Lepton-induced weak interactions in nuclei in the 1 to 3 GeV range

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Differential cross sections are obtained for the reactions $\nu_\mu + {}^{12}\mathrm{C} \rightarrow {}^{12}\mathrm{N} + \mu^-$ and $e^- + {}^{12}\mathrm{C} \rightarrow {}^{12}\mathrm{B} + \nu_e$ for incoming lepton energies of 1 and 3 GeV. The contributions of individual form factors to the differential cross section are also found. A discussion of the utility of these processes for studying the weak nuclear current is given.

The expected advent of electron accelerators such as CEBAF capable of producing monoenergetic beams of electrons up to 4 GeV and expandable to 6 GeV as well as the possibility that facilities may be constructed, such as LAMPF II, providing neutrino energies in the GeV range or greater make it interesting to examine electron and neutrino interactions in nuclei with the hope of studying the weak nuclear current.

The field of nuclear weak interactions has in general suffered from lack of data. In the $q^2 \approx 0$ range there are beta decay processes $N_i \rightarrow N_f + e^- + \bar{\nu}_e$. In the $|q^2| \approx m_\mu^2$ range a number of muon-capture reactions $\mu^- + N_i \rightarrow N_f + \nu_\mu$ have been observed. A few neutrino reactions[1,2] have also been run. However, there is no body of data which would permit a careful study of the weak nuclear interaction as a function of q^2 over a wide range of values.

It is with this in mind that we examine the question of what can be learned from reactions of the form $\nu_l + N_i \rightarrow N_f + l$, where l is a charged lepton and ν_l the corresponding neutrino. We shall also very briefly mention the reaction $e^- + N_i \rightarrow N_f + \nu_e$.

In a previous work[3] we considered the possibility of examining the differential cross section for the final state nucleus in an electron-induced weak process. Specifically we looked at $d\sigma/d\Omega_f$ for the reaction $e^- + {}^{12}\mathrm{C} \rightarrow {}^{12}\mathrm{B} + \nu_e$ in the 0.5–6 GeV range for E_e and obtained differential cross sections peaking in the 10^{-40} cm^2/sr range. This cross section is larger than would be anticipated from typical neutrino reactions in the 200–300 MeV range, which yield differential cross sections in the 10^{-40} cm^2/sr range. The reasons for this are easy to understand and will be discussed later. The difficulties associated with the observation of the final state nucleus include formidable background, particularly from electroproduction of pions, $e^- + N_i \rightarrow N_f + \nu_e + \pi^+$, as well as the difficulty of looking at relatively low energy nuclei. In this paper we therefore consider the possibility of examining the final state lepton. Clearly a final state high energy neutrino would not at present be observable so that we shall concentrate on the charged lepton final state, but for completeness we will present a final state neutrino cross section.

If we consider, for example, the process $\nu_\mu + {}^{12}\mathrm{C} \rightarrow {}^{12}\mathrm{N}_{(\mathrm{g.s.})} + \mu^-$ which is to a specific final state, we expect that the nuclear form factors will fall with increasing q^2. In principle, the W propagator grows as q^2 approaches m_W^2; however, in the energy range under consideration $|q^2|$ peaks at around 35 GeV^2, which is far below that which is needed to appreciably increase the size of the propagator. Thus it is adequate to represent the transition matrix element by

$$M = \frac{[G\cos(\theta_c)]}{\sqrt{2}} \bar{u}_{\mu^-} \gamma^\mu (1-\gamma_5) u_{\nu_\mu} \langle {}^{12}\mathrm{N} | J_\mu(0) | {}^{12}\mathrm{C} \rangle \;, \tag{1}$$

where $J_\mu(0) = V_\mu(0) - A_\mu(0)$ is the weak hadronic current. The nuclear matrix elements $\langle {}^{12}\mathrm{N} | A_\mu(0) | {}^{12}\mathrm{C} \rangle$ and $\langle {}^{12}\mathrm{N} | V_\mu(0) | {}^{12}\mathrm{C} \rangle$ may be written as[4]

$$\langle {}^{12}\mathrm{N} | V_\mu(0) | {}^{12}\mathrm{C} \rangle = -i\sqrt{2} m_i \epsilon_{\mu\delta\beta\gamma} q^\delta \xi^\beta Q^\gamma \frac{F_M(q^2)}{(2m_i 2m_p)} \;, \tag{2a}$$

$$\langle {}^{12}\mathrm{N} | A_\mu(0) | {}^{12}\mathrm{C} \rangle = \sqrt{2} m_i \left[\xi_\mu F_A(q^2) + q_\mu \xi\cdot q \frac{F_p(q^2)}{m_\pi^2} - Q_\mu \xi \cdot q \frac{F_E(q^2)}{(2m_i 2m_p)} \right] \;, \tag{2b}$$

where $q_\mu = P_{f\mu} - P_{i\mu}$ is the four-momentum transfer, $Q_\mu = P_{f\mu} + P_{i\mu}$, $P_{i\mu}$ and $P_{f\mu}$ are the ^{12}C and ^{12}N four-momenta, respectively, and m_f, m_i, m_π, and m_μ are the ^{12}N, ^{12}C, pion, and muon masses, while ξ_μ is the ^{12}N polarization vector. In writing Eqs. (2a) and (2b) we have assumed only first class currents.

The structures of Eqs. (2a) and (2b) are determined by Lorentz invariance for the spin and parity assignments of the initial and final states. The physics of the reaction is thus for the most part contained in the form factors F_A, F_M, F_E, and F_P. Almost all of the information available concerning these form factors is for $|q^2| \le m_\pi^2$. In this q^2 range, the weak magnetism form factor, F_M, may be written in dipole form as

$$F_M(q^2)=\frac{F_M(0)}{(1-q^2/M_M^2)^2}\ , \tag{3a}$$

$$F_M(0)=3.66\ , \tag{3b}$$

$$M_M^2 \approx 2.74 m_\pi^2\ . \tag{3c}$$

Equations (3a), (3b), and (3c) are extracted from electron scattering reactions $e^- + {}^{12}\mathrm{C} \to {}^{12}\mathrm{C}^* + e'^-$ via conserved vector current (CVC). In general, they are not expected to hold for high $|q^2|$. Nonetheless, we expect a falloff for each exclusive channel with increasing $|q^2|$ as more channels open, so we accept Eqs. (3a), (3b), and (3c) as an approximation because we are interested in an estimate of the cross sections and their general behavior.

A result[5] based on the nucleons-only impulse approximation leads to an estimate for $F_A(q^2)$ given by

$$\frac{F_A(q^2)}{F_A(0)} \approx \frac{F_M(q^2)}{F_M(0)}\ . \tag{4}$$

This estimate works well in the muon-capture range for a number of light nuclei,[6] and in the case of ^{12}C has been confirmed[7] at $q^2=-0.74m_\mu^2$. At $q^2 \approx 0$, $F_A(0)$ is obtained from beta-decay data so that $F_A(q^2)$ may be written as

$$F_A(q^2)=\frac{F_A(0)}{(1-q^2/M_A^2)^2}\ , \tag{5a}$$

$$F_A(0)=1.03\ , \tag{5b}$$

where $M_A^2 \approx M_M^2$.

The form factor $F_P(q^2)$ is more controversial. We write it in the form

$$F_P(q^2)=\frac{m_\pi^2 F_A(q^2)}{m_\pi^2-q^2}[1+\epsilon(q^2)]\ , \tag{6}$$

where $\epsilon(q^2)$ contains all corrections to simple pion pole dominance. The value of $\epsilon(q^2)$ has been estimated[8] by Kim and Primakoff to be $\epsilon(q^2=-0.74m_\mu^2) \approx -0.15$ for $^{12}\mathrm{C} \to {}^{12}\mathrm{N}$ transitions and to have a limiting value $\epsilon(-m_\mu^2) \approx -0.29$ for large nuclei. However, it will be seen that the differential cross section is extremely insensitive to $F_P(q^2)$ for the processes under consideration here where the incoming particle energy ranges from 1 GeV and above. Because all terms in the transition matrix squared containing F_P are proportional to the lepton mass squared, for the electron case, they are ignorable and even in the muon case, they are still generally of the order of a few percent.

Finally we consider $F_E(q^2)$. A calculation by Hwang and Primakoff[9] indicates that

$$\frac{F_E(q^2)}{F_E(0)} \approx \frac{F_M(q^2)}{F_M(0)}$$

and that

$$\frac{F_E(q^2)}{F_A(0)}=3.64\ .$$

We therefore take $F_E(q^2)$ as

$$F_E(q^2)=\frac{F_E(0)}{(1-q^2/M_E^2)^2}\ , \tag{7a}$$

$$F_E(0)=3.75\ , \tag{7b}$$

with $M_E^2 \approx M_M^2$. We are thus in a position to calculate $d\sigma/d\Omega_l$, the differential cross section of the outgoing lepton.

From Eqs. (1), (2), (3), (5), (6), and (7) the differential cross section may be calculated. We write

$$\frac{d\sigma}{d\Omega_l}=\frac{k m_i G^2\cos^2(\theta_C)\,|\mathbf{P}_l|}{16\pi^2 E_{l'}\left|m_i+E_{l'}\left[1-\frac{E_l}{|\mathbf{P}_l|}\cos\theta\right]\right|}\,|M|^2\ , \tag{8}$$

where l refers to the outgoing lepton (here a neutrino) and l' refers to the incoming lepton. The quantity $k=2$ for an incoming neutrino, and $k=1$ for an incoming massive lepton. The quantity $|M|^2$ may be given by

$$\begin{aligned}
|M|^2 = {} & \frac{F_M^2}{8m_i^2m_p^2}\{Q^2[(\nu\cdot l)^2-m_l^2(\nu\cdot l)]+(Q\cdot\nu)^2(\nu\cdot l-m_l^2)+(Q\cdot l)^2(\nu\cdot l)\}+F_A^2\left[\nu l+\frac{2(P_f\cdot\nu)(P_f\cdot l)}{m_f^2}\right] \\
& +\frac{F_P^2 m_l^2}{m_\pi^4}(\nu\cdot l)\left[2\nu\cdot l-m_l^2+\frac{(P_f\cdot\nu)^2+(P_f\cdot l)^2-2(P_f\cdot\nu)(P_f\cdot l)}{m_f^2}\right] \\
& +\frac{F_E^2}{16m_P^2m_i^2}[2(Q\cdot\nu)(Q\cdot l)-Q^2(\nu\cdot l)]\left[2\nu\cdot l-m_l^2+\frac{(P_f\cdot\nu)^2+(P_f\cdot l)^2-2(P_f\cdot\nu)(P_f\cdot l)}{m_f^2}\right] \\
& +\frac{F_MF_A}{m_im_p}[(\nu\cdot l-m_l^2)(Q\cdot\nu)+(\nu\cdot l)(Q\cdot l)]+\frac{F_AF_P}{m_\pi^2}(2m_l^2)\left[-(\nu\cdot l)+\frac{(P_f\cdot\nu)(P_f\cdot l)-(P_f\cdot\nu)^2}{m_f^2}\right] \\
& -\frac{F_AF_E}{2m_im_P}\left\{m_l^2(Q\cdot\nu)+\frac{P_f\cdot\nu-P_f\cdot l}{m_f^2}[(Q\cdot l)(P_f\cdot\nu)+(Q\cdot\nu)(P_f\cdot l)-(\nu\cdot l)(Q\cdot P_f)]\right\} \\
& +\frac{F_EF_P}{2m_\pi^2m_im_P}m_l^2(Q\cdot\nu)\left[2\nu\cdot l-m_l^2+\frac{(P_f\cdot\nu)^2+(P_f\cdot l)^2-2(P_f\cdot\nu)(P_f\cdot l)}{m_f^2}\right]
\end{aligned} \tag{9}$$

for the incoming lepton taken to be a neutrino. For the incoming lepton taken to be an electron or muon, the expression is very similar. In Fig. 1 we obtain the differential cross section for an incoming muon neutrino with E_ν at 1 GeV and 3 GeV, respectively, in terms of the laboratory angle for the outgoing μ^-. In Fig. 2 we do the same calculation for an incoming electron so that the final state is an outgoing neutrino. We note that the differential cross section yields almost the same results for an incoming electron neutrino as it does for a muon neutrino, and so we do not present results specifically for the electron neutrino case. In Fig. 3, we give results for the relative contributions of the form factors, $F_A(q^2)$, $F_M(q^2)$, $F_E(q^2)$, and $F_P(q^2)$, the differential cross section. We do this by setting all form factors except one to zero. Thus these curves are the result of contributions coming from the appropriate form factor alone. Finally, in Fig. 4 we give $|q^2|$ as a function of θ for 3 GeV and 1 GeV incoming muon neutrino cases.

As can be seen from Figs. 1, 2, and 4 for both neutrino- and charged-lepton-induced processes, the cross sections peak at the lowest $|q^2|$ values and all with increasing $|q^2|$ (or outgoing lepton angle). We furthermore note a pronounced peaking as the incoming lepton energy is increased. From Eqs. (8) and (9), $d\sigma/d\Omega$ increases approximately as $E_{l'}^2$ with incoming lepton energy so that at the lowest $|q^2|$ values, $d\sigma/d\Omega$ for a 3 GeV incoming lepton is approximately nine times as large as that for an incoming 1 GeV lepton. This can be seen at very small angles in Figs. 1 and 2. However, because the form factors are all dipole, and $|q^2|$ grows substantially more rapidly for the 3 GeV case than for the 1 GeV case, the cross sections fall much more rapidly with increasing angle as the incoming lepton energy increases. Thus an $E_{l'}$ increases, larger differential cross sections become compressed in smaller angular ranges. Thus the case of interest to us is clearly high $E_{l'}$ but low q^2.

Nonetheless, the $E_{l'}^2$ dependence is very attractive for these processes, yielding differential cross sections 2 to 3 orders of magnitude higher than the medium energy processes which have been considered.[10] This is true even for the 1 GeV incoming lepton case. Thus if it is possible to detect an outgoing charged lepton in a fairly narrow cone of less than 5 deg it may be possible to determine F_A as a function of q^2. This can be seen in Fig. 3 because in the very small angular range, contributions from the other form factors are sufficiently small to be initially ignorable. As we have mentioned before the q^2 dependence of F_A is identical to that of $F_M(q^2)$ in the nucleons-only impulse approximation and $F_M(q^2)$ is well known via CVC from electron scattering data. Sharp departures from Eq. (4) might be evidence for meson exchange current contributions.

If measurements in the $10^{-40}-10^{-41}$ cm^2/sr range are feasible, direct determination of the form factors $F_A(q^2)$,

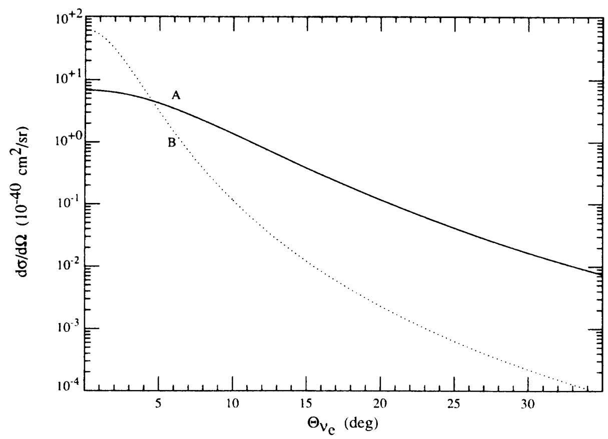

FIG. 2. Plot of the differential cross section for the reaction $e^- + {}^{12}C \rightarrow {}^{12}B + \nu_e$ as a function of outgoing muon laboratory angle. Curves A and B are for 1 GeV and 3 GeV incident electrons, respectively.

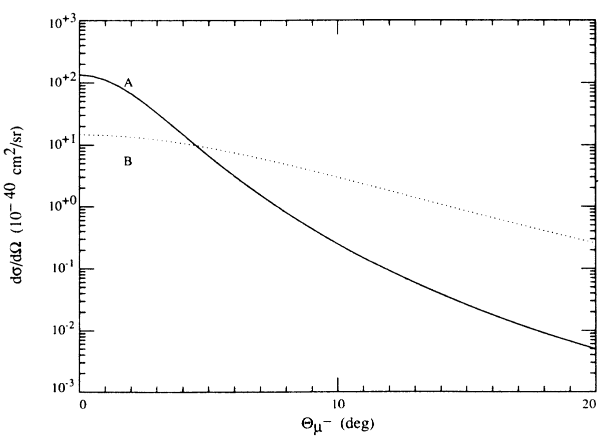

FIG. 1. Plot of the differential cross section for the reaction $\nu_\mu + {}^{12}C \rightarrow {}^{12}N + \mu^-$ as a function of outgoing muon laboratory angle. Curves A and B are for 3 GeV and 1 GeV incident neutrinos, respectively.

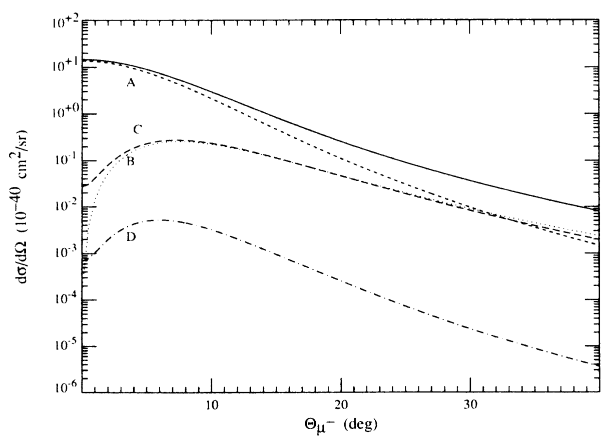

FIG. 3. Plot of the differential cross section for the reaction $\nu_\mu + {}^{12}C \rightarrow {}^{12}N + \mu^-$ as a function of outgoing muon laboratory angle for 1 GeV neutrinos. Curves A, B, C, and D refer to the contribution from F_A, F_M, F_E, and F_P, respectively.

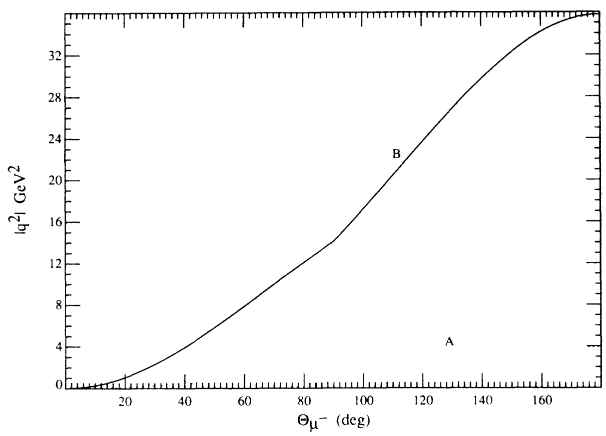

FIG. 4. Plot of $|q^2|$ as a function of outgoing muon laboratory angle. Curves A and B are for 1 GeV and 3 GeV incident neutrinos, respectively.

$F_M(q^2)$, and $F_E(q^2)$ as a function of q^2 over a range of q^2 would be possible. At angles of 15–25 deg the contributions from F_M and F_E each give rise to about 50% of the contribution from F_A. Thus by varying $E_{l'}$ and the angle, it should be possible to take sufficient measurements to determine F_A, F_M, and F_E. This has never been done before and again might yield useful information on nuclear structure, particularly on the role of meson exchange currents, or perhaps relativistic terms.

Finally, some attention should be given to the relative utility of the various lepton-induced weak cross sections. Electron beams up to the 4 GeV range will be available with the construction of CEBAF providing a precise electron energy which can be conveniently varied, thus simplifying the analysis of cross sections. However, as has been previously stressed, the only practical final state particle to measure is in this case the outgoing nucleus. Formidable background problems are encountered in such measurements. Currently, any measurement of the final state neutrino is unlikely. On the other hand, a neutrino beam give rise to a charged lepton in the final state which is easily observed[11,12] under conditions for which the differential cross section is large. However, neutrino beams are rarely pure, which greatly complicates the analysis of a cross section. Thus the present situation is unclear. However, the relatively large differential cross sections obtained from using energetic incoming leptons as projectiles is certainly worth further consideration and study.

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[4]S. L. Mintz, Phys. Rev. C **28**, 556 (1983). We remark that the method of calculation used here is the elementary particle model approach of Kim and Primakoff (see Ref. 5), where the initial and final nuclear states are treated as eigenstates of a strong Hamiltonian and electroweak interactions are added as perturbations. Structure effects then show up in the behavior of the form factors. For structureless particles, the form factors are constant and as the structure becomes manifest the form factors fall off more rapidly. The situation is in some respects similar to a description of a proton in terms of form factors and Dirac matrices sandwiched between spinors. The q^2 dependence of the form factors reflects the underlying quark structure. Therefore in the approach used here strong final state behavior is included, but electroweak final state interactions must be additionally included. Although clearly a working microscopic model is the most desirable situation, for the estimate we are seeking here our treatment should be reasonable. It is known to work very well for muon capture in the $q^2 \simeq -m_\mu^2$ region and gives results in reasonable agreement with other model calculations for neutrino reactions in the several hundred MeV range with maximal q^2 values of $-5m_\pi^2$ to $-6m_\pi^2$. Here we are interested in high incident neutrino energy but low q^2. For the 1 GeV case, for example, the cross section is maximal for q^2 comparable to that for muon capture, and when it has fallen over an order of magnitude, $q^2 \simeq -3m_\pi^2$, again within the range where we expect the model to be effective.

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[11]As θ increases so does q^2, so that as θ changes from 0° to 5°, q^2 increases to $-0.63m_\pi^2$. At 15°, $q^2=-3.105m_\pi^2$. Larger q^2 values lead to rapidly falling cross sections and more channels opening, making analysis too difficult. In general a change of 1 deg in angle corresponds to an average change in $|q|$ of 60 MeV.

[12]Recently an experiment in a few hundred MeV range for the reaction $\nu_\mu + {}^{12}C \rightarrow {}^{12}N_{(g.s.)} + \mu^-$ was undertaken. A sharp peak was noted for the ${}^{12}N_{(g.s.)}$ state. The largest background came from $\bar{\nu}$ impurities leading to the ${}^{12}B$ final state (T. Dombeck, private communication). If the charge of the final muon (electron) is observed these states will be readily distinguishable. As long as we restrict our case of interest to small q^2, this circumstance should persist.