Alpha-cluster model theory of ^{44}Ti and an effective two-body interaction

Shigeo Ohkubo
Department of Applied Science, Kochi Women's University, Kochi 780, Japan
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The $\alpha+{}^{40}$Ca system is investigated in the framework of the folding model using the Hasegawa-Nagata-Yamamoto effective two-body force. It is shown that the folding potential which can bind not only the higher nodal band with the $\alpha+{}^{40}$Ca cluster structure but also the ground-state band can describe well $\alpha+{}^{40}$Ca scattering in a wide energy range of $E_\alpha=18-100$ MeV. A critique of the α-cluster theory of ^{44}Ti using an effective interaction which cannot bind the ground-state band is given.

I. INTRODUCTION

The α-cluster correlation is essential in light nuclei.[1] Comprehensive study[2] of light nuclei from the viewpoint of molecule-like structure based on the Ikeda diagram has been shown to give a unified understanding of nuclear structure including shell-like states. The α-cluster structure study started from ^{8}Be in the 1960's, and was extended to the *sd*-shell region in the 1970's (Ref. 3) (^{20}Ne was a bridgehead of this study). The success of the α-cluster viewpoint encouraged the extension of the study to heavier nuclei and to higher excitation energy in the *sd*-shell nuclei. As a next step, initiated in the 1970's was an extension of the α-cluster viewpoint to the *fp*-shell region, typical of which is ^{44}Ti, which is an *fp*-shell analog of ^{20}Ne. However, this could not be done in a straightforward way. The viewpoints for approaching the α-cluster structure of ^{44}Ti can be classified as follows. First, the cluster model should be applied to ^{44}Ti, including the ground-state band.[4-7] Second, the model should be applied to the lowest excited levels with large α strength,[8] i.e., the 0^+ state at 8.54 MeV (Ref. 9) and the 1^- state at 11.7 MeV (Refs. 10 and 11), which are regarded as the parity doublet states. Third,[12-15] the viewpoint of the Münster group,[13-15] the model should be applied to the excited mixed-parity cluster states of 0^+ (11.2 MeV), 1^- (11.7 MeV), 2^+ (12.2 MeV), and 3^- (12.8 MeV) which are seen in high-resolution ^{40}Ca(α,α) ^{40}Ca scattering. Based on the first viewpoint, phenomenological potential models[6,7] and the microscopic model of the resonating group method (RGM) (Ref. 4) can reproduce the energy levels and electric transition probabilities between the ground-band states; which leads to the conclusion that the $\alpha+{}^{40}$Ca cluster picture still holds in ^{44}Ti. From the second viewpoint, Horiuchi[8] criticized the first viewpoint that the good agreement of the calculated results with the data is only superficial. In fact, however successful this approach may be, it possesses the essential defect of having the calculation locate the 1^- state with the well-developed cluster structure at a lower energy ($\sim$5 MeV) than the excited 0^+ state at 8.54 MeV in contradiction with the experiment. This contradiction is inevitable as long as the α-cluster model is applied to the ground band. This is the reason Horiuchi claims[8] that the α-cluster model should be applied to the excited 0^+ (8.54 MeV) and 1^- (11.7 MeV) states but not to the ground band. Along these lines is the third viewpoint based on the microscopic study by the Münster group[13-15] in that it states that the ground band should be excluded from the cluster model. However, they adopted the 0^+ state at 11.2 MeV as an α-cluster state instead of the 0^+ state at 8.54 MeV. (Mention should be made that the spin assignment of the state at 8.54 MeV is not conclusive.[9]) The Münster group claims that the well-developed mixed-parity cluster band can be reproduced by the microscopic model using the Brink-Boeker *B*1 force. On the other hand, however successful the second and third viewpoints may be, there arises a serious difficulty: The ground band is beyond the scope of the α-cluster model. This contradicts the fact that the ground band mostly occupies the model space (12,0) in the SU3 representation.[16-18] At any rate all of these viewpoints meet with a dilemma. Thus, the α-cluster model has been considered to be awkward when describing the nuclear structure of ^{44}Ti comprehensively, including the ground band. In Ref. 8 the author claims that the strong spin-orbit force in the *fp*-shell region breaks the [4] symmetry and makes it difficult for the α-cluster model to persist in the ground band.

Recently, however, Michel, Reidemeister, and Ohkubo[7] have shown that the α-cluster structure still persists in the ^{44}Ti nucleus including the ground band; namely, the first viewpoint is supported. They approached the long-standing dilemma about the structure of ^{44}Ti by noticing the importance of the phenomenon of α-particle scattering from ^{40}Ca. They attacked the problem from a higher energy region, i.e., from rainbow scattering to backward-angle anomaly (BAA), from fusion data, and then to the nuclear structure of ^{44}Ti; the fusion data of the $\alpha+{}^{40}$Ca system played an important role in resolving the dilemma. The key point is that the appearance of the $K=0_1^-$ band in the calculations midway between the ground state and the first excited 0^+ state with the $\alpha+{}^{40}$Ca cluster structure is not a defect of the cluster model. On the contrary, they claim that it should exist although it is not found in the data now available. The purpose of this paper is to show that the first viewpoint is reconfirmed, and that the second and the third viewpoints are not supported by investigating the $\alpha+^{40}\text{Ca}$ system using the folding model in a wide energy range from the bound state through 100 MeV of the incident α-particle energy.

In Sec. II, a folding model and an effective two-body force are described. In Sec. III, the angular distributions and excitation functions at the extreme back angle of the α-particle scattering from ^{40}Ca are analyzed, using the folding model, from the three viewpoints. In Sec. IV, discussions are given, and a summary is presented in Sec. V.

II. ALPHA-CLUSTER STRUCTURE IN ^{44}Ti AND THE FOLDING MODEL

The ground band of ^{44}Ti has been studied using the $\alpha+^{40}\text{Ca}$ cluster model.[4–7] It has been shown[4,6,7] that the energy levels and electric transition probabilities between the ground-band states can be well reproduced by the model. The fact that the $B(E2)$ values are reproduced without introducing any additional effective charge suggests that the α-cluster degree of freedom is important in the ground band, considering that the $(fp)^4$ shell model[18–21] needs a large effective charge of $\delta e \approx 0.5$ to reproduce the data. In Refs. 7 and 22 the $K=0^+$ band starting at $E_x = 11.2$ MeV has been suggested to be the higher nodal $\alpha+^{40}\text{Ca}$ cluster band whose relative motion is more excited state compared with the ground band: The 6^+, 8^+, 10^+, and 12^+ states of the band are observed as the peaks in the fusion excitation function. The α-cluster model which reproduces not only the higher nodal band, but also the ground band, inevitably produces the $K=0^-$ band midway. This has been, in fact, the controversy that has concerned for a long time the cluster model approach. The authors of Ref. 7 have pointed out that this is not a defect of the theory, but that the band could be considered to exist and should be searched for actively in experiments. This viewpoint will be confirmed if the predicted $K=0^-$ band states are experimentally found. However, this viewpoint is in contradiction with the second viewpoint (Horiuchi, Ref. 8) and the third one (the Münster group, Refs. 13–15). If the viewpoints of Münster and Horiuchi are valid, then their models should reproduce not only the α-cluster band but also the α-particle scattering data at the relevant energies. We show that their viewpoints collapse when their models are applied to the scattering phenomenon in a wide energy range including rainbow scattering. The Münster group and Horiuchi employed the microscopic theory of the generator coordinate method (GCM) and the RGM, and the adopted effective forces are Brink-Boeker $B1$ and Volkov No. 1 with $m=0.658$, respectively. On the other hand, the authors of Ref. 7 used a phenomenological potential of a squared Woods-Saxon form.

It has been shown[23] that the exchange effect in the resonating group treatment of the composite particle scattering can be represented equivalently by a local potential which is deep enough to incorporate the Pauli forbidden states: The equivalent potential is more similar to a folding potential in shape than a Woods-Saxon form. On the other hand, it has been shown[24] for the $\alpha+^{16}\text{O}$ system that a phenomenological potential determined in the analysis of the α-particle scattering in a wide energy region very much resembles the folding potential derived from the Hasegawa-Nagata-Yamamoto (HNY) force[25] which incorporates the Pauli-forbidden states and has been successful in describing the energy spectra with the α-cluster structure and the backward-angle anomaly in $\alpha+^{16}\text{O}$ scattering.[26] The appropriateness of the folding potential for the $\alpha+^{16}\text{O}$ system derived from the HNY force by slightly increasing the attractive parameter of the triplet-even state in the intermediate range has also been given by Tohsaki-Suzuki, Kamimura, and Ikeda[27] from a theoretical study in the resonating group method: The strength parameter Δ of the HNY force[25] seems to play the role of simulating the exchange effect of the resonating group method. Thus, both the phenomenological and microscopic studies have shown that the folding potential derived from the HNY force is reasonable for the $\alpha+^{16}\text{O}$ system. For the $\alpha+^{40}\text{Ca}$ system, Delbar *et al.*[28] have shown that a local potential, which is similar to a folding potential, describes the α-particle scattering from ^{40}Ca well. Therefore, it seems suitable to discuss the $\alpha+^{40}\text{Ca}$ system on the same footing, using the folding potential which incorporates the Pauli-forbidden states.

The folding model potential between α and ^{40}Ca is calculated by assuming that the intrinsic states of α and ^{40}Ca are described by the harmonic-oscillator wave functions with the same oscillator parameter. The folding potential derived from an effective two-body force,

$$v(r_{ij})=\sum_n [V_n^{13}P_{13}(ij)+V_n^{31}P_{31}(ij)+V_n^{33}P_{33}(ij)+V_n^{11}P_{11}(ij)]\exp(-\kappa_n r_{ij}^2)\ , \tag{1}$$

where $P(i,j)_{2T+1,2S+1}$ is the projection operator to the state with spin S and isospin T of the two-body system, is given as follows:

$$\begin{aligned} V_D(r)=&\sum_n (9V_n^{33}+3V_n^{13}+3V_n^{31}+V_n^{11})\left[\frac{80\nu}{80\nu+69\kappa_n}\right]^{3/2} \\ &\times\left\{10-\frac{600\kappa_n}{80\nu+69\kappa_n}+\frac{12\times10^3\kappa_n^2}{(80\nu+69\kappa_n)^2}+\left[\frac{2^5\times10^3\nu\kappa_n^2}{(80\nu+69\kappa_n)^2}-\frac{2^7\times10^4\nu\kappa_n^2}{(80\nu+69\kappa_n)^3}\right]r^2\right. \\ &\left.+\frac{2^{11}\times10^4\nu^2\kappa_n^2}{(80\nu+69\kappa_n)^4}r^4\right\}\exp\left[-\frac{80\nu\kappa_n}{80\nu+69\kappa_n}r^2\right]\ . \end{aligned} \tag{2}$$

The Coulomb potential is given by

$$V_{\text{Coul}}(r)=\frac{40e^2}{r}\text{erf}\left[\left[\frac{80}{69}\nu\right]^{1/2}r\right]-\frac{380e^2}{23\sqrt{\pi\nu}}\left[\frac{80\nu}{69}\right]^{3/2}\exp\left[-\frac{80\nu}{69}r^2\right]-\frac{2e^2}{\sqrt{\pi}\nu^2}\left[\frac{80\nu}{69}\right]^{7/2}r^2\exp\left[-\frac{80\nu}{69}r^2\right], \tag{3}$$

where

$$\text{erf}(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt\ .$$

We take an oscillator parameter $\nu=0.14$ fm^{-2} throughout this paper ($\nu=m\omega/2\hbar$). As an effective two-body interaction we adopt the Hasegawa-Nagata-Yamamoto force[25] with three ranges, which has a starting-energy dependence based on the reaction matrix theory. In the intermediate range of the triplet-even part of this force, the effect of the strong tensor force of the realistic nucleon-nucleon interaction is taken into account. This strength can be adjusted so as to incorporate the Pauli-forbidden states and to reproduce the binding energy of the relevant cluster structure.

III. ANALYSIS OF THE ELASTIC α SCATTERING FROM ^{40}Ca

In previous papers[7,22] we have shown that the phenomenological potential which was determined in order to describe the rainbow scattering, backward-angle anomaly,[28-33] and fusion data[34] reproduces the ground-state band and higher nodal band with the $\alpha+{}^{40}$Ca cluster structure in ^{44}Ti. In this section we will first show that the folding model which binds the ground-state band reproduces the elastic α-particle scattering well from ^{40}Ca (Refs. 28–33) up to $E_\alpha=100$ MeV. Second, we will show that the second and third viewpoints, which do not bind the ground-state band, are not supported from the analysis of the α-particle scattering from ^{40}Ca.

A. Folding potential with HNY500 force

Based on the first viewpoint, we take the HNY force with -500 MeV for V_2^{13} (abbreviated as HNY500) which was chosen not only to produce the ground-state band, but also to reproduce the experimental energy of the 0^+ state (11.2 MeV) of the higher nodal band correctly. Calculated energy levels which satisfy the Wildermuth condition $N=2n+l\geq 12$ are displayed in Fig. 1. At $E_\alpha=10-27$ MeV the fusion excitation function data for the $\alpha+{}^{40}$Ca system are available.[34] In Fig. 2 the calculated fusion cross sections are shown that use the same prescription and imaginary potential as Ref. 22. The general trend of the experimental data is reproduced by the calculation. The peaks of the oscillations are due to the broad resonances of the higher nodal band ($2n+l=14$). In Fig. 3 the calculated angular distributions of α-particle scattering from ^{40}Ca are compared with the experimental data in a wide energy range from $E_\alpha=18$ through 100 MeV. In the calculation the imaginary potential was assumed to be the same as for Delbar *et al.*[28] The characteristic oscillations at backward angles are reproduced well. The agreement with the data is good considering that no energy dependence is assumed for the real potential and that no adjustment of the imaginary potential is made. Especially the agreement up to $E_\alpha=61$ MeV is acceptably good even at the intermediate angles. At $E_\alpha=100$ MeV the cross sections are damped and the

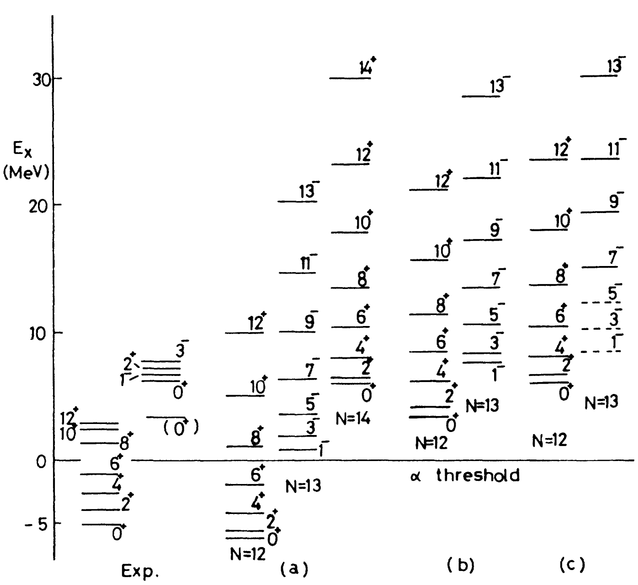

FIG. 1. Energy levels of ^{44}Ti calculated with the folding potentials of (a) HNY500, (b) HNY442, and (c) HNY422 are compared with the experimental data. The position of broader states is defined as the energy where the derivative of the phase shift with respect to the energy is the largest (when the phase shift does not pass through $\pi/2$, this position is indicated by an interrupted line).

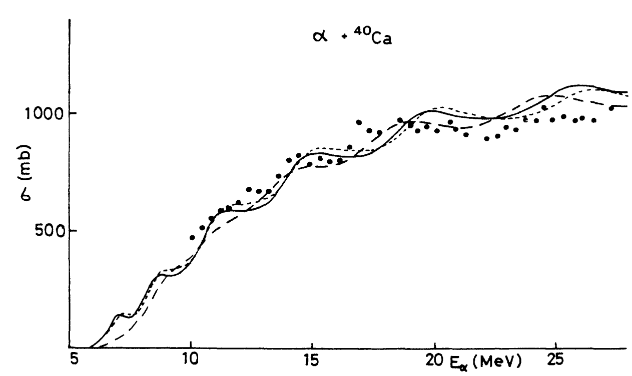

FIG. 2. Fusion excitation functions calculated with the HNY500 (solid lines), HNY442 (dashed lines), and HNY422 (dotted lines) folding potentials are compared with the experimental data (Ref. 34). The errors of the data, which are of the order of 5%, are not shown for clarity.

agreement is not good. However, this can be improved by adjusting the energy dependence of the imaginary potential (dashed lines in the figure). In Fig. 4 the calculated excitation functions of α-particle scattering from ^{40}Ca at $\theta = 180°$ are compared with the data.[35] The calculated result (solid lines) reproduces the gross structure of the data. The broad bumps are caused by the high-spin members of the higher nodal $(2n+l=14)$ and the second higher nodal $(2n+l=16)$ bands with the $\alpha + {}^{40}$Ca structure as well as the negative-parity higher nodal band $(2n+l=15)$.[36] Through the above analysis it is found that the folding model with the HNY500 force can describe both the α-particle scattering up to $E_\alpha = 100$ MeV and the α-cluster structure of ^{44}Ti in the bound and quasibound regions.

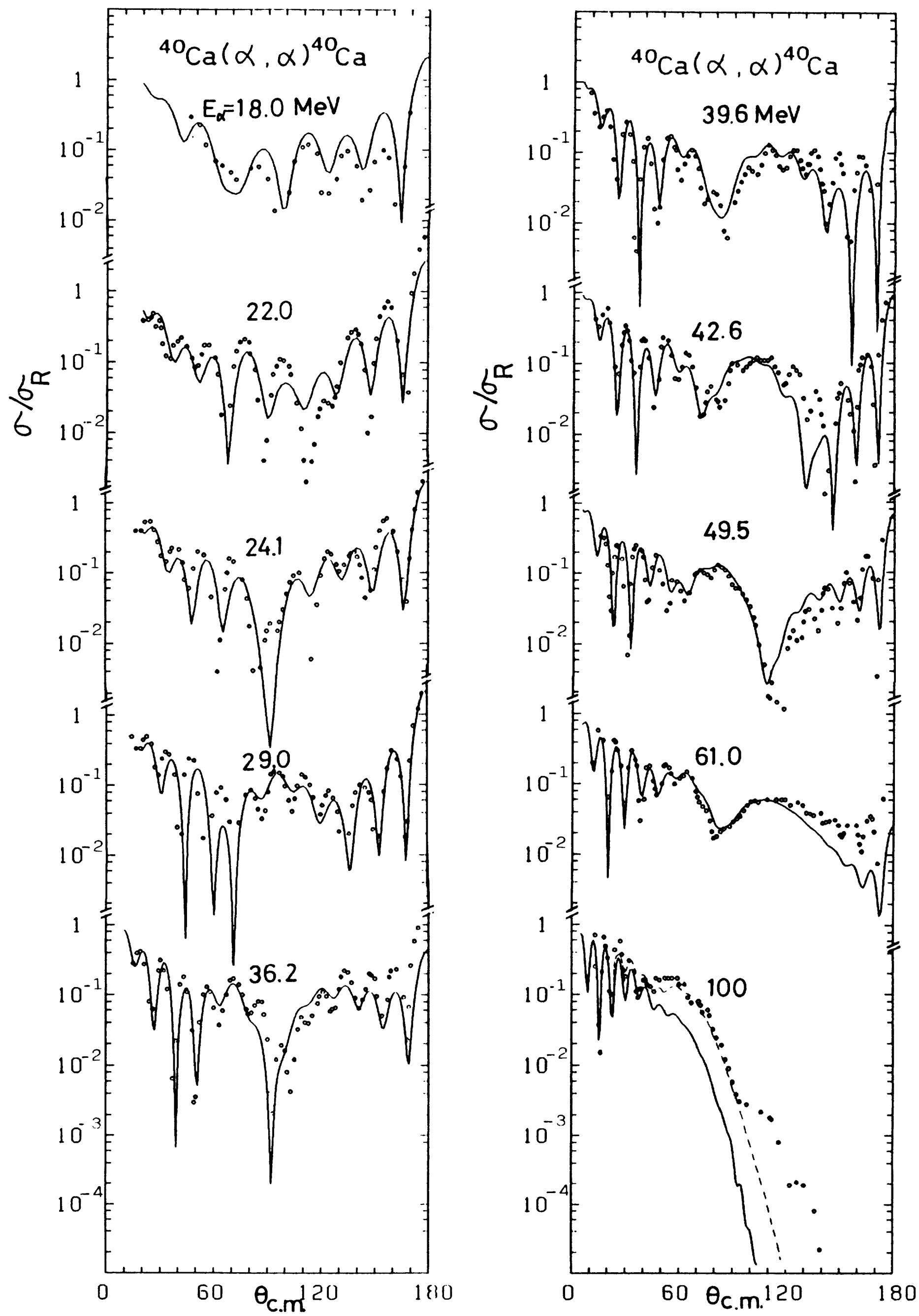

FIG. 3. Comparison of the theoretical cross sections calculated using the HNY500 folding potential (solid lines) with the experimental angular distributions (circles). For $E_\alpha = 100$ MeV, the calculation with a reduced imaginary potential ($W_0 = 26$ MeV) is also shown by dashed lines. Data are taken from Refs. 29, 31, and 32.

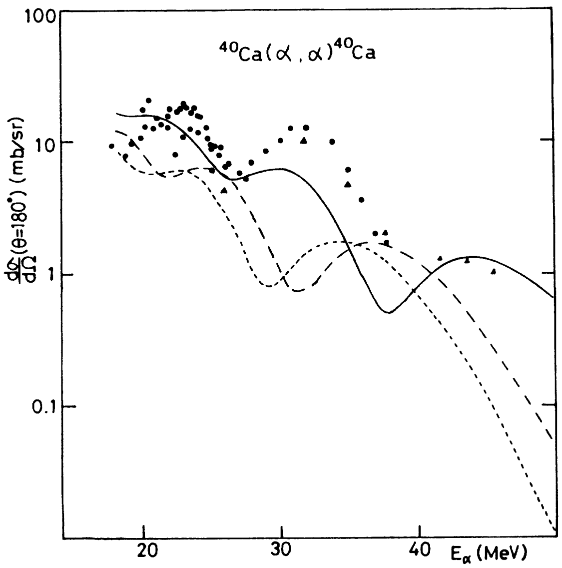

FIG. 4. The large-angle experimental excitation function (Ref. 35) is compared with the theoretical functions calculated at $\theta = 180°$ with the HNY500 (solid lines), HNY442 (dashed lines), and the HNY422 (dotted lines) folding potentials.

B. Folding potentials with HNY442 and HNY422 forces

Next, we investigate the $\alpha + {}^{40}$Ca system from the second viewpoint. We take the HNY force with -442 MeV for V_2^{13} (abbreviated as HNY442) so that the first Pauli-allowed ($2n + l = 12$) 0^+ state corresponds with the experimental (0^+) state at 8.54 MeV. The calculated 0^+ state has six nodes (not seven) in the relative wave function. As to the fusion excitation function as seen in Fig. 2, the agreement with the data is fair except for the peak at $E_\alpha \approx 11$ MeV. The oscillations are due to the broad resonances of $2n + l = 12$. As displayed in Fig. 5, the agreement of the calculated angular distributions of α-particle scattering from ^{40}Ca with the data is good at the lower energy of $E_\alpha = 18$ MeV. However, as the incident energy increases, the agreement with the data deteriorates, especially at the backward hemisphere. The agreement collapses drastically at $E_\alpha = 49.5$ MeV. Above this energy the structure of the calculated angular distributions is completely different from the experimental data. The deterioration of the agreement with the data at backward angles could be improved at the lower energy region by decreasing the strength of the imaginary potential. However, this kind of prescription does not serve to solve the matter at all above $E_\alpha = 49.5$ MeV because the disagreement is to be ascribed to a lack of attraction in the real part of the potential. In the HNY442 folding potential the rainbow scattering starts far below the experiment. This clearly indicates the inadequacy of the HNY442 folding potential. As seen in Fig. 4, the gross structure of the calculated excitation function is dissimilar to the data. The decrease of the cross sections becomes rapid at the higher energies, as already seen in the angular distributions in Fig. 5. As far as the magnitude of the cross sections at $\theta = 180°$ is concerned, it could be enhanced by reducing the strength of the imaginary potential used. However, this cannot change the situation that the rainbow scattering starts at a lower energy than in experiment.

From the third viewpoint, we have also studied the $\alpha + {}^{40}$Ca system by adopting the HNY422 force which was chosen so that the calculated first-allowed state corresponds with the experimental 0^+ state at 11.2 MeV. The calculated results are shown in Figs. 1, 2, 4, and 5. The calculations give results almost similar to those of the HNY442 case. As seen in Fig. 5, the agreement of the calculation with the scattering data collapses above $E_\alpha = 49.5$ MeV.

Through the above analysis, it has been found that both the HNY442 and HNY422 folding potentials cannot correctly reproduce the scattering data. Whether a folding potential can describe the rainbow scattering energy correctly is an important criterion for checking the appropriateness of the effective two-body force employed.

IV. DISCUSSION

The results of the HNY442 and HNY422 folding potentials are quite different from those of the HNY500 folding potential at both extreme ends of low and high energies: At the low-energy end the first two folding potentials give completely different energy level structure from the third as seen in Fig. 1, and at the high-energy end above $E_\alpha \approx 50$ MeV, the shape of the angular distributions becomes drastically different, reflecting the energy where the rainbow scattering starts. The authors of Refs. 7 and 22 have emphasized the importance of extending the potential determined at the rainbow scattering region to the lower energy region, namely the BAA region, to the fusion region and then to the bound (quasibound) state energy region, which made it possible to disentangle the long-standing dilemma about the α-cluster structure of ^{44}Ti. It has been difficult to know which viewpoint is correct as long as we are restricted to the bound-state energy region. The Münster group[14,15] has been advocating the view of the mixed-parity α-cluster band starting at 11.2 MeV. On the other hand, Horiuchi[8] maintains the (0^+) state at 8.54 MeV view. In order to reproduce the mixed-parity cluster band, the Münster group adopted the Brink-Boeker $B1$ force in their microscopic calculations while Horiuchi adopted the Volkov No. 1 force with $m = 0.658$ to reproduce the parity doublet states of (0^+) at 8.54 MeV and 1^- at 11.7 MeV. However, by extending the calculations using the models to the higher energy region above $E_\alpha \approx 50$ MeV, it is found that the models fail in reproducing the onset energy of the rainbow scattering. The angular distributions of α-particle scattering from ^{40}Ca at less than $E_\alpha \approx 30$ MeV could be somehow reproduced by the models, although the fit to the data is not satisfactory. However, the serious difficulty is that they cannot give the rainbow scattering energy correctly, which causes the disagreement of the calculated angular distributions with the data at the relevant energies. Therefore, the angular distributions at the high-energy region is a crucial test to discriminate between the models.

The second difficulty of the second and third viewpoints is that they say nothing about the structure of the ground band. Although it has been shown in the literature[16–18] that the dominant component of the ground-band states is (12,0) in the SU3 representation, in these viewpoints the first Pauli-allowed 0^+ state which has six nodes in the relative wave function is attributed to the excited state at $E_x = 8.54$ MeV or $E_x = 11.2$ MeV, and there appears no band corresponding to the experimental ground band. These viewpoints cannot explain why the models[4,6,7,18] [which have (12,0) components] dominantly reproduce the essential energy-level structure and $B(E2)$ values of the ground-state band (it was claimed that this is superficial in Ref. 8).

We should mention here the experimental state 1^- at 11.7 MeV advocated by the second and third viewpoints. It is very difficult to interpret this state as having a simple

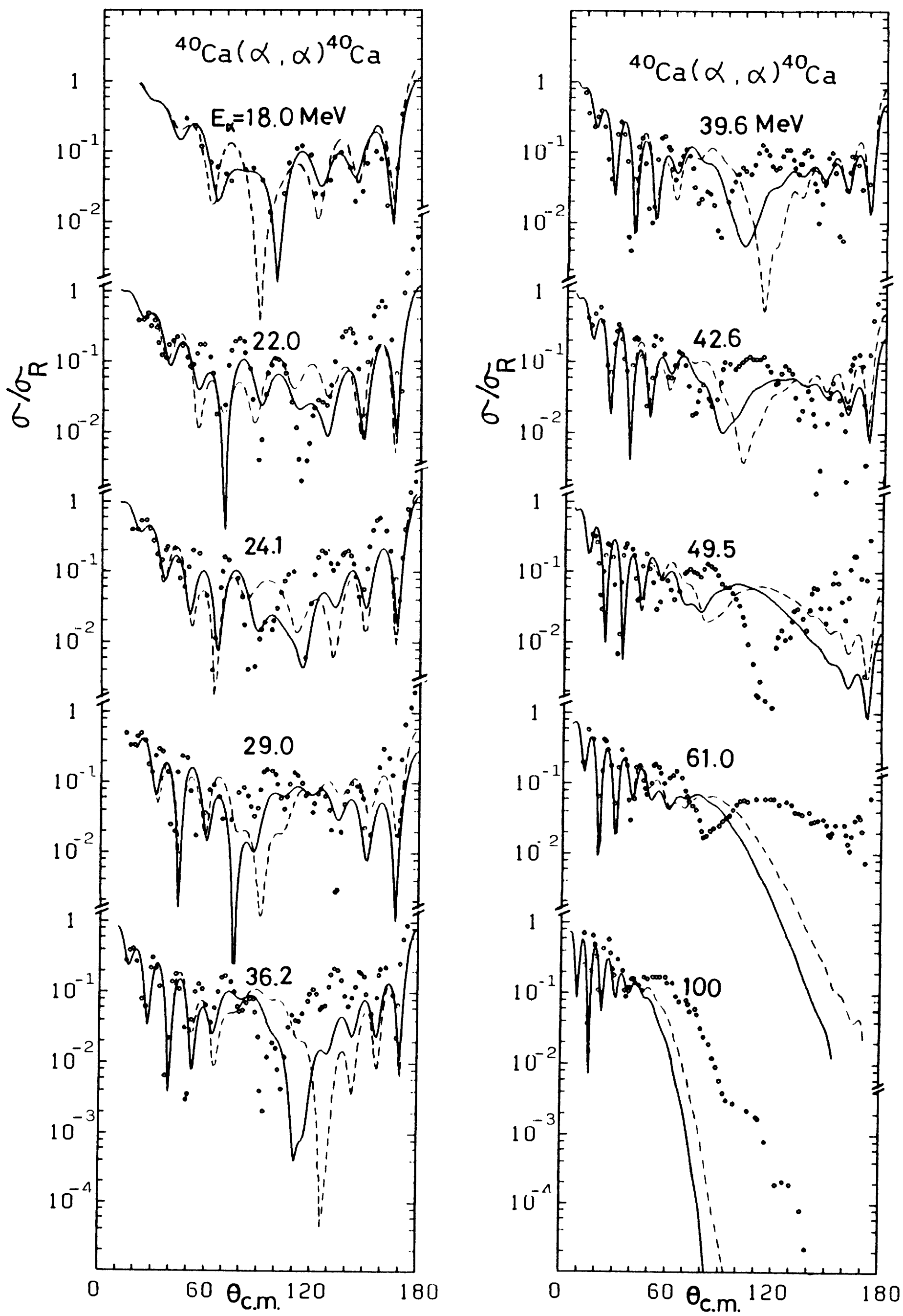

FIG. 5. The same as Fig. 3, but for the theoretical cross sections calculated with the HNY442 (dashed lines) and HNY422 (solid lines) folding potentials.

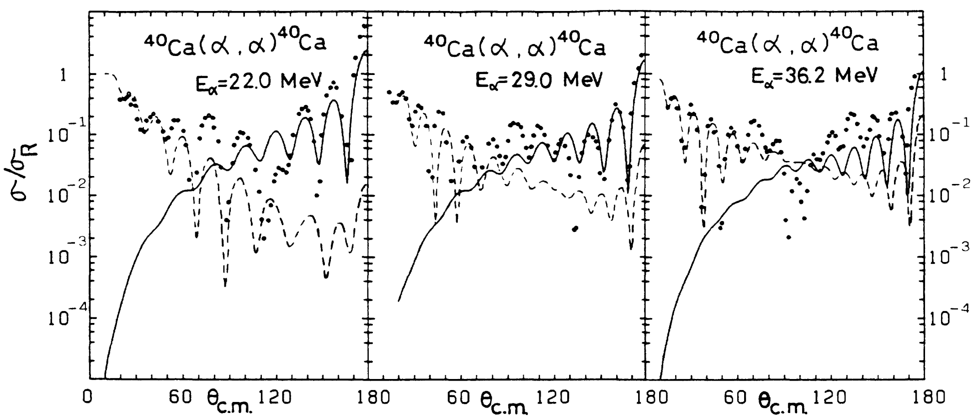

FIG. 6. Angular distributions calculated by the HNY500 folding potential $E_\alpha = 22$, 29, and 36.2 MeV are decomposed into internal (solid lines) and barrier (dashed lines) wave contributions.

$\alpha + {}^{40}$Ca (g.s.) cluster because of the incompatibility of its very small width ($\Gamma = 40$ keV) with its position with respect to the $l = 1$ barrier. As for the 3^- state at 12.8 MeV extracted by Frekers *et al.*[10] its spin determination is far less conclusive and should be confirmed independently.

As far as we are concerned, with the fusion data alone or the angular distributions of BAA in a limited energy range, it is difficult to determine which potential is adequate. In Ref. 22 it was shown that the oscillations of the fusion excitation function can be reproduced equally well by the optical potentials with discrete ambiguity. It has been also shown that many optical potentials[14,28,33,37,38] can give a good description of the experimental angular distributions of BAA. As shown in Fig. 5, even the HNY422 folding potential can give a qualitative description of the angular distributions at $E_\alpha < 30$ MeV. Therefore, what is important in checking the appropriateness of a potential is whether it can describe the bound states, quasibound states, fusion reactions, BAA, and rainbow scattering in a unified way.

Next, we discuss the mechanism of the oscillations in the fusion excitation function and that of the angular distributions of BAA. In the HNY500 case, the peaks of the oscillations are caused by the broad resonances of the higher nodal band, ($2n + l = 14$). On the other hand, in the HNY422 and HNY442 cases, the peaks of the oscillations are brought about by the first-allowed band of $2n + l = 12$. It is noted that although the character of the two bands is quite different from that of HNY500, there is no distinctive difference in the calculated fusion excitation functions in Fig. 2. This is because the gross behavior of the phase shifts is not very different in the low-energy region; a remarkable difference appears at the higher energies. In the energy range of $E_\alpha = 18-30$ MeV, although the shape of the angular distributions calculated with the HNY500 and HNY422 folding potentials does not differ much, the magnitude of the cross sections in the latter case is damped at the backward angles compared with the former. This is due to a lack of attraction in the HNY422 folding potential. It has been shown[36] that the BAA is caused by the high-spin members of the higher nodal bands of $2n + l = 14$, $2n + l = 15$, and $2n + l = 16$. To see this is the case in our HNY500 potential, it is useful to decompose the angular distributions and the scattering amplitudes at $\theta = 180°$ in terms of internal and barrier waves.[36,39] The calculated results using the quantum decomposition method[40] are shown in Fig. 6 at $E_\alpha = 22$, 29, and 36.2 MeV. The backward enhancement of the angular distributions is brought about by the internal waves, which are the high-spin members of the higher nodal bands. It is noted that almost the same partial waves in the broad resonant states contribute to the backward angular distributions at $E_\alpha \lesssim 30$ MeV in the three cases of HNY500, HNY442, and HNY422.

Finally, we show the folding potentials of HNY500, HNY442, and HNY422 in Fig. 7. For comparison, the phenomenological potential D180 (Ref. 22), (which describes the $\alpha + {}^{40}$Ca scattering in a wide energy range) and the equivalent local potential obtained by Horiuchi[8] in the microscopic calculation are also shown. The values of the volume integral per nucleon pair, $J_v/4A$ for the HNY500, HNY442, and HNY422 folding potentials, are 336 MeV fm^3, 285 MeV fm^3, and 268 MeV fm^3, respectively. The value for HNY500 potential is close to that for the unique phenomenological potential D180, 350 MeV fm^3. As seen in Fig. 7, the shape of the HNY500 potential is quite similar to the D180 potential, although it is slightly deeper than the latter in the inner region and is slightly shallower in the intermediate region. On the other hand, the shape of the HNY442 and HNY422 folding potentials is quite different from that of the HNY500 and D180 potentials. Since they are of shorter range compared with the HNY500 and D180 potentials, the values of the volume integral for these two are close to that of the potentials of the shallower family, 280 MeV fm^3. It is noted that the equivalent local potential of Horiuchi gives a value of the volume integral—330 MeV fm^3 at $E_\alpha = 0$ MeV[8]—which is almost as large

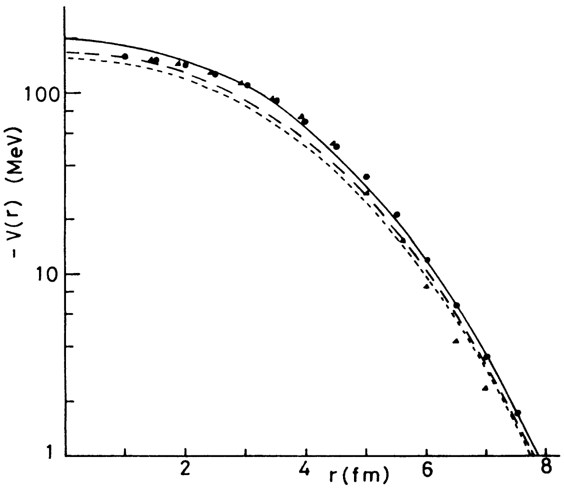

FIG. 7. Comparison of the folding potentials of HNY500 (solid lines), HNY442 (dashed lines), and HNY422 (dotted lines) with the phenomenological potential D180 (filled circles) (Refs. 7 and 22) and with the equivalent local potential (triangles) of Horiuchi (Ref. 8).

as that of the HNY500. On the other hand, the former potential cannot correctly describe the nuclear structure of ^{44}Ti and α-particle scattering from ^{40}Ca, as was shown in Sec. III. In fact, Horiuchi's equivalent local potential is of shorter range as seen in Fig. 7, reflecting that the Volkov No. 1 force with a large value of $m=0.658$ was adopted so as to reproduce the excited (0^+) state at 8.54 MeV. It seems these two points, namely the short-ranged character and the large volume integral of 330 MeV fm^3, are incompatible. It is quite difficult to understand how this large volume integral is obtained from Horiuchi's equivalent local potential based on the second viewpoint. The equivalent potentials of the Münster group (Figs. 1 and 2 in Ref. 15) also give a value of the volume integral which belongs to the shallower family potential. The potential is also much shallower at the outer region than the D180 potential. The short-range character of the potentials, which is related to the collapse in reproducing the angular distributions at the higher energy region, is caused by the requirement that the relative wave function of the first-allowed state is pushed outward to enhance clustering.

V. SUMMARY

We have shown that the folding model with the HNY force reproduces the α-cluster structure of ^{44}Ti including the energy level structure, the fusion excitation function, and the angular distributions in a wide energy range of $E_\alpha=18-100$ MeV. It is essential to use a two-body effective interaction which can bind the energy levels of the ground-state band of ^{44}Ti. These potentials, based on the second and third viewpoints that the α-cluster model should be applied to the excited 0^+ state, 8.54 MeV or 11.2 MeV, cannot reproduce the experimental angular distributions at $E_\alpha \gtrsim 50$ MeV and onset energy of the rainbow scattering. On the other hand, the first viewpoint inevitably predicts the $K=0_1^-$ band with the $\alpha+{}^{40}$Ca cluster structure just above the threshold. It is highly desirable to search for the $K=0_1^-$ band in experiment. The present study suggests that when it is difficult to determine an effective interaction to be adopted in the structure study in the bound (quasibound) state as was the case in ^{44}Ti, it would be useful to determine an interaction through a systematic analysis of α-particle scattering at the higher energy region including rainbow scattering.

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