PHYSICAL REVIEW C VOLUME 38, NUMBER 5 NOVEMBER 1988

Renormalizability of effective pionic Lagrangians in the nuclear medium

W. M. Alberico,[a] R. Cenni,[b] A. Molinari,[a] and P. Saracco[b]
[a]Dipartimento di Fisica Teorica, Universitá di Torino and Istituto Nazionale di Fisica Nucleare, Sezione di Torino, Torino, Italy
[b]Istituto Nazionale di Fisica Nucleare, Sezione di Genova, and Dipartimento di Fisica, Universitá di Genova, 16146 Genova, Italy
(Received 29 December 1987)

We show the renormalizability of a Lagrangian of nucleons and pions interacting via a pseudoscalar coupling in the presence of a medium. We extend the proof to the effective Lagrangian obtained from the previous one through the functional integration over the fermionic fields. Notably in the latter case the renormalization takes place within each order of the corresponding loop expansion. This result is of significance in view of the connection between the loop expansion and the conventional many-body approximation methods.

I. INTRODUCTION

In a recent paper[1] we dealt with a system of pions and nucleons with functional techniques. Such an approach is quite convenient since it allows us to project the whole problem into a purely pionic or purely nucleonic space. Accordingly we have been able to describe the response of our system to an external electromagnetic field in terms of pionic degrees of freedom only, which is indeed appropriate for intermediate energy nuclear physics where the pions are known to play a quite relevant role. Technically this aim has been achieved through the introduction of an effective bosonic Lagrangian, which not only exactly embodies the dynamics of the problem, but also naturally leads to a loop expansion grouping together, at each order, important classes of many-body diagrams.

In Ref. 1, however, no attention has been payed to the renormalizability of the approach, notwithstanding that the pseudoscalar pion-nucleon ($N\pi$) Lagrangian, the one we are concerned with, yields divergencies in a perturbative framework. This point seems to us worthy of consideration. Indeed, although it is well known that a pseudoscalar $N\pi$ Lagrangian is renormalizable in the vacuum, yet one would like to ascertain whether the same remains true in the presence of a medium and, even more so, how the renormalizability is realized order by order in the loop expansion of the effective theory, an outcome expected, but far from evident.

Until now the renormalizability of a theory in the presence of a Fermi sea has not been much explored. For all practical purposes the (unphysical) divergent terms have been simply omitted, with the prescription of replacing, at the same time, the bare masses and coupling constants with the physical ones. A notable exception to this pragmatic attitude is represented by the quantum hadrodynamics (QHD) theory of nuclear matter and finite nuclei[2-4] (see also, however, the recently carried out investigation on the Bonn potential[5]).

In the context of QHD the importance of verifying the unfolding of renormalizability for selected classes of many-body perturbative diagrams, the ones, for example, associated with the Hartree-Fock[3] or the Bethe-Salpeter[4] equations, has been stressed. Our approach should be viewed along the same line of thought, but rather than focusing on a given set of many-body diagrams, suggested by intuition, we concentrate on those naturally arising in the loop expansion of an effective Lagrangian.

This procedure appears to us preferable on two counts: Firstly, the effective Lagrangian is not assumed, but rather deduced, with functional methods, from the standard $\gamma_5 N\pi$ Lagrangian; secondly, the loop expansion significantly improves upon the standard perturbation theory since it allows to classify, in a coherent framework, the corrections to the many-body contributions appearing in leading order.

Clearly our work, being restricted to the treatment of nucleons and pions, is far from complete. It offers, however, a scheme which could be of some guidance when more realistic dynamical situations, encompassing a variety of mesons, arise.

The present paper is organized as follows. In Secs. II and III we explore, within the standard perturbative frame, the influence of the medium on the renormalization mechanism holding in the vacuum. We confine ourselves to treating this item mainly at the one-loop level, partly because this is sufficient for introducing the definitions needed in the general proof given in the following section, and partly to avoid the repetition of all the details of the Bogoliubov-Parasyuk[6-8] renormalization method. For the latter we just sketch the proof of its validity in the presence of a medium. In Sec. IV we carry out by induction the general proof of the renormalizability, at *each* order of the loop expansion, of our effective bosonic Lagrangian.

II. THE ELEMENTARY AND THE EFFECTIVE LAGRANGIANS

In this paper we assume as fundamental the Lagrangian

$$\mathcal{L}=\mathcal{L}_0+\mathcal{L}_{\rm int} , \tag{1}$$

where

$$\mathcal{L}_0=\bar\psi(i\partial\!\!\!/-M)\psi+\tfrac{1}{2}(\partial_\mu\mathbf{\Phi})^2-\frac{m_\pi^2}{2}\mathbf{\Phi}^2 \tag{2}$$

and

$$\mathcal{L}_{\rm int}=-ig\bar\psi\gamma_5\boldsymbol{\tau}\psi\cdot\mathbf{\Phi} , \tag{3}$$

a pseudoscalar pion-nucleon coupling being assumed. The associated generating functional reads

$$\mathbf{Z}[\mathbf{J},\eta,\bar\eta]=\frac{1}{\mathcal{N}}\int\mathcal{D}[\psi,\bar\psi,\mathbf{\Phi}]\exp\left[i\int d^4x[\mathcal{L}(\psi,\bar\psi,\mathbf{\Phi})+\bar\psi\eta+\bar\eta\psi+\mathbf{J}\cdot\mathbf{\Phi}]\right] , \tag{4}$$

where the presence of a Fermi sea is not explicitly apparent, being hidden in the boundary conditions for the integration variables $\psi(\mathbf{r},t),\bar\psi(\mathbf{r},t)$ as $t\to\pm\infty$.[9,10] As a consequence one can formally manipulate the path integral without worrying about the nature of the system: Whether it is the vacuum or nuclear matter is can be specified afterwards.

However, in discussing the perturbative expansion, the boundary conditions on the fields become of relevance (for an explicative example in the bosonic case see Ref. 11). In fact, in this framework, the generating functional (4) is conveniently rewritten as follows:

$$\mathbf{Z}[\mathbf{J},\eta,\bar\eta]=\exp\left[i\int d^4x\,\mathcal{L}_{\rm int}\left[\frac{\delta}{\delta\mathbf{J}},\frac{\delta}{\delta\eta},\frac{\delta}{\delta\bar\eta}\right]\right]\mathbf{Z}_0[\mathbf{J},\eta,\bar\eta] , \tag{5}$$

$\mathbf{Z}_0$ being given by (4) with the free Lagrangian $\mathcal{L}_0$ replacing the full one. Therefore when the functional integration for the free generator $\mathbf{Z}_0$ is explicitly carried out, then the boundary conditions on the fermionic fields should be specified through the particular fermion Green's functions one uses. Thus the Feynman propagator

$$S_F^{(v)}(q)=\frac{1}{q\!\!\!/-M+i\eta} \tag{6}$$

is the appropriate one for the vacuum, whereas in the presence of a filled Fermi sea the single-particle propagator reads instead

$$\begin{aligned}S_F^{(m)}(q)&=\frac{q\!\!\!/+M}{2E_q}\left[\frac{\theta(q-k_F)}{q_0-E_q+i\eta}+\frac{\theta(k_F-q)}{q_0-E_q-i\eta}-\frac{1}{q_0+E_q-i\eta}\right]\\&=S_F^{(v)}(q)+2\pi i\delta(q_0-E_q)\theta(k_F-q)\frac{q\!\!\!/+M}{2E_q} ,\end{aligned} \tag{7}$$

k_F being the Fermi momentum. Obviously the difference between (6) and (7), which disappears in the limit $k_F\to 0$, only concerns particles with $q\le k_F$, the ones propagating as holes.

Also when an effective action $S_B^{\rm eff}$ is treated with a semiclassical expansion, the boundary conditions on ψ and $\bar\psi$ are explicitly required. The derivation of $S_B^{\rm eff}$ has been carried out in Ref. 1 via a Gaussian integration over the fermionic fields in the generating functional (4). This, dropping for simplicity the external sources η and $\bar\eta$, was found to be given by the following expression:

$$\mathbf{Z}[\mathbf{J}]=\frac{1}{\mathcal{N}}\int\mathcal{D}[\mathbf{\Phi}]\exp\left[i\left[S_B^{\rm eff}+\int d^4x\,\mathbf{J}\cdot\mathbf{\Phi}\right]\right] , \tag{8}$$

with

$$S_B^{\rm eff}=\int d^4x\left[\tfrac{1}{2}(\partial_\mu\mathbf{\Phi})^2-\frac{m_\pi^2}{2}\mathbf{\Phi}^2\right]+i\,{\rm tr}\sum_{n=1}^{\infty}\frac{1}{n}\left[ig\gamma_5\boldsymbol{\tau}\cdot\mathbf{\Phi}S_F\right]^n . \tag{9}$$

The above formula can be recast into the more convenient form

$$S_B^{\rm eff}=\tfrac{1}{2}\int d^4x\,d^4y\,\mathbf{\Phi}(x)\Delta_0^{-1}(x-y)\mathbf{\Phi}(y)+i\sum_{n=1}^{\infty}\frac{1}{n}\int d^4x_1\cdots d^4x_n C_n^{l_1\cdots l_n}\Phi_{l_1}(x_1)\cdots\Phi_{l_n}(x_n) , \tag{10}$$

where $\Delta_0(x-y)$ is the free pion propagator and the coefficients C_n play the role of nonlocal, elementary vertices of the effective theory *exactly* embodying the nucleon dynamics of the model. Their explicit expression reads

$$C_n^{l_1\cdots l_n}(x_1\cdots x_n)=(ig)^n\,{\rm tr}S_F(x_1-x_2)\gamma_5\tau^{l_1}S_F(x_2-x_3)\gamma_5\tau^{l_2}\ldots S_F(x_n-x_1)\gamma_5\tau^{l_n} \tag{11}$$

and, as before, the boundary conditions on the fermionic fields are automatically included in the appropriate Green's functions entering into (11).

Thus, from a formal point of view, the transition from the vacuum to the medium simply amounts to replacing $S_F^{(v)}$ with $S_F^{(m)}$ whenever a fermion propagator is involved.

In passing we note that in Eq. (11) the tadpole term C_1 actually vanishes, except in the presence of pion condensation. Notably this situation of broken translational invariance can also be handled within the present framework by means of an appropriate field translation.

III. THE RENORMALIZABILITY OF PERTURBATION THEORY IN THE MEDIUM

Let us now face the problem of renormalizability. As is well known, the Lagrangian (1) leads to a meaningless theory already in the vacuum, thus entailing an ill-defined generating functional (4); as a consequence the same will occur in the medium, and the generating functional (8) will be ill-defined as well. Of course the Lagrangian (1), including a Φ^4 term, is renormalizable in the vacuum by introducing an appropriate set of counterterms and a suitable regularization procedure. This aim is achieved by replacing (1) with the new Lagrangian

$$\begin{aligned}\mathcal{L} &= \bar\psi(i\not\partial - M)\psi - \delta M\bar\psi\psi + (Z_2-1)\bar\psi(i\not\partial - M)\psi \\ &\quad + \tfrac{1}{2}(\partial_\mu\Phi)^2 - \tfrac{1}{2}m_\pi^2\Phi^2 - \tfrac{1}{2}\delta m_\pi^2\Phi^2 \\ &\quad + (Z_3-1)\tfrac{1}{2}[(\partial_\mu\Phi)^2 - m_\pi^2\Phi^2] \\ &\quad + ig\bar\psi\gamma_5\tau\psi\cdot\Phi + i\delta g\bar\psi\gamma_5\tau\psi\cdot\Phi - \lambda\Phi^4 - \delta\lambda\Phi^4 ,\end{aligned} \tag{12}$$

the counterterms being fixed through the identification of M, m_π, g and λ (this term being usually set to zero in nuclear physics) with the physical masses and coupling constants. The expansion of the counterterms in powers of $\hbar$, e.g.,

$$\delta M = \hbar\delta M^{(1)} + \hbar^2\delta M^{(2)} + \cdots \tag{13}$$

allows us then to cancel the divergences order by order, thus regularizing the elementary diagrams of the theory. For the Lagrangian (12) these are shown in Fig. 1.

Notably this procedure remains valid in the presence of a medium, since the technique outlined above fixes the counterterms in such a way that they can no longer be altered by the medium. This can be seen by considering, for example, the regularization in the vacuum and in the medium of the elementary diagram (a) of Fig. 1. The latter corresponds to the inverse propagator $\Gamma_{\bar\psi\psi}(p)$ (p being the four-momentum of the fermionic line).

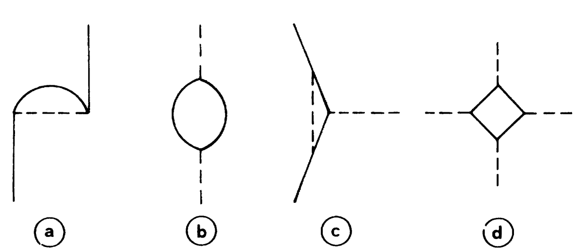

FIG. 1. Elementary diagrams of Lagrangian (11).

At the tree level one has (superscripts indicate orders in $\hbar$)

$$\Gamma^{(0)}_{\bar\psi\psi(v)}(p) = \not p - M , \tag{14}$$

and, to first order in $\hbar$,

$$\begin{aligned}\Gamma^{(1)}_{\bar\psi\psi(v)}(p) &= \not p - M - \Sigma^{(1)}_{(v)}(p) - \hbar\delta M^{(1)} \\ &\quad + (Z_2-1)^{(1)}\hbar(\not p - M) ,\end{aligned} \tag{15}$$

where the standard expansion

$$\begin{aligned}\Sigma^{(1)}_{(v)}(p) &= \Sigma^{(1)}_{(v)}(\not p = M) + (\not p - M)\Sigma'^{(1)}_{(v)}(\not p = M) \\ &\quad + \tfrac{1}{2}(\not p - M)^2\Sigma^{(1)}_r(p)\end{aligned} \tag{16}$$

holds for self-energy $\Sigma^{(1)}_{(v)}(p)$. In (16) the first two terms on the right-hand side (rhs) are divergent and a regularization procedure needs to be introduced for their practical evaluation, while the third one is regular by itself. Accordingly the divergences are canceled by setting, after the insertion of (16) and (15),

$$\hbar\delta M^{(1)} = -\Sigma^{(1)}_{(v)}(\not p = M) , \tag{17}$$

$$\hbar(Z_2-1)^{(1)} = -\Sigma'^{(1)}_{(v)}(\not p = M) ; \tag{18}$$

what is left out is just the renormalized inverse propagator to the order $\hbar$,

$$\Gamma^{(1)}_{\bar\psi\psi(v)}(p) = \not p - M - \tfrac{1}{2}(\not p - M)^2\Sigma^{(1)}_r(p) . \tag{19}$$

Obviously the whole self-energy correction is negligible for nucleons close to the mass shell, since the factor $(\not p - M)^2$ is then very small.

Let us now evaluate $\Gamma_{\bar\psi\psi(m)}$, to the same order in $\hbar$, in the presence of a nuclear medium. For this purpose we utilize the appropriate fermion propagator (7) in the internal line, getting, instead of (16),

$$\Sigma^{(1)}_{(m)}(p) = \Sigma^{(1)}_{(v)}(p) + \tilde\Sigma^{(1)}(p) , \tag{20}$$

where

$$\begin{aligned}\tilde\Sigma^{(1)}(p) &= -6i\pi g^2 \int \frac{d^4q}{(2\pi)^4}\delta(p_0 - q_0 - E_{p-q}) \\ &\quad \times \theta(k_F - |\mathbf{p}-\mathbf{q}|) \\ &\quad \times \mathrm{tr}\left[\gamma_5\frac{\not p - \not q + M}{2E_{p-q}}\gamma_5\frac{1}{q^2 - m_\pi^2 + i\eta}\right] .\end{aligned} \tag{21}$$

By repeating then the same steps leading from (15) to (19), one ends up with the customary expression

$$\begin{aligned}\Gamma^{(1)}_{\bar\psi\psi(m)} &= \not p - M - \tilde\Sigma^{(1)}(p) - \tfrac{1}{2}(\not p - M)^2\Sigma^{(1)}_r(p) \\ &\simeq \not p - M - \tilde\Sigma^{(1)}(p) ,\end{aligned} \tag{22}$$

which shows that the values for $\delta M^{(1)}$ and $(Z_2-1)^{(1)}$ are fixed once and for all by the renormalization in the vacuum, the additional contribution stemming from the medium being regular. It is worth noticing that the latter can be recast in the following form:

$$\tilde{\Sigma}^{(1)}(p)=-3\frac{g^2}{4M^2}\int\frac{d^3q}{(2\pi)^3}\frac{\theta(k_F-|\mathbf{p}-\mathbf{q}|)}{(p_0-E_{p-q})^2-\mathbf{q}^2-m_\pi^2+i\eta} \tag{23}$$

which is recognized as the Fock contribution to the nucleon self-energy arising from the one-pion exchange (OPE) potential. As for the other diagrams of Fig. 1 the technique for their regularization is perfectly analogous to the one illustrated above.

It thus appears that, for the Lagrangian we are concerned with, the renormalizability of the theory in the vacuum implies as well the disappearance of the unphysical divergences when the fields are embedded in a medium as far as the simplest (primitive) diagrams are concerned.

For the diagrams not primitively divergent (i.e., still diverging even when an internal line is broken) the renormalization problem is solved in the vacuum by the so-called R operation of Bogoliubov and Parasyuk,[6,7] which remains valid even in the presence of a medium.

Here we shall limit ourselves to briefly sketch the proof of this statement. For this purpose let us consider a one-particle irreducible (proper) diagram $\Gamma_{(n)}$ with n external (incoming or outgoing) fermionic lines. The regularized integrand of $\Gamma_{(n)}$ in the vacuum is given by the Bogoliubov recurrence formula

$$R_{\Gamma^0_{(n)}}=I_{\Gamma^0_{(n)}}+\sum_{\gamma_i\cap\gamma_j=\phi}I_{\Gamma^0_{(n)}/\{\gamma_1\cdots\gamma_n\}}\left[-\prod_{i=1}^{n}T_{\gamma_i}R_{\gamma_i}\right], \tag{24}$$

the notations being those of Refs. 8 and 12 where R and I denote the renormalized and unrenormalized integrands of $\Gamma^0_{(n)}$ (the index 0 referring to the vacuum). In (24) $\Gamma^0_{(n)}/\{\gamma_1\cdots\gamma_n\}$ corresponds to a diagram whose internal lines are shrinked to a point when they belong to a given internal subdiagram γ_i and the factor $(-\prod_{i=1}^{n}T_{\gamma_i}R_{\gamma_i})$ essentially represents the contributions arising from the counterterms (these being set by the theory in the vacuum). Quantum-field theory asserts then that $R^0_{\Gamma_{(n)}}$ may be integrated yielding a finite result.

In the presence of a medium we again replace each $S_F^{(v)}$ with $S_F^{(m)}$ and write

$$R_{\Gamma_{(n)}}=I_{\Gamma_{(n)}}+\sum_{\gamma_i\cap\gamma_j=\phi}I_{\Gamma_{(n)}/\{\gamma_1\cdots\gamma_n\}}\left[-\prod_{i=1}^{n}T_{\gamma_i}R_{\gamma_i}\right], \tag{25}$$

the counterterms contributions remaining the *same* as in Eq. (24). We now make use of Eq. (7), expand the products, and collect all the terms appearing both on the rhs and on the left-hand side (lhs) of (25) as follows. First we group together all the terms associated with the Feynman propagators in the vacuum. Obviously they will reproduce Eq. (24), thus giving a finite contribution to $R_{\Gamma_{(n)}}$. Next we consider on the lhs the terms with *only one* $S_F^{(v)}$ replaced, in a well-defined place, by the medium corrected expression $S_F^{(m)}-S_F^{(v)}$. From the expansion on the rhs we correspondingly pick up all the diagrams with just one $S_F^{(m)}-S_F^{(v)}$ in the *same* place (obviously if such terms still exist after the shrinking of the subdiagrams to a point). We are thus led to consider new integrands which, apart from the coefficient of the $S_F^{(m)}-S_F^{(v)}$ term, are just the ones appearing in the diagrams with two more external fermionic lines in the vacuum. For the latter the Bogoliubov recurrence formula applies again, a finite result ensuing.

The extension of this procedure to the terms with any finite number of $S_F^{(m)}-S_F^{(v)}$ allows one to establish a fully regularized theory in the medium, an outcome to be expected, since ultraviolet divergencies in the vacuum should not be affected by the finite Fermi momentum k_F.

IV. THE RENORMALIZABILITY OF THE EFFECTIVE THEORY IN THE MEDIUM

The proof of the renormalizability of the effective bosonic Lagrangian $\mathcal{L}_B^{\text{eff}}$ entering into the action (10) is not trivial since $\mathcal{L}_B^{\text{eff}}$ entails an intricate rearrangement of the elementary diagrams of the Lagrangian (1).

To tackle this problem we first notice that $\mathcal{L}_B^{\text{eff}}$ embodies an infinite number of elementary effective vertices, nonlocal in nature, whose regularization requires the introduction of as many counterterms. The latter, however, can all be derived from the finite set of counterterms already included in (12), since the path integral over the fermionic fields is properly defined (and can be exactly performed) only when use is made of the Lagrangian (12). Accordingly one gets the following bosonic effective action:

$$\tilde{S}_B^{\text{eff}}=\int d^4x\left[\tfrac{1}{2}(\partial_\mu\Phi)^2-\frac{m_\pi^2}{2}\Phi^2-\frac{\delta m_\pi^2}{2}\Phi^2+(Z_3-1)\left[\tfrac{1}{2}(\partial_\mu\Phi)^2-\frac{m_\pi^2}{2}\Phi^2\right]-(\lambda+\delta\lambda)\Phi^4\right]$$
$$+i\sum_{n=1}^{\infty}\frac{1}{n}\int d^4x_1\ldots d^4x_n\tilde{C}_n^{l_1\cdots l_n}(x_1\ldots x_n)\Phi_{l_1}(x_1)\ldots\Phi_{l_n}(x_n), \tag{26}$$

which replaces (10). Next we recall, and this is the crucial point, that in terms of the bare fermion propagator

$$\tilde{S}_F(p)=\frac{1}{Z_2(\not{p}-M)-\delta M+i\eta}, \tag{27}$$

which includes the counterterms as well, the new expansion coefficients in (26) read

$$\tilde{C}_n^{l_1\cdots l_n}(x_1\ldots x_n)=[i(g+\delta g)]^n \mathrm{tr}\tilde{S}_F(x_1-x_2)\gamma_5\tau^{l_1}\ldots\tilde{S}_F(x_n-x_1)\gamma_5\tau^{l_n}\ , \tag{28}$$

thus being highly nonlocal in nature. In other words the elementary vertices (28) of the effective Lagrangian possess an internal structure, entailing an asymptotic behavior in momentum space, basically set by the nucleon mass M, quite different from the one associated with the elementary Lagrangian (1). As a consequence the standard renormalization rules appear now unsuitable: indeed even the notion of a primitively divergent diagram and of surface divergence becomes hard to define for $\mathcal{L}_B^{\mathrm{eff}}$. Accordingly a new renormalization scheme should be devised to replace the R operation.

We look for this new procedure in the framework of the loop expansion to which $\tilde{S}_B^{\mathrm{eff}}$ naturally leads when the bosonic mass characterizing (1) is small ($m_\pi \ll M$), and we shall utilize a recursive scheme to show how the cancellation of the divergencies takes place order by order in the loop expansion. We then start by splitting, in the effective vertices $\tilde{C}_n$ of $\tilde{S}_B^{\mathrm{eff}}$, the finite contributions and the counterterms and by assigning to each counterterm an appropriate order in the loop expansion.

For this purpose we first set

$$\tilde{C}_n^{l_1\cdots l_n}(x_1\ldots x_n)=C_n^{l_1\cdots l_n}(x_1\ldots x_n)+\delta C_n^{l_1\cdots l_n}(x_1\ldots x_n)\ , \tag{29}$$

the $C_n^{l_1\cdots l_n}$ being given by (11). The $\delta C_n^{l_1\cdots l_n}$ are then obtained by inserting into (28) the expansion of the bare propagator (27) around the renormalized one (6)

$$\tilde{S}_F(p)=S_F(p)\sum_{k=0}^{\infty}\sum_{s=0}^{k}\begin{bmatrix}k\\ s\end{bmatrix}\left[\frac{1-Z_2}{\not p-M+i\eta}\right]^{k-s}\left[\frac{\delta M}{\not p-M+i\eta}\right]^{s}, \tag{30}$$

yielding

$$\begin{aligned}\tilde{C}_n^{l_1\cdots l_n}(x_1\ldots x_n)=i^n\sum_{m=0}^{n}\begin{bmatrix}n\\ m\end{bmatrix}g^{n-m}\delta g^{m}\,\mathrm{tr}\int\frac{d^4p_1}{(2\pi)^4}e^{-ip_1(x_1-x_2)}S_F(p_1)\\ \times\sum_{k_1=0}^{\infty}\sum_{s_1=0}^{k_1}\begin{bmatrix}k_1\\ s_1\end{bmatrix}\left[\frac{1-Z_2}{\not p_1-M+i\eta}\right]^{k_1-s_1}\left[\frac{\delta M}{\not p-M+i\eta}\right]^{s_1}\gamma_5\tau^{l_1}\cdots\\ \times\mathrm{tr}\int\frac{d^4p_n}{(2\pi)^4}e^{-ip_n(x_n-x_1)}S_F(p_n)\\ \times\sum_{k_n=0}^{\infty}\sum_{s_n=0}^{k_n}\begin{bmatrix}k_n\\ s_n\end{bmatrix}\left[\frac{1-Z_2}{\not p_n-M+i\eta}\right]^{k_n-s_n}\\ \times\left[\frac{\delta M}{\not p_n-M+i\eta}\right]^{s_n}\gamma_5\tau^{l_n}\ ,\end{aligned} \tag{31}$$

where the $C_n^{l_1\cdots l_n}$ of Eq. (11) are recovered for $m=k_1=\cdots=k_n=0$ and all the remaining terms define the infinite series for the effective counterterms δC_n. It is easy to recognize that the latter embody at least one factor δg, or (Z_2-1), or δM.

For sake of illustration a few δC_n for the case $n=6$ are diagrammatically shown in Fig. 2, where a dot denotes (Z_2-1) or δM [both of these counterterms enter into the renormalization of the inverse propagator $\Gamma_{\bar\psi\psi}$, as shown in Eqs. (14)–(19)] and a circle denotes δg [this counterterm is needed for renormalizing the elementary diagram (c) of Fig. 1].

In Fig. 3, instead, we display the first C_n ($n\le 6$). Since in the effective action (26) the diagrams (a) and (c) of this figure multiply Φ^2 and Φ^4, respectively, their divergences at the tree level will be regularized (together with the mass term and the $\lambda\Phi^4$ interaction) by the standard counterterms δm_π^2 and (Z_3-1) for C_2 and $\delta\lambda$ for C_4. Concerning diagrams (b) and (d), they are vanishing since, in the vacuum, the n-odd diagrams are identically zero for parity constraints. Finally all the remaining C_n ($n\ge 6$) are regular, as is easily verified by counting powers.

Thus, with the standard renormalization of the elementary pionic lines and vertices, the action (26) can be recast in the following form:

$$\begin{aligned}\tilde{S}_B^{\mathrm{eff}}=\tfrac{1}{2}\int d^4x\,d^4y\,\Phi(x)\Delta_0^{-1}(x-y)\Phi(y)+i\sum_{n=2}^{\infty}\frac{1}{n}\int d^4x_1\ldots d^4x_n C_{n,\mathrm{reg}}^{l_1\cdots l_n}(x_1\ldots x_n)\Phi_{l_1}(x_1)\ldots\Phi_{l_n}(x_n)\\ +i\sum_{n=2}^{\infty}\tfrac{1}{2}\int d^4x_1\ldots d^4x_n\delta C_n^{l_1\cdots l_n}(x_1\ldots x_n)\Phi_{l_1}(x_1)\ldots\Phi_{l_n}(x_n)\ ,\end{aligned} \tag{32}$$

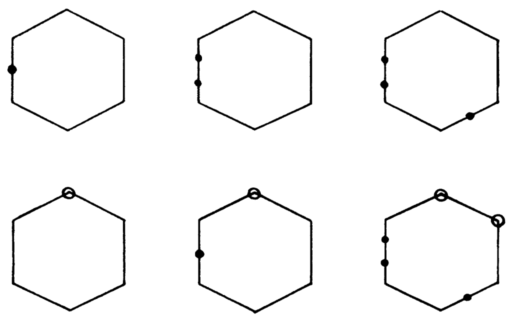

FIG. 2. Examples of δC_n ($n=6$).

the regularized C_n (actually those with $n=2$ and 4) being labeled with the suffix reg. These coefficients yield the tree approximation of the loop expansion, which is thus finite.

It should be noticed that since the terms of the action (32) quadratic in Φ contain $C_{2,\text{reg}}$ as well, the $\hbar$ expansion will no longer coincide with our loop expansion.

The structure of the latter is best illustrated by considering the one-particle (pion) irreducible diagrams, which describe the effective vertices of the theory. Their tree approximation, as already mentioned, simply coincides with the $C_{n,\text{reg}}$. Higher orders in the loop expansion are then conveniently viewed by shrinking the fermionic loops, namely $C_{n,\text{reg}}$ (see Fig. 3), down to a point and by connecting the elementary vertices thus obtained with dressed pionic propagators.

The divergences arising in this scheme are compensated by the appropriate choice of the counterterms δC_n, whose order in the loop expansion is set by the number of factors δM, $(1-Z_2)$ or δg they embody (note that we choose here to have only divergent contributions in our counterterms, i.e., a soft regularization scheme). For the sake of illustration two lowest order (one-loop) diagrams contributing to the effective vertices are displayed in Fig. 4.

To understand now how the regularization mechanism works, it is essential to consider the internal structure of the elementary vertices. All the diagrams (but for trivial permutations) contributing to Fig. 4(a) in the $n=6$ case, counterterms included, are displayed in Fig. 5, where a dashed line represents a dressed pion (the $C_{2,\text{reg}}$ insertion up to infinite order being understood). These diagrams are obtained by adding a dressed pionic line either to a fermion propagator [Fig. 5(a)], or between two distant ones [Fig. 5(c)], or across a pion-fermion vertex [Fig. 5(b)]. Now, clearly, the self-energy insertion of diagram

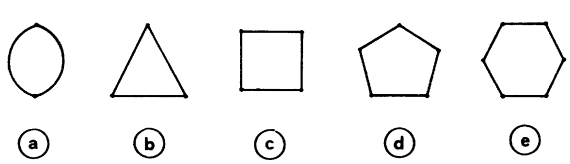

FIG. 3. Illustration of C_n for $2 \le n \le 6$.

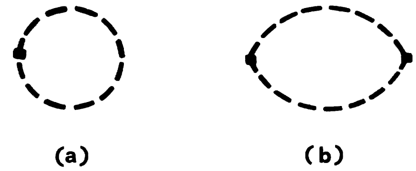

FIG. 4. Two examples of one-loop contributions to the effective vertices.

(a) is regularized by means of a suitable combination with (d), whereas the divergences of the vertex correction (b) are eliminated by the counterterm displayed in (e). Finally diagram (c) is regular by itself.

We remind the reader that a dressed pionic line corresponds to an infinite series of terms (Fig. 6 illustrates the case of the self-energy insertion) of which only the first one is truly divergent and needs the appropriate counterterms δC_n to be renormalized.

Finally the diagram of Fig. 4(b), divergent for truly pointlike vertices, is in fact seen to be regular by powers counting, since the two pions entering into each $C_{n,\text{reg}}$ are necessarily connected by at least one fermion propagator.

By extending the same arguments to higher orders in the loop expansion, the renormalization procedure can be consistently carried out and the feasibility of the renormalization procedure can then be proved by induction.

Consider indeed the n-loop diagrams, assumed to be renormalized. The $(n+1)$-loop diagrams follow from the previous ones in five different ways:

(1) by adding a dressed pionic line closed on a fermion propagator in one $C_{n,\text{reg}}$ (self-energy insertion);

(2) by adding a dressed pionic line closed at one vertex in one $C_{n,\text{reg}}$ (vertex correction);

(3) by adding a counterterm insertion [$\delta g, \delta M$ or $(1-Z_2)$];

(4) by adding a dressed pionic line between two fermion propagators separated by more than one vertex in one $C_{n,\text{reg}}$ [see, e.g., diagram (c) of Fig. 5];

(5) by adding a dressed pionic line between two different $C_{n,\text{reg}}$.

Now a one-to-one correspondence can be established between the diagrams of class (1) or (2) and the counter-

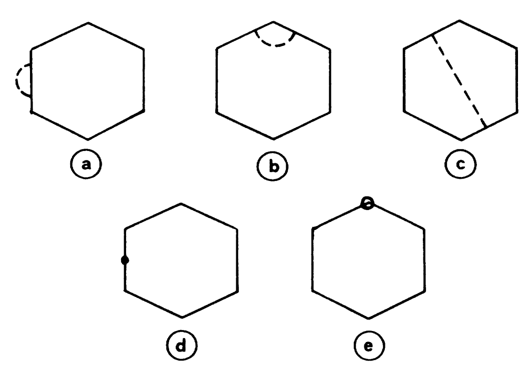

FIG. 5. One-loop order renormalization of C_0.

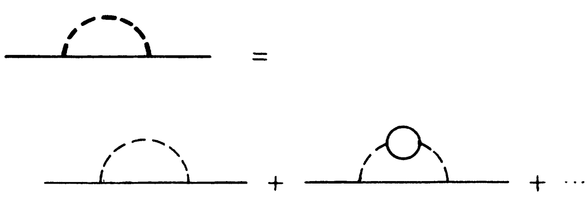

FIG. 6. Self-energy insertions with the dressed pion.

term insertions of class (3) such as to wash out the relative divergences. Diagrams of classes (4) and (5) are, instead, necessarily regular by themselves. The renormalizability of the theory at the $(n+1)$ loops level then follow.

This, although expected, is a nice result. Indeed, even if the Lagrangian (1) is known to be renormalizable order by order in $\hbar$, one should realize that our loop expansion includes, to a given order, an infinite series of diagrams which correspond to different powers of $\hbar$.

Now let the fields be embedded in the nuclear medium. We have already seen that this requires us to replace $S_F^{(v)}$ [Eq. (6)] with $S_F^{(m)}$ [Eq. (7)] in all the elementary diagrams, which play the role of the building blocks of the loop expansion in the nuclear matter. Furthermore, we have shown that the medium corrections arising from the differences between $S_F^{(v)}$ and $S_F^{(m)}$ are always finite and do not effect the interplay between the counterterms and the divergent pieces of the elementary vertices. Thus we conclude that our bosonic theory can be explicitly renormalized *order by order* in the loop expansion within a medium.

Obviously the self-energy and the vertex corrections of the vacuum are quite negligible in the context of nuclear physics; in this domain only those diagrams are left which truly correspond to medium corrections, their physical significance being well established.

V. CONCLUSIONS

This paper has mainly been concerned with the problem of the renormalizability in the medium of the effective bosonic Lagrangian deduced from (1) via functional integration over the fermionic fields.

The importance of $\mathcal{L}_B^{\text{eff}}$ for intermediate (and high) energy nuclear physics is twofold. First $\mathcal{L}_B^{\text{eff}}$ explicitly deals with degrees of freedom most relevant in that energy domain, namely the pions. Second, viewing the nucleus as a collection of nucleons and pions, one would obviously like, as a first step, to deal with the fermions as accurately as possible. This is just what one achieves with path integrals, which *exactly* engrain the fermionic dynamics in $\mathcal{L}_B^{\text{eff}}$.

Clearly the effective Lagrangian one obtains turns out to be a complicated object and its renormalizability, although expected, is far from being straightforward. Indeed renormalizability is a matter of asymptotic behavior at infinite momentum, and the latter is strongly affected by the structure endowed in the vertices of $\mathcal{L}_B^{\text{eff}}$.

Yet we have been able to prove that our effective Lagrangian is renormalizable order by order in the loop expansion. Such a framework groups a wide set of diagrams of great relevance in nuclear physics [in particular the random-phase approximation (RPA) ones] in the class of the quantum fluctuations around the classical mean-field (Hartree-Fock) solution. At the same time it yields, as already mentioned in the Introduction, a prescription for identifying the set of diagrams to be considered in the next order beyond the RPA. As a consequence, the loop expansion appears particularly suitable in dealing with Lagrangians embodying light mesons, those carrying the long-range nucleon-nucleon correlations.

When heavier mesons come into play, then the many-body problem is best compared with the well-known hole-line expansion, which is based on the ladder diagrams of the Bethe-Salpeter equation. In this connection we would like to mention a remarkable feature of our approach, namely that a straight perturbative treatment to second order of our effective Lagrangian, beyond being finite, would embody, among other contributions, the ladder diagrams as well.

It thus appears that our approach complements the one of Serot and Walecka,[2] who have repeatedly stressed the importance of utilizing renormalizable Lagrangians in the context of the many-body problem. For example, Horowitz and Serot[4] have shown that the relativistic Brueckner Bethe-Salpeter method leads to sensible results when based on a renormalized Lagrangian, which allows us to self-consistently account for the full contribution of the Dirac sea. Of course one can alternatively obtain finite results by treating the divergencies associated with the singular behavior at short distances of the nuclear forces with a form factor characterized by an *ad hoc* cutoff Λ. It seems to us of significance that this scope could be achieved as well in our approach, in second-order perturbation theory, the ultraviolet behavior being naturally controlled by the nucleon mass M.

In conclusion, our analysis suggests that an appropriate procedure for treating relativistic, bound Fermi systems should be to apply the loop expansion to the light mesons and the perturbative expansion to the heavy ones. As a preliminary, effective Lagrangians for both kinds of particles should be derived via functional integration over the fermionic fields.

[1]W. M. Alberico, R. Cenni, A. Molinari, and P. Saracco, Ann. Phys. (N.Y.) **174**, 131 (1987).
[2]B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. **16**, 1 (1986).
[3]A. F. Bielajew and B. D. Serot, Ann. Phys. (N.Y.) **156**, 215 (1984).
[4]C. J. Horowitz and B. D. Serot, Nucl. Phys. **A464**, 613 (1987).
[5]Ch. Elster, W. Ferchländer, K. Holinde, D. Schutte, and R. Machleidt, Phys. Rev. C **37**, 1647 (1988).

[6]N. N. Bogoliubov and O. S. Parasyuk, Acta Math. **97**, 227 (1957).
[7]N. N. Bogoliubov and D. V. Shirkov, *Introduction to the Theory of Quantized Fields* (Interscience, New York, 1959).
[8]W. Zimmerman, in *Lectures on Elementary Particles and Quantum Fields Theory,* Proceedings of the Brandeis Summer Institute, 1970, edited by S. Deser, M. Grisaru, and H. Pendleton (MIT, Cambridge, Mass., 1970).
[9]L. D. Faddeev and A. A. Slavnov, *Gauge Fields, Introduction to Quantum Theory* (Benjamin-Cummings, New York, 1980).
[10]P. Saracco, Ph.D. thesis, University of Genova, 1987.
[11]R. Cenni and P. Saracco, Lett. Nuovo Cimento **41**, 173 (1984).
[12]C. Itzykson and J. B. Zuber, *Quantum Field Theory* (McGraw-Hill, New York, 1980).