Comparison of average s-wave resonance spacings from proton and neutron resonances

H. Vonach, M. Uhl, and B. Strohmaier
Institut für Radiumforschung und Kernphysik, Universität Wien, Vienna, Austria

B. W. Smith and E. G. Bilpuch
Duke University and Triangle Universities Nuclear Laboratory, Duke Station, Durham, North Carolina

G. E. Mitchell
North Carolina State University, Raleigh, North Carolina
and Triangle Universities Nuclear Laboratory, Duke Station, Durham, North Carolina
(Received 29 August 1988)

Average s-wave resonance spacings were derived from high-resolution proton resonance data for 11 odd-Z nuclei in the mass range $A=41$–67 and compared with the average s-wave spacings obtained from neutron resonances for their odd-N isobars. After careful correction for missing levels the neutron and proton results are consistent; proton resonance data can be used to increase the data base for nuclear level densities. After correction to a common excitation energy, the spacings show a correlation with the shell correction energy E_{shell}, especially for $A=41$, 55, and 59, where data exist for three isobars with quite different shell correction energies. The dependence of the average level spacing $\overline{D}$ on E_{shell} is approximately exponential, in good agreement with the theoretical models of Ignatyuk *et al.* and Kataria *et al.* The data also have been used to calculate asymptotic a parameters in these two models. The mass dependence of these parameters can be described by the simple relation $a=0.137A$ with an rms deviation of 4.3%. Attempts to describe the data within the Fermi gas model resulted in much larger fluctuations of the parameters with A and Z. Finally, the data were compared to the predictions of the microscopic Fermi gas model, using the single particle level scheme of Seeger and Perisho. At present, the quality of this description is still somewhat worse than with the phenomenological models of Ignatyuk *et al.* and Kataria *et al.*

I. INTRODUCTION

Average s-wave neutron resonance spacings have long been one of the main sources of nuclear level density information.[1–3] A large amount of high-resolution proton resonance data has been accumulated;[4] however, these data have not yet been systematically analyzed in terms of average resonance spacings. The purpose of this paper is to use this additional information: to extract average s-wave resonance spacings in a systematic way from the existing proton resonance data and to compare these results with the spacings obtained from the neutron resonance data for neighboring isobars. For experimental reasons proton resonance data suitable for this purpose exist only for odd-Z nuclei [from (p,p) and (p,p') studies on even-even nuclei] up to mass 67. Accordingly in this study we shall derive average resonance spacings for 11 odd-Z nuclei in the mass range $A=41-67$ from proton resonance data and for the corresponding odd-N isobars from neutron resonance data. Comparison of these two types of data permits a check on the procedures (e.g., correction for missing levels) used in the analysis of such resonance data. This comparison also allows a direct study of the influence of shell effects on nuclear level densities, as level densities of different isobars of odd-mass nuclei should be the same except for shell effects.

In Sec. II we describe the procedure used to analyze the neutron resonance data, while in Sec. III the corresponding analysis for the proton data is described. For the former we might have used existing information;[4] however, the earlier analyses have not always followed the most accurate procedures for the extraction of average spacings. Therefore we have reanalyzed the neutron resonance data by fitting the Porter-Thomas distribution to the neutron widths; a similar procedure was used with the proton resonance data.

In Sec. IV a detailed comparison of the proton and neutron resonance data is presented, with special emphasis on shell effects in nuclear level densities. In Sec. V nuclear level density parameters are derived for all nuclei studied, both in the framework of the Fermi gas model[3] and in the models of Ignatyuk *et al.*[5] and of Kataria *et al.*,[6] which explicitly include shell effects.

Section VI presents a comparison of proton and neutron resonance data with calculations in the frame of the microscopic Fermi gas model including consideration of pairing, i.e., a model with no easily adjustable parameters. The results of these calculations are also compared with those of the model of Ignatyuk *et al.*[5] Sec. VII summarizes the main conclusions from the present work.

Although the mass range studied is rather limited, this range contains an important region of structural materials. Thus the level density parameters derived in this work are also quite important for applied purposes.

II. AVERAGE NEUTRON RESONANCE SPACINGS

The most recent compilation of neutron resonance data and evaluation of average s-wave resonance spacings[4] uses the experimental data obtained up to 1980 and simple staircase plots in order to estimate the average resonance spacing $\bar{D}$. Since that time, improved data have been published for a number of nuclei studied in this work.[7–10] In addition a number of computer programs have been developed[10] which allow the estimation of the fraction of missed levels in an experimental sequence.

Therefore it was decided to perform a new determination of the average s-wave spacings for odd-N nuclei in the mass range $A=41-67$ using the code STARA.[11,12] This code performs a maximum likelihood fit to the observed reduced neutron widths, assuming a Porter-Thomas distribution with a diffuse detection threshold. This detection threshold may also have a linear dependence on neutron energy. From the fit, we obtain the most likely value of the average reduced neutron width, a quantity specifying the detection threshold, and an estimate of the fraction of resonances lost because of the finite detection threshold. This latter quantity will be called MF (missing fraction). In addition the fit to the width distribution allows an excellent visual check of the compatibility of the resonance widths with the assumed Porter-Thomas distribution. The distribution of the reduced neutron widths is given by the relation

$$P(\Gamma_n^0/\bar{\Gamma}_n^0)=\exp[-\Gamma_n^0/2\bar{\Gamma}_n^0]/(2\pi\Gamma_n^0/\bar{\Gamma}_n^0)^{1/2}\ , \quad (1)$$

with Γ_n^0 equal to the reduced neutron width and $\bar{\Gamma}_n^0$ equal to the average reduced neutron width.

This distribution of the reduced neutron widths can be transformed into a uniform distribution of values in the range $0\le u\le 1$ if the new variable u is defined by

$$u=\mathrm{erfc}(\Gamma_n^0/2\bar{\Gamma}_n^0)\ . \quad (2)$$

Thus the set of data points $\{u_i,E_{ni}\}$ corresponding to the individual resonances will be uniformly distributed in the (u,E_n) plane, except for the region $u<u_c$ which corresponds to reduced neutron widths below the detection threshold. The small increase of level density with neutron energy E_n is neglected.

Figure 1 shows the distributions of the neutron resonances in the (u,E_n) plane for two examples: a recent measurement[7] of ^{58}Ni and the older resonance data of Ref. 4. The pictures clearly show the quality of resonance data which is achievable at present, but also demonstrate the need for a considerable correction for undetected weak resonances in most older measurements (which are all that are available for most nuclei).

The uncertainty in the calculated $\bar{D}$ values was assumed to arise from three sources.

(i) The first is the statistical error due to the finite number of resonances in the energy range studied. A Wigner distribution is assumed for the nearest-neighbor spacing distribution for levels of the same spin and parity (here $\frac{1}{2}^+$); this results in a fractional error

$$\left[\frac{\Delta\bar{D}}{\bar{D}}\right]_1=\left[\frac{4-\pi}{\pi}\right]^{1/2}\frac{1}{\sqrt{N_{\mathrm{corr}}}}\ , \quad (3)$$

$$N_{\mathrm{corr}}=\frac{N}{1-\mathrm{MF}}\ , \quad (4)$$

with N the observed number of resonances and MF the fraction of resonances below the detection threshold.

(ii) The second source of the uncertainty is the statistical error due to the uncertainty in the quantity MF from visual inspection of the (u,E_n) plots described above. This uncertainty was estimated to be 25% of MF. The fractional error is

$$\left[\frac{\Delta\bar{D}}{\bar{D}}\right]_2=0.25\frac{\mathrm{MF}}{1-\mathrm{MF}}\ . \quad (5)$$

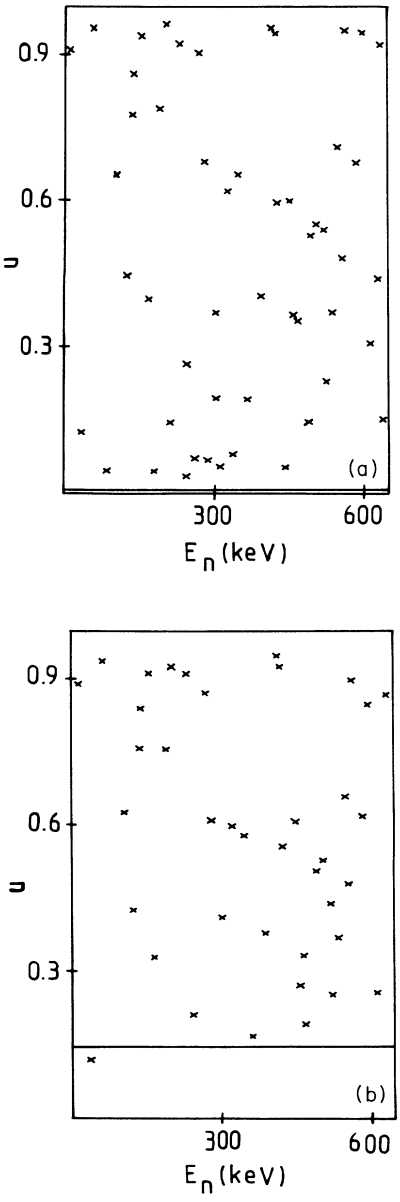

FIG. 1. Distribution of neutron resonances in the (u,E_n) plane for ^{58}Ni + n. The more recent data (a) are from Ref. 7, while the older data (b) are from Ref. 4. The horizontal lines give detection thresholds u_c estimated by the code STARA.

(iii) The third source is the statistical error in the actual number of levels in the undetected range of the Porter-Thomas distribution. Here the normal statistical error is applied—contrary to contribution $(\Delta\bar{D}/\bar{D})_1$. This fractional error is

$$\left[\frac{\Delta\bar{D}}{\bar{D}}\right]_3 = \left[\frac{\mathrm{MF}}{N_{\mathrm{corr}}}\right]^{1/2} . \tag{6}$$

The total fractional uncertainty of the $\bar{D}$ values was obtained by combining the three contributions in quadrature:

$$\frac{\Delta\bar{D}}{\bar{D}} = \left[\left[\left(\frac{4-\pi}{\pi}+\mathrm{MF}\right)\Big/N_{\mathrm{corr}}+\left[\frac{0.25\mathrm{MF}}{1-\mathrm{MF}}\right]^2\right]^{1/2} . \tag{7}$$

The results from this analysis are summarized in Table I. Columns 2–5 list the data source, the neutron separation energy, the energy range of the neutron resonance measurements, and the number of s-wave resonances identified in this energy range. Column 6 lists the missing fraction MF and columns 7–9 list our results for the average level spacing $\bar{D}$ as well as the values of $\bar{D}$ from Refs. 4 and 3.

As the table shows, there are large discrepancies between the $\bar{D}$ values derived in Ref. 3, Ref. 4, and the present work. The $\bar{D}$ values used by Dilg *et al.*[3] for their parametrization of the back-shifted Fermi gas model exceed our values on average by a factor of 1.4. This is mainly because the need for missing level corrections had not been generally recognized at the time of Ref. 3. The same comment also applies to the resonance data used by Gilbert and Cameron in Ref. 2. Thus it seems clear that the most common level density parameters underestimate the nuclear level densities at the neutron binding energy by about a factor of 1.4 (at least in the mass range $A=41-67$ studied here) and that these parametrizations should be updated. Such a change will probably result in an increase of the a parameters by about 4–5 %.

It should also be noted that there are considerable discrepancies between our values and those published in Ref. 4, which is the most recent compilation of neutron resonance data. There are several reasons for this.

(i) First, corrections were made for missing levels based on the Porter-Thomas distribution. The approximate method of staircase plots used in Ref. 4 cannot detect any neutron-energy-independent missing level fraction [such as that present in the data of Fig. 1(b)].

(ii) Second, in a number of cases (e.g., ^{43}Ca, ^{45}Ca) the results in Ref. 4 are based on data sets which were probably "contaminated" with p-wave resonances to a considerable extent, as indicated by our (u,E_n) plots for those isotopes.

(iii) Finally, in the case of ^{55}Cr careful measurements[8] published after the cutoff date of Ref. 4 proved that the earlier data were rather bad and contained a large number of spurious resonances.

Thus it appears that a new careful analysis of the data compiled in Ref. 4 and all later publications is needed to make optimum use of the existing neutron resonance data.

III. AVERAGE PROTON RESONANCE SPACINGS

The proton resonance data used to obtain the $\frac{1}{2}^+$ level densities were obtained in a series of measurements which

TABLE I. Average s-wave neutron resonance spacings of odd-A nuclei with $A=41-67$.

Nucleus	Data source	B_n (MeV)	Energy range (keV)	Number of resonances	MF	$\bar{D}$ (keV)	$\bar{D}$ (Ref. 4) (keV)	$\bar{D}$ (Ref. 3) (keV)
^{41}Ar	Mughabghab[a]	6.10	0–630	7	0.22	70.2±17.0		90
^{41}Ca	Mughabghab[a]	8.36	0–1400	31	0.28	32.6±4.3	45±6	45
^{43}Ca	Mughabghab[a]	7.93	0–580	27[b]	0.25	15.8±2.1	8.6±1.0	28
^{45}Ca	Mughabghab[a]	7.41	0–570	21	0.13	24.1±3.2	16±2	33
^{47}Ti	Mughabghab[a]	8.88	0–370	12	0.19	25.0±4.4	20±8	22
^{49}Ti	Mughabghab[a]	8.14	0–360	16	0.19	18.3±2.9	13±3	25
^{51}Cr	Mughabghab[a]	9.26	0–600	41	0.11	13.3±1.3	15±2	21
^{55}Cr	Agrawal[c]	6.25	0–900	15	0.09	54.4±8.2	15±2	66
^{55}Fe	Mughabghab[a]	9.30	0–500	23	0.17	18.0±2.4	13±2	20
^{57}Fe	Mughabghab[a]	7.65	0–900	41	0.14	19.2±1.9	17±2	25
^{59}Fe	Mughabghab[a]	6.58	0–350	11[d]	0.20	25.4±4.9	35±15	
^{59}Ni	Perey[e]	9.00	0–650	52	0.00	12.5±0.9	13.7±2	22
^{61}Ni	Perey[f]	7.82	0–450	30	0.07	13.9±1.5	16±2.5	17
^{63}Ni	Mughabghab[a]	6.84	0–600	30	0.25	15.0±2.0	19.1±3.6	19
^{67}Zn	Garg[g]	7.05	0–380	80	0.16	4.0±0.3	4.7±0.4	6

[a]Reference 4.
[b]14 weak resonances for which the $l=0$ assignment in Ref. 4 appeared doubtful were not used.
[c]Reference 8.
[d]5 weak resonances for which the $l=0$ assignment in Ref. 4 appeared doubtful were not used.
[e]Reference 7.
[f]Reference 9.
[g]Reference 10.

were primarily on $1f$-$2p$ shell nuclei. These data are unique in terms of comprehensiveness and of energy resolution. They were obtained as a part of a systematic study of the fine structure of analog states. Most of the data is contained in a review,[13] although several nuclei were studied later. References to specific nuclei are given in Table II.

In keeping with the approach described in Sec. II, we have accepted the spin assignments and proton widths as listed in the earlier references, but have analyzed the width distribution to determine the fraction of levels missed (MF). Reexamination of all our proton resonance results determined that the data are sufficient to provide reliable level density estimates for 11 odd-Z nuclei.

In a given sequence we assume that above an energy E_0 *all* levels with laboratory width greater than some minimum width ($\Gamma > \Gamma_{\min}$) are observed. With this assumption the *largest* minimum γ^2 value for the sequence can be determined, and a maximum fraction of missing levels obtained. This γ^2 value corresponds to the maximum reduced width value of the $\Gamma_{\min}$ level, which occurs at the lower energy bound E_0. From this width one can determine the maximum fraction of missing levels for the section of the sequence with $E > E_0$. Since the data extend over a wide energy range, and the Coulomb penetrability changes rapidly with energy, the fraction of levels missed depends strongly on the choice of E_0.

We considered how this maximum fraction behaves versus energy (over the whole energy range of the sequence), using the minimum observed laboratory width as $\Gamma_{\min}$. In practice, a value of approximately 30% missing levels seemed to be a good compromise. The reduced widths were assumed to follow the Porter-Thomas distribution. The value for the minimum width was then used, together with the average reduced width γ^2, to determine the MF. Although this procedure appears to work reliably, the method does depend crucially upon the minimum width observed.

We therefore used a second method to determine MF. The energy range is determined as before, yielding a set of N reduced proton widths. We then take the set of N reduced widths and sample (with replacement) at random N times from this set. The resulting set of γ^2 values are then analyzed as before to obtain MF. The entire procedure is repeated many times. One then obtains a distribution of MF values. The average value gives MF and the spread provides an estimate of the error in MF. This statistical procedure is called the bootstrap method;[14] an example of the application of this statistical approach in nuclear physics is given by Shriner *et al.*[15] In 10 of the 11 nuclei studied, the two methods for determining MF agreed. However, for ^{66}Zn the two methods give significantly different results.

Since we believe the second procedure to be less sensitively dependent on the value of one width, we adopt the bootstrap procedure to obtain the final values for MF. These results are listed in Table II. Columns 2–6 list the data source(s), the proton separation energy, the energy range of the proton resonance data, the number of s-wave resonances identified in this energy range, and the value of the missing fraction (MF). The average s-wave level spacing $\overline{D}$ (and its error) is listed in column 7.

The errors in the level density were determined in the same manner as described in Sec. II, except that the uncertainty in MF was obtained from the bootstrap calculation. Since the bootstrap distribution is not always completely symmetric, we consider the width of the central 68% of the distribution to be equal to 2σ.

IV. COMPARISON OF NEUTRON AND PROTON RESONANCE SPACINGS

The average resonance spacings derived in Secs. II and III correspond to quite different excitation energies in the range of 6–10 MeV (see Tables I and II). As a first step all data are converted to a common excitation energy; this energy was chosen to be 8 MeV. For this purpose it is necessary to use some model for the energy dependence of the nuclear level densities. For reasons discussed later we used the level density model of Ref. 6 and converted our s-wave spacings to 8 MeV by multiplying with the factor

$$R = \rho(\tfrac{1}{2}^+,\ U = 8\ \text{MeV}) / \rho(\tfrac{1}{2}^+,\ U = U_{\text{exp}})\ ,$$

calculated with the level density parameters of Ref. 6. For the uncertainty of this conversion we assumed a max-

TABLE II. Average s-wave proton resonance spacings for odd-A nuclei with $A = 41$–67.

Nucleus	Reference	B_p (MeV)	Energy range (MeV)	Number of resonances	MF	$\overline{D}$ (keV)
^{41}K	16	7.80	2.210–2.598	25	0.26	11.5±1.5
^{43}Sc	17	4.92	2.152–2.981	26	0.15	27.0±3.3
^{45}Sc	18	6.89	2.118–3.006	79	0.19	9.1±1.7
^{55}Mn	19	8.07	2.167–2.686	47	0.36	7.1±0.7
^{57}Co	20	6.03	2.438–3.140	28	0.23	19.4±2.4
^{57}Co	21	6.03	3.121–4.001	56	0.15	13.3±1.1
^{59}Co	20,22	7.37	2.589–3.104	82	0.31	4.3±0.4
^{63}Cu	23	6.12	2.715–3.097	44	0.32	5.9±0.7
^{47}V	24	5.17	2.088–3.068	22	0.19	36.0±4.8
^{49}V	25	6.76	2.496–3.102	46	0.19	10.6±1.0
^{51}V	26	8.05	2.242–2.949	68	0.24	7.9±0.6
^{67}Ga	27	5.27	3.045–3.255	52	0.38	2.5±0.2

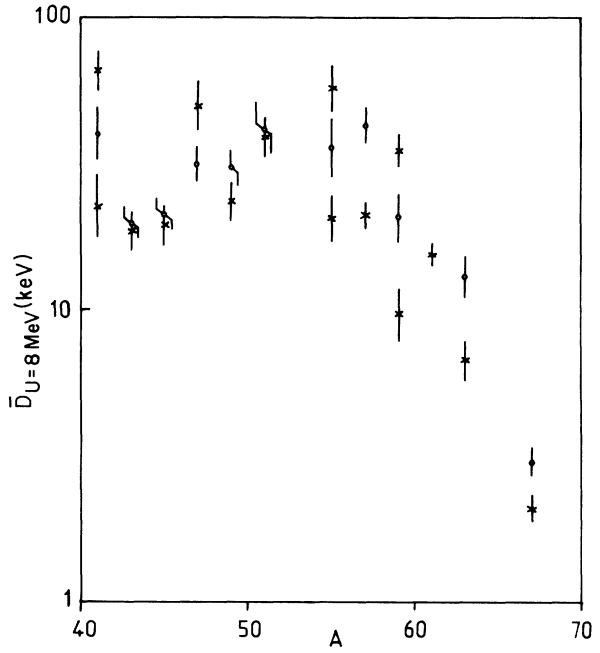

FIG. 2. Mass dependence of average s-wave spacings of odd nuclei for excitation energy $U=8$ MeV derived from proton resonances (circles) and from neutron resonances (crosses).

imum error of 10% in the nuclear temperatures. This error in the temperature results in uncertainties in the conversion factor R of 1–21% depending on the energy difference between the actual data and the reference energy of 8 MeV. This error was added quadratically to the errors given in Tables I and II. Figure 2 shows the values

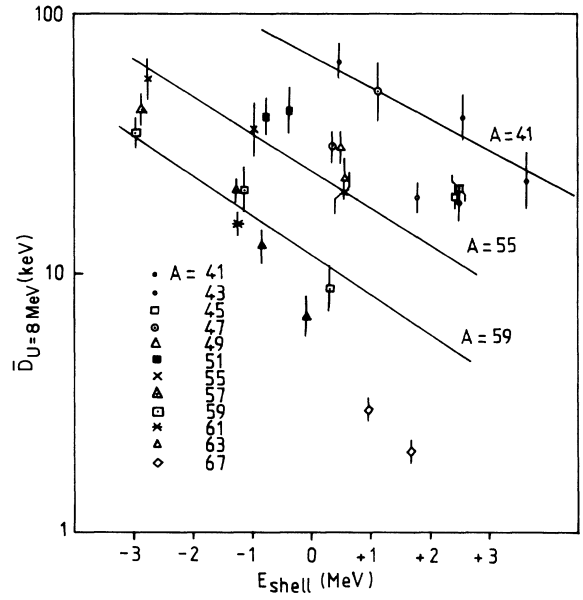

FIG. 3. Correlation between average s-wave resonance spacings at $U=8$ MeV and the ground-state shell correction energy E_{shell}. The straight lines are estimates to guide the eye for the dependence of $\bar{D}$ ($U=8$ MeV) on the shell correction energy for $A=41$, 55, and 59.

$\bar{D}$ (8 MeV), obtained in this way as a function of mass number A. As the figure shows, there are still rather large differences between the level densities between different isobars and there is also no smooth dependence on A, as predicted by simple level density models. However, even from this figure it can be recognized that there is no obvious discrepancy between the level densities derived from proton and neutron resonance data.

A better understanding of the data can be obtained if the data for the various isobars are plotted as a function of the ground-state shell correction E_{shell} (defined in Sec. V) as shown in Fig. 3. For all cases where there are data for several isobars with different correction energies (e.g., for $A=41$, 55, and 59) one sees a clear correlation of the $\bar{D}$ (8 MeV) values with the shell correction energy E_{shell}. This approximately exponential dependence of $\bar{D}$ on E_{shell} is qualitatively in agreement with the level density models of Kataria *et al.*[6] and Ignatyuk *et al.*[5]

V. DETERMINATION OF LEVEL DENSITY PARAMETERS

The experimental average resonance spacings were used to extract level density parameters which can be used for cross section calculations. In the following we consider simple level density models which are based on the methods of statistical mechanics.[28] The state density $\omega(U)$ as function of the excitation energy U is

$$\omega(U)=\frac{1}{(2\pi)^{3/2}D^{1/2}(U)}\exp[S(U)]\ , \tag{8a}$$

where $S(U)$ is the entropy and $D(U)$ is the determinant related to the saddle point approximation. The density of levels with spin I around excitation energy U is given in terms of the state density $\omega(U)$ and the spin cutoff parameter $\sigma(U)$:

$$\rho(U,I)=\frac{2I+1}{2\sqrt{2\pi}\sigma^3(U)}\exp\left[-\frac{(I+1/2)^2}{2\sigma^2(U)}\right]\omega(U)\ . \tag{8b}$$

For the Fermi gas model (FG model), assuming equidistant single particle states with a density g and an empirical pairing correction Δ, the following relations hold:

$$\begin{aligned} a&=\frac{\pi^2}{6}g\ ,\\ U-\Delta&=at^2\ ,\\ S(U)&=2at\ ,\\ D(U)&=\frac{18}{\pi^4}a^{1/2}(U-\Delta)^{5/2}\ . \end{aligned} \tag{9a}$$

The spin cutoff factor is related to the temperature t and to the average value $\langle m^2\rangle$ of the square of the angular momentum projection of the single particle states

$$\sigma^2(U)=g\langle m^2\rangle t=\frac{1}{\hbar^2}\Theta t\ . \tag{9b}$$

The quantity Θ can be regarded as the moment of inertia of the nucleus under consideration.

Calculations based on realistic single particle states

predict for the level density shell effects which may be pronounced at lower excitation energy and tend to disappear at higher energies. Among the simple semiempirical models which account for such energy-dependent shell effects we selected the models of Ignatyuk, Smirenkin, and Tishin[5] (IST model) and of Kataria, Ramamurthy, and Kapoor[6] (KRK model). Both of them relate the shell effects of the level densities to the ground-state shell correction

$$E_{\text{shell}}=M_{\text{exp}}-M_{\text{drop}} , \tag{10}$$

defined as the difference of the experimental mass M_{exp} and the liquid drop mass M_{drop}.[29]

The IST model[5] employs in Eqs. (9a) and (9b) an energy-dependent a parameter

$$a(U)=\tilde{a}\left[1+E_{\text{shell}}\frac{[1-\exp(-\gamma U)]}{U}\right] . \tag{11}$$

Here, $\tilde{a}$ is the asymptotic a parameter which should exhibit no shell effects. The parameter γ was determined by comparison with calculations using more realistic single particle states. This model was later refined in Refs. 30 and 31 by accounting for collective enhancement and for pairing correlations.

The KRK model[6] starts from a Fourier expansion of the shell fluctuations of the single particle state density. The last three relations of Eq. (9a) are replaced by

$$U-\Delta=\tilde{a}t^2+E_{\text{shell}}\left[\frac{\pi^2\omega^2t^2\cosh(\pi\omega t)}{\sinh^2(\pi\omega t)}-1\right] ,$$

$$S(U)=2\tilde{a}t+\frac{E_{\text{shell}}}{t}\left[\frac{\pi^2\omega^2t^2\cosh(\pi\omega t)}{\sinh^2(\pi\omega t)}-\frac{\pi\omega t}{\sinh(\pi\omega t)}\right] , \tag{12a}$$

$$D(U)=\frac{18}{\pi^4}\tilde{a}^{1/2}(U-\Delta)^{5/2} .$$

Again $\tilde{a}$ represents the asymptotic a parameter and

$$\omega=\omega_0A^{1/3} ,$$

where A is the mass number, is related to the major shell spacing. For the spin cutoff factor the following expression is proposed:

$$\sigma^2(U)=\frac{1}{\hbar^2}\theta\frac{S(U)}{2\tilde{a}} , \tag{12b}$$

where $S(U)$ is the entropy defined in Eq. (12a).

Ramamurthy *et al.*[32] pointed out the close relation be-

TABLE III. The resulting level density parameters.

Individual parameters	FG model a (MeV^{-1})	IST model $\gamma=0.40A^{-1/3}$ MeV^{-1} $\tilde{a}$ (MeV^{-1})	KRK model $\omega=0.17A^{+1/3}$ MeV^{-1} $\tilde{a}$ (MeV^{-1})
^{41}Ar	6.80±0.27	5.56±0.22	5.31±0.24
^{41}K	6.09±0.10	5.35±0.09	5.30±0.10
^{41}Ca	5.99±0.11	5.84±0.11	5.83±0.11
^{43}Ca	7.35±0.14	6.42±0.12	6.35±0.13
^{43}Sc	7.09±0.13	6.41±0.11	6.37±0.12
^{45}Ca	7.44±0.14	6.51±0.12	6.42±0.13
^{45}Sc	6.76±0.07	5.94±0.06	5.89±0.06
^{47}Ti	6.35±0.15	5.97±0.14	5.93±0.15
^{47}V	6.68±0.13	6.54±0.13	6.54±0.13
^{49}Ti	7.21±0.16	6.99±0.15	6.98±0.15
^{49}V	6.81±0.08	6.63±0.08	6.63±0.08
^{51}V	6.42±0.06	6.55±0.06	6.55±0.06
^{51}Cr	6.56±0.08	6.85±0.09	6.85±0.08
^{55}Cr	7.36±0.16	7.13±0.16	7.10±0.16
^{55}Mn	6.71±0.08	7.07±0.08	7.07±0.08
^{55}Fe	6.57±0.11	7.73±0.14	7.67±0.12
^{57}Fe	7.54±0.10	8.09±0.11	8.08±0.10
^{57}Co	6.76±0.07	7.97±0.08	7.88±0.08
^{59}Fe	8.56±0.23	8.42±0.23	8.41±0.23
^{59}Co	7.45±0.08	7.93±0.09	7.92±0.08
^{59}Ni	6.97±0.06	8.30±0.08	8.19±0.07
^{61}Ni	7.82±0.11	8.40±0.12	8.39±0.11
^{63}Ni	8.68±0.15	8.65±0.15	8.65±0.15
^{63}Cu	8.18±0.12	8.56±0.12	8.56±0.12
^{67}Zn	10.07±0.09	9.19±0.08	9.15±0.09
^{67}Ga	9.94±0.09	9.43±0.09	9.42±0.09
Average parameters			
α (MeV^{-1})	0.138	0.138	0.137
χ^2 (MeV^{-2})	14.642	2.111	2.090

tween the IST and the KRK models. In this context the authors proposed the following weak dependence on the mass number A of the parameter γ of the IST model:

$$\gamma=\gamma_0 A^{-1/3} ,$$

which we adopted in this paper.

From the average s-wave resonance spacings listed in Tables I and II the level density parameters a and $\tilde{a}$ of the three previously mentioned models were deduced. We assumed that at the relevant excitation energies the level density is parity independent. For the moment of inertia Θ in Eqs. (9b) and (12b) we used the rigid body value $(1/\hbar^2)\Theta=0.0150A^{5/3}$ and for the pairing correction Δ the values proposed by Gilbert and Cameron.[2] The results are listed in Table III. In view of the relatively narrow mass range we tried to represent the asymptotic a parameters as a linear function $\tilde{a}=\alpha A$ of the mass number A. The constants α and γ_0 or ω_0 of the IST and the KRK model, respectively, were found by minimizing the quantity

$$\chi^2=\sum_i (\tilde{a}_i-\alpha A_i)^2 ,$$

where the subscript i refers to the individual nuclei. The resulting parameters γ_0 and ω_0 are listed in the second line of Table III and hold for the following individual asymptotic a parameters. An analogous fit was also performed for the parameters a of the FG model. The results of the linear least-squares fits are shown in the lower part of Table III. Figure 4 and the χ^2 values of Table III show that, in contrast to the parameter a of the FG model, the asymptotic level density parameter $\tilde{a}$ of the IST and KRK models are quite well described by the relation $\tilde{a}=\alpha A$. Both models yield very similar results with a mean-square deviation of the individual asymptotic a parameters from the relation $\tilde{a}=\alpha A$ of about 4.3%. The spread of the $\tilde{a}$ values, however, is still definitely larger than the values due to the experimental errors of the quantities $\bar{D}$ (see last line of Table III). This indicates genuine deviations of the true $\tilde{a}$ values of a few percent from a smooth behavior.

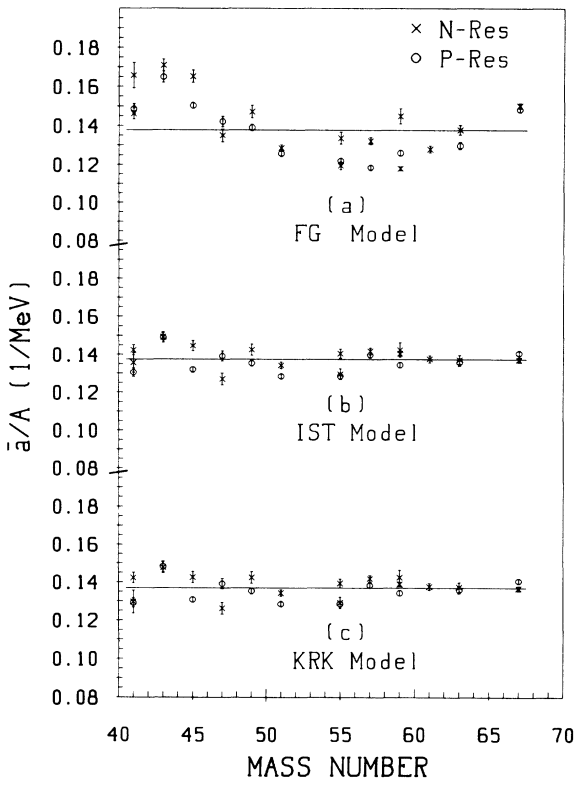

FIG. 4. Ratio of the deduced level density parameters to the mass number A (crosses for neutron and circles for proton resonances) and the results of the least-squares fits (solid curves) described in the text. For the FG model (a) the quantity $\tilde{a}$ stands for the parameter a of Eq. (9a) while for the IST model (b) and the KRK model (c) $\tilde{a}$ represents the asymptotic level density parameter $\tilde{a}$ defined in Eqs. (11) and (12a).

The errors listed for a and $\tilde{a}$ reflect only those of the underlying quantities $\bar{D}$. Additional uncertainties result from the neglect of a parity dependence of the level density as indicated by the data of Agrawal *et al.*[8] Accounting for the reported difference of the densities of positive and negative parity resonances with spin $I=\frac{1}{2}$ of ^{55}Cr decreases the respective parameters a and $\tilde{a}$ of Table III by about 5%. As an indication of the effects of uncertainties of the moment of inertia Θ we mention that a reduction of this quantity by a factor of 2 decreases the parameter α for all three considered models by about 12%.

If combined with a constant temperature form for the lower excitation energies, as suggested by Gilbert and Cameron,[2] these models can be used to deduce level densities for application oriented cross section calculations. The IST and KRK models offer clear advantages compared to the FG model. Apart from providing a more realistic energy dependence of the level densities, the simple relation between the parameter $\tilde{a}$ and the mass number allows a more reliable assessment of this parameter for nuclei with no resonance data.

We also determined level density parameters for two other simple models: the back-shifted Fermi gas model and the extended IST model described in Refs. 30 and 31. In the former case the level density parameters a show similar features as those displayed in Fig. 3(a) and the back shifts also exhibit considerable scatter. The asymptotic a parameters resulting from the extended IST model are well represented by the relation $\tilde{a}=\alpha A$. However, the numerical values of the constant α ($\alpha=0.093$ with and $\alpha=0.113$ without consideration of vibrational enhancement) and consequently also the asymptotic level densities differ from those of the original IST model.

VI. COMPARISON OF THE RESONANCE SPACINGS WITH PREDICTIONS OF THE MICROSCOPIC FERMI GAS MODEL

The nuclear level spacings determined from both neutron and proton resonance experiments were also compared with the corresponding results of a microscopic theory which includes the nuclear pairing interaction. The formalism of the code used[33] is described in Ref. 34. For the calculation of the single particle levels, the parameters of Seeger and Perisho[35] were used. Since all nu-

clei considered have odd mass, the presence of one unpaired nucleon must be accounted for in the calculation. As the blocking effect due to the odd particle is not included in the computer program, an empirical method of considering this effect was chosen. Such procedures were suggested by Behkami and Huizenga.[36] We made the calculations for the odd-A nuclei, using only 50% of the Gilbert-Cameron pairing energies[2] for that type of nucleon whose number is odd, and 100% for the other type. The resulting level densities were also compared to those calculated for the adjacent $A+1$ even-even nuclei with full pairing energies. The energy shift resulting from the above parameter choice which accounts for the additional quasiparticle is around 2 MeV.

According to our calculations, both parities contribute equal amounts to the level density at the projectile binding energy. A deviation of ρ^+/ρ^- at the binding energy from unity had been obtained by Mengoni *et al.*[37] and is physically more satisfactory.

The $\overline{D}$ values resulting from the microscopic Fermi gas calculation are presented in Table IV for s-wave proton and neutron resonances. The ratio of calculated to experimental $\overline{D}$ has a mean value of 1.02 for proton resonances with a standard deviation of 0.46, and 1.21 for neutron resonances with a standard deviation of 0.75. There is a correlation of merely 0.07 between the ratio of theoretical to measured $\overline{D}$ and average excitation energy, indicating that there is not a systematic problem with the slope of the level density.

TABLE IV. Comparison of experimental and calculated (microscopic Fermi gas model) proton and neutron resonance spacings of odd-A with nuclei with $A=41-67$.

Nucleus	Average excitation energy (MeV)	$\overline{D}_{exp}$[a] (keV)	$\overline{D}_{calc}$ (keV)
^{41}K	10.21	11.5±1.5	11.2
^{43}Sc	7.49	27.0±3.3	52.4
^{45}Sc	9.45	9.1±0.7	10.0
^{47}V	7.75	36.0±4.8	14.8
^{49}V	9.56	10.6±1.0	9.4
^{51}V	10.65	7.9±0.6	9.0
^{55}Mn	10.50	7.1±0.7	3.9
^{57}Co	8.82	19.4±2.4	29.7
^{57}Co	9.59	13.3±1.1	18.6
^{59}Co	10.22	4.3±0.4	4.9
^{63}Cu	9.03	5.9±0.7	4.4
^{67}Ga	8.42	2.5±0.2	1.0
Nucleus	Average excitation energy (MeV)	$\overline{D}_{exp}$[b] (keV)	$\overline{D}_{calc}$ (keV)
^{41}Ar	6.42	70.2±17.0	54.1
^{41}Ca	10.80	32.6±4.3	29.4
^{43}Ca	8.22	15.8±2.1	23.6
^{45}Ca	7.70	24.1±3.2	38.8
^{47}Ti	9.07	25.0±4.4	13.4
^{49}Ti	8.32	18.3±2.9	64.6
^{51}Cr	9.56	13.3±1.3	17.9
^{55}Cr	6.70	54.4±8.2	25.7
^{55}Fe	9.55	18.0±2.4	31.5
^{57}Fe	8.10	19.2±1.9	22.0
^{59}Fe	6.76	25.4±4.9	24.9
^{59}Ni	9.33	12.5±0.9	14.9
^{61}Ni	8.10	13.9±1.5	15.0
^{63}Ni	7.14	15.0±2.0	14.4
^{67}Zn	7.24	4.0±0.3	1.8

[a]From the publications quoted in Table II.
[b]From the publications quoted in Table I.

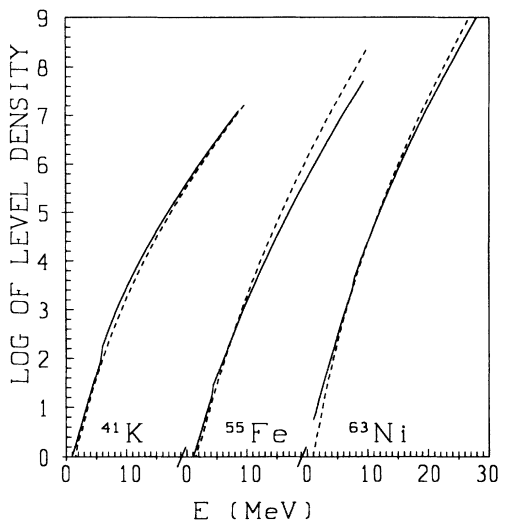

FIG. 5. Comparison of level densities from the microscopic FG model (solid lines) and the IST model (dashed lines) for ^{41}K, ^{55}Fe, and ^{63}Ni.

A comparison of the level densities resulting from the microscopic Fermi gas model and the model of Ignatyuk *et al.*[5] for 3 of the 26 nuclei considered is displayed in Fig. 5. The agreement is surprisingly good, given that no adjustment procedure has been used for the microscopic calculations. The discontinuity[38] of the pre-exponential factor (determinant) in the level density at the critical energy is due to an inaccuracy in the formalism[39] in the superconducting phase and should be of minor influence on the resonance spacings. The relevant energies are above the critical energies except for ^{61}Ni where the discontinuity is small.

VII. CONCLUSIONS

The results of this paper can be summarized as follows.

(i) High-resolution proton resonance data can be used to determine nuclear level densities with about equal accuracy as achieved in modern neutron resonance experiments.

(ii) Comparison of average s-wave spacings derived from both proton and neutron resonance data clearly show that both types of data give consistent results, indicating that the problem of missing levels can be handled adequately in both cases.

(iii) After conversion to a common excitation energy the spacings show a definite correlation with the shell correction energy E_{shell}, especially for $A=41$, 55, and 59 where data exist for three isobars with quite different

shell correction energies. The dependence of the average level spacing $\overline{D}$ on E_{shell} is approximately exponential, in good agreement with the IST (Ref. 5) and KRK (Ref. 6) models.

(iv) The observed average resonance spacings can be reasonably well described within the phenomenological IST and KRK level density models which contain an empirical shell correction, and the microscopic Fermi gas model including pairing. Attempts to describe the data with a simple Fermi gas model produced much poorer results.

(v) When the results from the phenomenological IST and KRK models in which two adjustable parameters are fitted to the data, and the microscopic Fermi gas calculations are compared, the quality of the description of the experimental resonance spacings by the former is somewhat superior. This can be estimated quantitatively by comparing the rms values of the quantity $\ln(\overline{D}_{\text{exp}}/\overline{D}_{\text{calc}})$ for the two cases. For the microscopic Fermi gas model a value of 0.51 is obtained from the data in Table IV. For the IST and KRK models this quantity can be derived from the spread of the $\tilde{a}/A$ values around the straight-line fits in Fig. 4. From the observed rms value of 4.3% and the sensitivity of the level density to the $\tilde{a}$ value one obtains an rms value of $\ln(\overline{D}_{\text{exp}}/\overline{D}_{\text{calc}})$ of about 0.34, considerably less than the value of 0.51 obtained for the microscopic Fermi gas calculations. Thus for estimating level densities for nuclei in the mass range studied in this work for which no resonance data are available it still seems best to use the IST or KRK model. Further refinements of the microscopic Fermi gas model may change this situation.

This work was supported in part by the U.S. Department of Energy, Office of High Energy and Nuclear Physics, under Contract Nos. DE-AC05-76ER01067 and DE-FG05-88ER40441.

[1]E. Erba, U. Facchini, and E. Saetta-Menichella, Nuovo Cimento **22**, 1237 (1961).
[2]A. Gilbert and A. W. G. Cameron, Can. J. Phys. **43**, 1446 (1965).
[3]W. Dilg *et al.*, Nucl. Phys. **A217**, 269 (1973).
[4]S. F. Mughabghab, M. Divadeenam, and N. E. Holden, *Neutron Cross-Sections* (Academic, New York, 1981), Vol. 1, Part A.
[5]A. V. Ignatyuk, G. N. Smirenkin, and A. S. Tishin, Sov. J. Nucl. Phys. **21**, 255 (1975).
[6]S. K. Kataria, V. S. Ramamurthy, and S. S. Kapoor, Phys. Rev. C **18**, 549 (1978).
[7]C. M. Perey *et al.*, in *Proceedings of the International Conference on Nuclear Science and Technology, Santa Fe, 1985,* edited by P. G. Young *et al.* (Gordon and Breach, New York, 1986), p. 1639.
[8]A. M. Agrawal, J. B. Garg, and J. Harvey, Phys. Rev. C **30**, 1880 (1984).
[9]C. M. Perey *et al.*, Phys. Rev. C **27**, 2556 (1983).
[10]J. B. Garg *et al.*, Phys. Rev. C **24**, 1922 (1981).
[11]Nuclear Energy Agency Newsletter, No. 27, 1982.
[12]F. H. Fröhner, Kernforschungszentrum Karlsruhe Report No. KFK 3553, 1983.
[13]E. G. Bilpuch, A. M. Lane, G. E. Mitchell, and J. D. Moses, Phys. Rep. **28C**, 145 (1976).
[14]B. Efron, SIAM Rev. **21**, 460 (1979).
[15]J. F. Shriner, Jr., G. E. Mitchell, and E. G. Bilpuch, Nucl. Instrum. Methods **A254**, 139 (1987).
[16]G. A. Keyworth, G.C. Kyker, Jr., E. G. Bilpuch, and H. W. Newson, Nucl. Phys. **89**, 590 (1966).
[17]W. M. Wilson, E. G. Bilpuch, and G. E. Mitchell, Nucl. Phys. **A271**, 49 (1976).
[18]W. M. Wilson, E. G. Bilpuch, and G. E. Mitchell, Nucl. Phys. **A245**, 262 (1975).
[19]J. D. Moses, H. W. Newson, E. G. Bilpuch, and G. E. Mitchell, Nucl. Phys. **A175**, 556 (1971).
[20]D. P. Lindstrom, H. W. Newson, E. G. Bilpuch, and G. E. Mitchell, Nucl. Phys. **A168**, 37 (1971).
[21]W. A. Watson III, E. G. Bilpuch, and G. E. Mitchell, Phys. Rev. C **24**, 1992 (1981).
[22]D. P. Lindstrom, H. W. Newson, E. G. Bilpuch, and G. E. Mitchell, Nucl. Phys. **A187**, 481 (1972).
[23]J. C. Browne, H. W. Newson, E. G. Bilpuch, and G. E. Mitchell, Nucl. Phys. **A153**, 481 (1970).
[24]N. H. Prochnow, H. W. Newson, E. G. Bilpuch, and G. E. Mitchell, Nucl. Phys. **A199**, 571 (1973).
[25]N. H. Prochnow, H. W. Newson, E. G. Bilpuch, and G. E. Mitchell, Nucl. Phys. **A194**, 353 (1972).
[26]N. H. Prochnow, H. W. Newson, E. G. Bilpuch, and G. E. Mitchell, Nucl. Phys. **A213**, 134 (1973).
[27]K. B. Sales, G. E. Mitchell, E. G. Bilpuch, and C. R. Westerfeldt, J. Phys. G **7**, 1405 (1981).
[28]J. R. Huizenga and L. G. Moretto, Annu. Rev. Nucl. Sci. **22**, 427 (1972).
[29]W. D. Meyers and W. S. Swiatecki, Ark. Fys. **36**, 343 (1967).
[30]A. V. Ignatyuk, K. K. Istekov, and G. N. Smirenkin, Sov. J. Nucl. Phys. **29**, 450 (1979).
[31]O. D. Grudzevich, A. V. Ignatyuk, V. I. Plyaskin, and A. V. Zelenetsky, Proceedings of the International Conference on Nuclear Data for Science and Technology, Mito, Japan, 1988 (unpublished).
[32]V. S. Ramamurthy, S. K. Kataria, and S. S. Kapoor, Proceedings of the IAEA Advisory Group Meeting on Basic and Applied Problems of Nuclear Level Density, Brookhaven, 1983, edited by M. R. Bhat, Brookhaven National Laboratory Report No. BNL-NCS-52694, 1983, p. 187.
[33]S. M. Grimes, code NILSSIG (unpublished).
[34]L. G. Moretto, Nucl. Phys. **A182**, 641 (1972).
[35]P. Seeger and R. C. Perisho, Los Alamos Scientific Laboratory Report No. LA-3751, 1967.
[36]A. N. Behkami and J. R. Huizenga, Nucl. Phys. **A217**, 78 (1973).
[37]A. Mengoni, F. Fabbri, and G. Maino, Nuovo Cimento **94A**, 297 (1986).
[38]L. G. Moretto, Nucl. Phys. **A185**, 145 (1972).
[39]A. V. Ignatyuk, Proceedings of the Consultants' Meeting on the Use of Nuclear Theory in Neutron Nuclear Data Evaluation, Trieste, 1975, International Atomic Energy Agency Report No. IAEA-190, 1976, p. 211.