Test of the O(6) character of nuclei near $A = 130$

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(Received 16 May 1988)

The lifetimes of the lowest 2^+ and 4^+ levels can distinguish between the O(6) and U(5) dynamical symmetries of the interacting boson model. The data for the $A = 130$ nuclei confirm the proposed O(6) character of these nuclei.

One of the most attractive features of the interacting boson model (IBM) (Ref. 1) is the existence of three dynamic symmetries corresponding to the subgroup chains of U(6) which contain the rotation group O(3). The U(5) and O(6) symmetries have the subgroups O(5) and O(3) in common:[2,3]

$$\mathrm{U}(6)\supset\mathrm{U}(5)\supset\mathrm{O}(5)\supset\mathrm{O}(3),\quad \mathrm{U}(5) \tag{1}$$

and

$$\mathrm{U}(6)\supset\mathrm{O}(6)\supset\mathrm{O}(5)\supset\mathrm{O}(3),\quad \mathrm{O}(6)\ . \tag{2}$$

These two dynamic symmetries correspond to particular geometric models. U(5) corresponds to the spherical vibrator and O(6) to the gamma-soft rotor, respectively.[4] Evidence for an extensive region of nuclei near $A = 130$ which were consistent with the O(6) symmetry was given in Ref. 5. A strong similarity to the previously known O(6)-like nuclei in the Pt region[6] was found. This evidence was based on excitation energies and $B(E2)$ branching ratios for low-lying levels of nuclei around $A = 130$.

Subsequently Leviatan *et al.*[7] investigated the properties of nuclei belonging to the U(5)-O(6) transition region. They noted that both group chains (1) and (2) contain the O(5) and O(3) groups and emphasized that the energies and $B(E2)$ branching ratios of levels with $\sigma = N$ can establish only an O(5) dynamic symmetry and cannot distinguish between the U(5) and O(6) dynamic symmetries because a highly anharmonic U(5) spectrum can resemble an O(6) level pattern.

Thus the question of whether the $A = 130$ nuclei are an example of the O(6) or of the U(5) symmetry had to be considered an open one. It is the purpose of this paper to decide this question. To begin with we will briefly review the arguments of Leviatan *et al.*, which show the difficulties to distinguish between O(6) and U(5) on the basis of a limited set of energies alone. The energies of the U(5) symmetry are given by the formula[1,8]

$$E = E_0 + \epsilon n_d + \alpha n_d(n_d+4) + 2\beta\tau(\tau+3) + 2\gamma L(L+1)\ , \tag{3}$$

where E_0 is constant for a given nucleus, n_d and $n_d(n_d+4)$ are the eigenvalues of the linear and quadratic Casimir operators, respectively, of U(5), and $\tau(\tau+3)$ and $L(L+1)$ are the eigenvalues of the Casimir operators of O(5) and O(3), respectively.

On the other hand, the energies of the O(6) symmetry are[3,8]

$$E = E_0 + 2\eta\sigma(\sigma+4) + 2\beta'\tau(\tau+3) + 2\gamma' L(L+1)\ . \tag{4}$$

The second term is the eigenvalue of the quadratic Casimir operator of O(6), while the other terms have identical counterparts in (3). The U(5) representations are labeled by the d-boson number n_d, and those of O(6) by σ, with $\sigma = N, N-2, \ldots, 0(1)$; N is the total boson number. The states forming each representation are classified by the labels of irreducible representations of O(5) (denoted τ) and O(3) (denoted L). In both limits τ is the boson seniority and L is the angular momentum. The subset of levels (representation) of O(6) characterized by the condition that the distinct quantum numbers N and σ fulfill $N = \sigma$, which are also lowest in energy, shows great similarity with the subset of U(5) states characterized by the condition $n_d = \tau$, as can be seen from Eqs. (3) and (4). In fact, as shown in Ref. 7, the coefficients in (3) can be chosen in such a way that the U(5) excitation energies of states with $n_d = \tau$ are identical to those of the $\sigma = N$ representation of O(6). This will happen if $\epsilon = -\alpha$ and $\alpha + 2\beta = 2\beta'$. In addition, the U(5) states with $\tau = n_d - 2$ need to be pushed up by a suitable choice of a negative β. The second term in Eq. (4) is irrelevant to this comparison, since $\sigma = N$ is kept constant.

Thus level energies can serve to prove the O(6) symmetry only if levels with $\sigma < N$ are found, and if they obey the O(6) energy formula. $B(E2)$ branching ratios of transitions between two seniority (τ) multiplets will depend only on O(5) and O(3) quantum numbers, i.e., on τ and L. Therefore, such branching ratios do not distinguish between the O(6) and U(5) dynamic symmetries.[7]

Evidence for O(6) symmetry can, however, be provided by the $\Delta\sigma = 0$ selection rule[3,6] forbidding $E2$ transitions between different O(6) representations. We can take as an example the 2_5 excited state in ^{196}Pt which, in the O(6) limit, is a $\tau = 2$ state of the $\sigma = N-2$ representation.[9] The only state to which it can decay is the 2_4 state which has the same σ and $\tau=1$. The experiment shows indeed that this is the only strong $E2$ transition from the 2_5 state. In a U(5) description of ^{196}Pt the 2^+ level could be assigned to the $n_d=4$ multiplet: The $E2$ selection rule is $n_d=\pm 1$. Consequently the transitions $2_5 \rightarrow 3_1$ and $2_5 \rightarrow 4_2$ should be allowed, contrary to the experimental evidence. Unfortunately, such data are not available in the $A=130$ region. As will be seen in the following, absolute $B(E2)$ values can also distinguish O(6) and U(5), however.

Reference 5 shows that the nuclei around $A=130$ display spectra and $E2$ branching ratios very similar to ^{196}Pt. Since the latter nucleus has now been conclusively shown[6,9] to manifest O(6) symmetry, there is a presumptive argument to also associate the empirical structure of the $A=130$ region with this symmetry rather than with a highly anharmonic U(5) symmetry. However, the experimental evidence of a Ref. 5 mainly showed that nuclei in this region display an O(5) symmetry. In order to distinguish between the O(6) and U(5) symmetry it is necessary to inspect data which can unambiguously distinguish U(5) from O(6). It is the purpose of this paper to do so for the $A=130$ region.

Absolute $B(E2)$ values for transitions between states of the ground-state representation ($\sigma=N$) can distinguish the O(6) vs U(5) dynamic symmetry.[3,8] In particular, since the d-boson structures of U(5) and O(6) are different, the dependence of $B(E2)$ values on spin within a band differs substantially in the two dynamic symmetries. The following ratio, $R(4^+;g)$, between absolute $B(E2)$ values of states in the ground-state band (g.s.b.),

$$R(4^+;g)=B(E2;4^+\rightarrow 2^+)/B(E2;2^+\rightarrow 0^+)\ ,$$

is particularly sensitive to the type of dynamic symmetry. In order to calculate the $E2$ transitions we use the usual $E2$ operator of the IBM-1,

$$T(E2)=e[d^\dagger s+s^\dagger \tilde{d}+\chi(d^\dagger \tilde{d})^{(2)}]\ .$$

Since we consider only $\Delta\tau=-1$ transitions, the term $(d^\dagger \tilde{d})^{(2)}$ cannot bring any contribution because its selection rule is $\Delta\tau=0,\pm 2$. Therefore, the quantities we consider are independent of the value of χ,

$$R(4^+;g)=2(N-1)/N$$

for U(5) and

$$R(4^+;g)=\tfrac{10}{7}(N-1)(N+5)/[N(N+4)]$$

for O(6), where N is the total boson number.[3]

This dissimilarity of ratios of $B(E2)$ values of the ground band for the U(5) and the O(6) limits can be qualitatively understood by considering the d-boson structure of wave functions. An example is given in Table I, for $N=6$, expressed in a simple basis in which only the n_d quantum number need be specified. It is clear by inspection that, in U(5), $\langle n_d \rangle=0$, 1, and 2, for the 0^+, 2^+, and 4^+ states of the ground band, respectively. In O(6), however, these expectation values are larger and, more importantly, they increase more slowly with spin. Since $B(E2)$ values for low-spin states ($n_d \ll N$) in a U(5) (or n_d) basis are proportional to the number of d bosons in the initial state, it is clear that the yrast $B(E2)$ values will increase more slowly in O(6) than in U(5) and therefore, that R_4 [O(6)] $< R_4$ [U(5)].

Similar arguments apply to other absolute $B(E2)$ ratios. As an example we consider the transition from the 6^+ state of the gamma band to the 4^+ state of the gamma band,

$$R(6^+;\gamma)=B(E2;6^+\rightarrow 4^+)/B(E2;2^+\rightarrow 0^+)\ .$$

One finds

$$R(6^+;\gamma)=\tfrac{15}{11}(2N-6)/N$$

for U(5) and

$$R(6^+;\gamma)=0.31(2N-6)(2N+14)/[N(N+4)]$$

for O(6).

These relations have been obtained by using formulas (5) and (15) of Ref. 2 and (5) and (27) of Ref. 3. We note again that the results for the U(5) symmetry are independent of the value of χ.

A comparison of experimental $B(E2)$ ratios with the U(5) and O(6) predictions is given in Table II. Absolute experimental $B(E2)$ values were mainly extracted from recoil distance Doppler-shift lifetime measurements. Although in some cases the standard deviations of $R(4;g)$ are rather large, we notice that the data exclude U(5), but they are in overall agreement with O(6). Actually the experimental ratios are slightly smaller than the O(6) values in a rather systematic fashion. This cannot be due to an O(6)-U(5) transition. We have so far no explanation for this deviation. We were able to compare just one experimental value of $R(6;\gamma)$ with theory, which agrees with the O(6) value.

TABLE I. Some O(6) and U(5) wave functions in an n_d basis ($N=6$).

Level	symmetry	Wave function amplitudes							
	n_d	0	1	2	3	4	5	6	$\langle n_d \rangle$
0_1	U(5)	1	0	0	0	0	0	0	0
	O(6)	0.433	0	0.750	0	0.491	0	0.094	2.14
2_1	U(5)	0	1	0	0	0	0	0	1
	O(6)	0	0.612	0	0.732	0	0.299	0	2.43
4_1	U(5)	0	0	1	0	0	0	0	2
	O(6)	0	0	0.768	0	0.627	0	0.134	2.86

TABLE II. Comparison of experimental $B(E2)$ ratios with the U(5) and O(6) predictions.

$B(E2)$ ratios Nucleus	N	$R(4;g)_{\rm exp}$	$R(4;g)_{\rm theory}$ O(6)	U(5)	Ref. (Expt.)
^{120}Xe	10	1.46(20)	1.38	1.80	10
^{124}Xe	8	1.29(15)	1.35	1.75	11,12
^{126}Ba	9	1.12(20)	1.37	1.78	13
^{128}Ba	8	1.03(14)	1.35	1.75	14
^{130}Ba	7	0.90(13)	1.34	1.71	15
^{130}Ce	10	1.35(18)	1.38	1.80	16
		$R(6;\gamma)_{\rm exp}$	$R(6;\gamma)_{\rm theor}$		
^{128}Ba		1.05(22)	0.97	1.70	14

One may ask how reliable the data in Table II are. The quoted standard deviations are mainly statistical. As shown in Ref. 17, the main source of systematic errors is the so-called side feeding. This includes the continuum feeding from many highly excited primary states, as well as discrete feeding from unobserved weakly populated levels. As has been discussed in Refs. 17 and 10, only coincidence lifetime experiments can really give us detailed information on side feeding lifetimes and thus determine the corrections to the lifetime. One such experiment has recently been carried out in this region of nuclei for ^{120}Xe (Ref. 10), and reliable values of the side feeding lifetimes to the 8^+ and 10^+ states of the yrast band were obtained. These data allowed for tests of various sensible models for the side feeding lifetimes. In particular it was shown that an upper limit to the side feeding lifetime of a level is given by the average discrete line feeding lifetime to this level. With this model the lifetime data for the nucleus ^{128}Ba were analyzed in Ref. 14. As the side feeding fraction of the 2^+ and 4^+ levels are rather small, and their lifetimes are relatively long, the fitted lifetimes are insensitive to the assumed side feeding hypothesis.[14] Since the other nuclei considered in Table II have been produced in similar nuclear reactions, we conclude that the influence of side feeding is also small in these cases. Thus we think that the quoted values of the ratio $R(4,g)$ in the table are reliable.

In conclusion, we have shown that experimental absolute $B(E2)$ values in the ground-state band of nuclei around $A=130$ exclude the U(5) symmetry but they are in overall agreement with the O(6) symmetry. Thus the suggestion in Ref. 5 that the $A=130$ nuclei are a good example of the O(6) dynamic symmetry of the IBM-1 is strongly suggested. A small but apparently systematic downward deviation of the data from the O(6) values has still to be understood. We hope that this paper stimulates new accurate lifetime measurements in this region.

The authors would like to thank A. Arima for stimulating discussions. This work was funded by the German Federal Minister for Research and Technology (BMFT) under Contract No. 06OK272 and by Contract DE-AC02-76CH00016 with the U.S. Department of Energy.

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