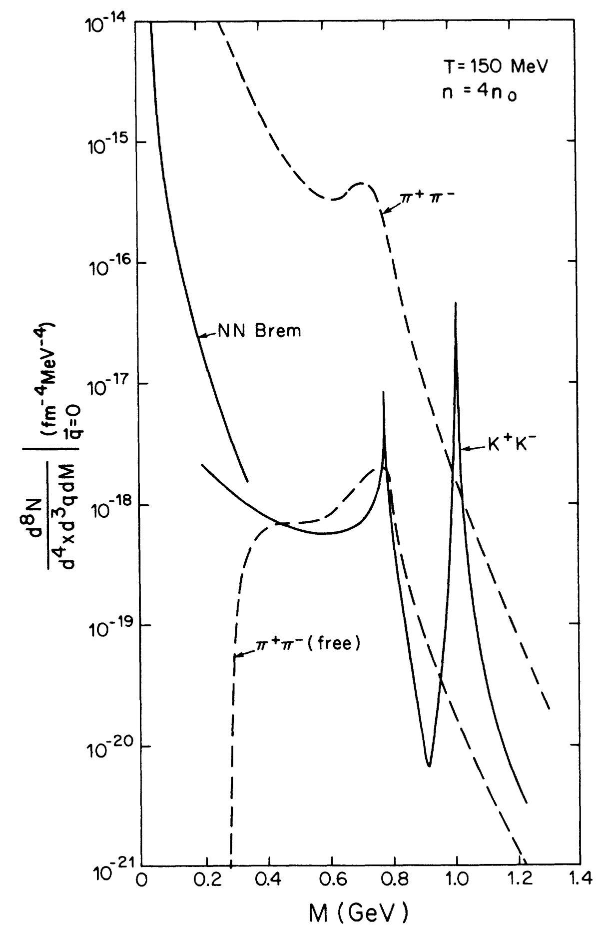

FIG. 4. Thermal dilepton production rates from $np\to npe^+e^-$, from $K^+K^-\to e^+e^-$ with the interacting kaon dispersion relation, and from $\pi^+\pi^-\to e^+e^-$ using both free and interacting pion dispersion relations.

In Fig. 4 we plot various rates at $T=150$ MeV and $n=4n_0$. For kaon annihilation we use the dispersion relation from Eqs. (3) and (4). At this density kaons are massless. Hence, K^+K^- can annihilate not only through the ϕ resonance but also through the $\rho-\omega$ resonances. Compared to pion annihilation using the free-space pion dispersion relation there is a slight enhancement of the ρ peak, but now the ϕ peak towers a factor of 1000 above the pion rate. Below the ρ peak the yield from kaons is masked by the yields from pions and from $np \to npe^+e^-$. When the pion dispersion relation from Ref. 3 is used instead the picture is drastically different. Pion annihilation now dominates everything except for a very narrow spike at the ϕ resonance. Kaons are almost completely masked.

We should remark that the vector meson masses and widths may change at high temperature[11] and this will change the electromagnetic form factors. Such temperature dependence is extremely interesting and should be studied soon. Thus Figs. 2–4 should be taken as schematic only and do not provide the final answer. Nevertheless, the basic features should survive.

IV. CONCLUSION

We have found that $N\bar{N}$ annihilations are comparable to or hidden by $\pi^+\pi^-$ annihilations, even with a free-space dispersion relation for pions. The prospects for detecting K^+K^- annihilations are better, especially around the ρ-ω and ϕ regions. The contribution from np virtual bremsstrahlung seems to be important only at small invariant mass. This should be firmed up by relaxing the soft-photon approximation. Such an effort is now underway.

Our main conclusion is the same as that reached in Ref. 3: The mass spectrum of back-to-back e^+e^- pairs in the range $100-200<M<900-1000$ MeV is primarily a source of information on pion dynamics in hot and dense nuclear matter.

Clearly there is more theoretical work to be done. As mentioned in Ref. 3, the rate for $\pi N \to Ne^+e^-$ still needs to be estimated. On the experimental side results are now becoming available from the DLS (dilepton spectrometer) experiment at the Bevalac. There may also be corresponding experiments at the SIS and the Alternating Gradient Synchrotron. The study of dilepton emission will reveal features of strongly interacting many-body systems that were up to now inaccessible.

This work was supported by the Department of Energy under Contract DE-FG02-87ER40328 and by a grant from the Minnesota Supercomputer Institute.

[1]See, for example, L. P. Csernai and J. I. Kapusta, Phy. Rep. **131**, 223 (1986).

[2]See, for example, *Quark Matter '86*, Proceedings of the 5th International Conference on Ultrarelativistic Nucleus-Nucleus Collisions, Pacific Grove, California, 1986, edited by L. S. Schroeder and M. Gyulassy [Nucl. Phys. **A461**, Nos. 1 and 2 (1987)].

[3]C. Gale and J. Kapusta, Phys. Rev. C **35**, 2107 (1987); Nucl. Phys. **A471**, 35c (1987).

[4]D. B. Kaplan and A. E. Nelson, Phys. Lett. B **175**, 57 (1986).

[5]G. E. Brown, K. Kubodera, and M. Rho, Phys. Lett. B **192**, 273 (1987).

[6]B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. **16** (1986).

[7]G. J. Gounaris and J. J. Sakurai, Phys. Rev. Lett. **21**, 244 (1968).

[8]F. M. Renard, Phys. Lett. **47B**, 361 (1973).

[9]G. Bassompierre, M. A. Schneegans, G. Binder, G. Gissinger, S. Jacquey, P. Dalpiaz, P. F. Dalpiaz, C. Peroni, and L. Tecchio, Nuovo Cimento **73A**, 347 (1983).

[10]R. Machleidt, K. Holinde, and Ch. Elster, Phys. Rep. **149**, 1 (1987).

[11]R. D. Pisarski, Phys. Lett. **110B**, 155 (1982).