Charge form factors and root mean square radii of ^{3}He and ^{3}H with the new Bonn potential

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We calculate the charge form factors and the rms charge radii of ^{3}H and ^{3}He using the wave functions generated from the solutions of the Faddeev equations in momentum space with the new Bonn one-boson-exchange momentum space potential. For the charge radii, we obtain, $r_c(^3\mathrm{H})=1.72$ fm and $r_c(^3\mathrm{He})=1.89$ fm, which are in excellent agreement with the experimental data. However, our results for the trinucleon charge form factors in the impulse approximation indicate no improvement with regard to the experimental situation.

I. INTRODUCTION

In 1960 Faddeev published a rigorous method of solving the nonrelativistic three-body problem.[1] His original work became widely known in 1965 when an English translation of his book on his method was published.[2] With the advent of better computers in the late sixties, the Purdue[3,4] and Utrecht[5] groups developed several numerical techniques for solving the Faddeev equations for the ^{3}H and ^{3}He ground states with simple local nucleon-nucleon (NN) potentials. In a series of papers[6–12] during the early 1970's, the Purdue group published the results of their Faddeev calculations of the low-energy bound-state properties of ^{3}H and ^{3}He using a "realistic" NN potential [the Reid soft-core (RSC) potential][13] using the numerical techniques developed by the Utrecht group.[5] These results showed that the nonrelativistic model with the RSC potential gives a reasonable description of most of the low-energy bound-state properties of ^{3}H and ^{3}He [binding energy of ^{3}H,[6,9,10] the root mean square (rms) charge radii,[6,12] asymptotic normalizations,[8,12] magnetic moments,[7,11] etc.]. However, their results also showed small discrepancies ($10\sim15\%$) with the experimental data: (1) The calculated triton binding energy is smaller by ~1 MeV, (2) the calculated rms charge radii are too large, and (3) the calculated asymptotic normalization ratio between the S and D states,[14] C_2/C_0, is too small. Subsequent refined Faddeev calculations[15–19] using the RSC potential and other realistic NN potentials [Argonne V14 (AV14),[20] super-soft-core C (SSCC),[21] de Tourreil-Rouben-Sprung (TRS),[22] and Paris[23] potentials] have confirmed the above discrepancies. In addition, there are other known discrepancies: (4) The calculated Coulomb energies for ^{3}He are smaller than 640 keV compared to the ^{3}H-^{3}He binding energy difference of 764 keV,[24,25] and (5) the calculated spin-doublet neutron-deutron (nd) scattering length[26,27] is too large by approximately a factor of 2 as compared to the experimental value[28] of 0.65 ± 0.4 fm.

Recently, the Bonn meson-exchange model,[29] which takes into account the various "physical" mesons observed in nature, has been proposed. Indeed, the Bonn meson-exchange picture of the two-nucleon interaction may be an effective low-energy limit of a more fundamental quark-gluon picture of quantum chromodynamics.[29,30] Being a full meson field theoretic potential, it has an added advantage in that one can in principle calculate the contributions from meson-exchange currents to various electromagnetic processes in a consistent manner. Furthermore, it has already been applied to nuclear-matter calculations with very promising results.[30]

In recent Faddeev calculations with the new Bonn potential, Brandenburg *et al.*[31,32] obtained a triton binding energy, $E_b(^3\mathrm{H})=8.34$ MeV, using the momentum space version (OBEPQ) of the potential, and Sasakawa[33] obtained $E_b(^3\mathrm{H})=8.33$ MeV using the coordinate space version (OBEPR) of the potential. This is to be compared to the Reid soft-core and the Paris potential predictions of $E_b(^3\mathrm{H})=7.35$ (Ref. 19) and 7.64 MeV,[17,33] respectively, and to the experimental value of $E_b(^3\mathrm{H})=8.48$ MeV.[34] Sasakawa[33] also reports other trinucleon calculations with Bonn OBEPR including the asymptotic normalization ratio and the rms charge radii. In addition, Klepacki *et al.*[35] have calculated the asymptotic normalization constants with Bonn OBEPQ which prove to be consistent with the results of Sasakawa.[33] In light of the above calculations, it remains to be seen how the Bonn OBEPQ potential predicts the charge form fac-

tors and the rms charge radii of ^{3}He and ^{3}H. We emphasize that the OBEPQ and OBEPR potentials are not identical [for example, the $J=1$ tensor coupling parameter ϵ_1 at 325 MeV (Ref. 36) is different between the two].

In this paper, we present the results of our calculation of the charge form factors and the charge radii of ^{3}He and ^{3}H using the wave functions generated from the solutions of the Faddeev equations with the new Bonn OBEPQ potential and also with the RSC potential for comparison. In Sec. II we first give a brief history of the experimental results, and a synopsis of the previous theoretical efforts. After providing a description of our wave functions and our numerics, we then present our results for the charge form factors and compare them with the experimental data. Isospin decomposition of the charge form factors is also made and is compared with the recent data. In Sec. III we first discuss the inconsistencies in the experimentally extracted values of $r_c(^3\mathrm{H})$ and $r_c(^3\mathrm{He})$, and discuss possible reasons for the inconsistencies. Then we present our results for $r_c(^3\mathrm{H})$ and $r_c(^3\mathrm{He})$. Conclusions and a summary are given in Sec. IV.

II. CHARGE FORM FACTORS

The first measurements of the charge form factors of ^{3}He and ^{3}H, $F_{\mathrm{ch}}^{^3\mathrm{He}}(Q^2)$ and $F_{\mathrm{ch}}^{^3\mathrm{H}}(Q^2)$, were carried out by Collard *et al.*[37] in 1965 for momentum transfer squared Q^2, $1.0\le Q^2\le 8.0$ fm^{-2}, and the corresponding rms charge radii of $r_c(^3\mathrm{He})=1.87\pm0.05$ fm and $r_c(^3\mathrm{H})=1.70\pm0.05$ fm were extracted. Since then, more measurements[38–46] of $F_{\mathrm{ch}}^{^3\mathrm{He}}(Q^2)$ and $F_{\mathrm{ch}}^{^3\mathrm{H}}(Q^2)$ have been made for 0.032 fm$^{-2}\le Q^2\le 100$ fm^{-2} and the corresponding rms radii, $r_c(^3\mathrm{He})$ and $r_c(^3\mathrm{H})$, have been extracted.[38–44,47] In 1970 McCarthy *et al.*[38] measured $|F_{\mathrm{ch}}^{^3\mathrm{He}}(Q^2)|$ for 0.032 fm$^{-2}\le Q^2\le 20$ fm^{-2} and found the first minimum, $|F_{\mathrm{ch}}^{^3\mathrm{He}}(Q^2_{\mathrm{min}})|=0$, at $Q^2_{\mathrm{min}}\approx 11$ fm^{-2} and the secondary maximum $|F_{\mathrm{ch}}^{^3\mathrm{He}}(Q^2_{\mathrm{max}})|\approx 6\times10^{-3}$ at $Q^2_{\mathrm{max}}\approx 16\sim 19$ fm^{-2}.

Since then, the first minimum and the secondary maximum of $|F_{\mathrm{ch}}^{^3\mathrm{He}}(Q^2)|$ have been investigated with various theoretical models.[6,10,48,49] Most of these theoretical calculations predict much smaller values of the secondary maximum compared to the experimental value.[38] Earlier calculations were based on the impulse approximation using the variational method,[48] truncated,[49] and approximate Faddeev solutions[6] with the Reid soft-core potential[13] effective in $^1S_0$ and $^3S_1-{}^3D_1$ partial-wave states ($J\le 1$). They all underestimated $|F_{\mathrm{ch}}^{^3\mathrm{He}}(Q^2_{\mathrm{max}})|$ by factors of $8\sim10$. Later in 1975 the Purdue group[10] improved the previous Faddeev calculation[6] of $|F_{\mathrm{ch}}^{^3\mathrm{He}}(Q^2)|$ for $Q^2\le 21$ fm^{-2} with $J\le 1$ and obtained the first minimum at $Q^2\approx 14$ fm^{-2} and the secondary maximum at $Q^2_{\mathrm{max}}\approx 18$ fm^{-2} with $|F_{\mathrm{ch}}^{^3\mathrm{He}}(Q^2_{\mathrm{max}})|\approx 1.4\times10^{-3}$, which is still small by a factor of ~4 compared to the measured value. Since then, there have been many other Faddeev calculations[17–19,50–54] of $|F_{\mathrm{ch}}^{^3\mathrm{He}}(Q^2)|$ using the RSC potential[13] as well as other realistic nucleon-nucleon (NN) potentials. More refined Faddeev calculations with the RSC potential have been recently carried out by the Hannover group[17] and the Los Alamos-Iowa group[19] keeping all $J\le 2$ (18-channel)[17] and all $J\le 4$ (34-channel)[19] NN partial-wave states. Their calculated values of $|F_{\mathrm{ch}}^{^3\mathrm{He}}(Q^2)|$ are very similar to the Purdue groups's original 5-channel ($J\le 1$) calculations[10] with the RSC potential.[13]

Experimental measurements of $|F_{\mathrm{ch}}^{^3\mathrm{H}}(Q^2)|$ have been very few[37,42,43] due to the difficulties associated with handling ^{3}H targets. Most recently in 1985 Juster *et al.*[43] measured $F_{\mathrm{ch}}^{^3\mathrm{H}}(Q^2)$ for $0.3\le Q^2\le 22.9$ fm^{-2} and found the first minimum of $|F_{\mathrm{ch}}^{^3\mathrm{H}}(Q^2)|$ at $Q^2=12.6\pm0.4$ fm^{-2}.

Our charge form factors, for ^{3}He and ^{3}H are defined as

$$F_{\mathrm{ch}}(Q^2)=\frac{1}{Z}\int\int\int d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r}_3 \Psi_A^*(\mathbf{r}_1\mathbf{r}_2\mathbf{r}_3) \times\left[\sum_{i=1}^{3} e^{i\mathbf{Q}\cdot\mathbf{r}_i}\rho_i(\mathbf{Q})\right] \times\Psi_A(\mathbf{r}_1\mathbf{r}_2\mathbf{r}_3)\ , \tag{1}$$

where Q is the three-momentum transferred to the nucleus, Z is the number of protons in the nucleus, and $\Psi_A(\mathbf{r}_1\mathbf{r}_2\mathbf{r}_3)$ is the antisymmetric ground-state wave function in configuration space. The charge operator $\rho_i(\mathbf{Q})$ is[55]

$$\rho_i(\mathbf{Q})=\hat{\mathbf{e}}_i(Q^2)-Q^2\hat{\mathbf{e}}_i(Q^2)/8M^2 -(i\mathbf{Q}\cdot\boldsymbol{\sigma}_i\times\mathbf{k}_i)[2\hat{\boldsymbol{\mu}}_i(Q^2)-\hat{\mathbf{e}}_i(Q^2)]/4M^2\ , \tag{2}$$

where the last two terms are the Darwin-Foldy and spin-orbit terms, respectively, and are relativistic corrections to the first term. Corrections due to meson-exchange currents are ignored. The quantities $\boldsymbol{\sigma}_i$ and $\mathbf{k}_i$ are the spin and momentum of the ith nucleon, and M is the nucleon mass. The operators $\hat{\mathbf{e}}_i(Q^2)$ and $\hat{\boldsymbol{\mu}}_i(Q^2)$ are defined as

$$\hat{\mathbf{e}}_i(Q^2)=\tfrac{1}{2}[1+\tau_z(i)]F_{\mathrm{ch}}^p(Q^2) +\tfrac{1}{2}[1-\tau_z(i)]F_{\mathrm{ch}}^n(Q^2) \tag{3}$$

and

$$\hat{\boldsymbol{\mu}}_i(Q^2)=\tfrac{1}{2}\mu_p[1+\tau_z(i)]F_{\mathrm{mag}}^p(Q^2) +\tfrac{1}{2}\mu_n[1-\tau_z(i)]F_{\mathrm{mag}}^n(Q^2) \tag{4}$$

with $\tau_z(i)$ as the z component projection of the ith nucleon's isospin. $F_{\mathrm{ch}}^{p(n)}(Q^2)$ and $F_{\mathrm{mag}}^{p(n)}(Q^2)$ are the proton (neutron) charge and magnetic form factors, and for the present calculation we used the parametrization of nucleon form factors of Iachello *et al.*[56] and Blatnik and Zovko.[57] $\mu_{p(n)}$ is the proton (neutron) anomalous magnetic moment in nuclear magnetons. Using the Jacobi coordinates $\mathbf{x}_i=\mathbf{r}_j-\mathbf{r}_k$, $\mathbf{y}_i=\mathbf{r}_i-\frac{1}{2}(\mathbf{r}_j+\mathbf{r}_k)$, $\mathbf{R}=\frac{1}{3}(\mathbf{r}_i+\mathbf{r}_j+\mathbf{r}_k)$, and after integrating over the center-of-mass coordinate R, Eq. (1) becomes

$$F_{\mathrm{ch}}(Q^2)=\frac{3}{Z}\int\int d\mathbf{x}_i d\mathbf{y}_i \Psi_A^*(\mathbf{x}_i\mathbf{y}_i) \times e^{i\mathbf{Q}\cdot 2/3\mathbf{y}_i}\rho_i(Q^2)\Psi_A(\mathbf{x}_i\mathbf{y}_i)\ , \tag{5}$$

where the momentum conserving δ function is suppressed. After Fourier transforming the configuration-space wave function $\Psi_A(\mathbf{x}_i\mathbf{y}_i)$ to the momentum-space wave function $\Psi_A(\mathbf{p}_i\mathbf{q}_i)$, we obtain

$$F_{\rm ch}(Q^2)=\frac{3}{Z}\int\int d\mathbf{p}_i d\mathbf{q}_i \Psi_A^*(\mathbf{p}_i\mathbf{q}_i')\rho_i(Q^2)\Psi_A(\mathbf{p}_i\mathbf{q}_i)\ , \tag{6}$$

where $\mathbf{q}_i'=\mathbf{q}_i+\frac{2}{3}\mathbf{Q}$, and $\mathbf{p}_i$ and $\mathbf{q}_i$ are the conjugate momenta to the Jacobi coordinates $\mathbf{x}_i$ and $\mathbf{y}_i$, respectively.

For a given two-nucleon potential, we solve the Faddeev equations in momentum space to obtain the Faddeev amplitude $\Psi_F(\mathbf{pq})$. The totally antisymmetric wave function $\Psi_A(\mathbf{pq})$ is generated from $\Psi_F(\mathbf{pq})$ by the following equation:[58]

$$\begin{aligned}\Phi_A(\mathbf{pq})&=(1+P)\Psi_F(\mathbf{pq})\\&=\sum_{\alpha=1}^{N}\Psi_\alpha^D(pq)\phi_\alpha(\hat{\mathbf{p}}\hat{\mathbf{q}})\\&\quad+\sum_{\alpha=1}^{M}\Psi_\alpha^{\rm ex}(pq)\phi_\alpha(\hat{\mathbf{p}}\hat{\mathbf{q}})\ ,\end{aligned} \tag{7}$$

where P is the sum of the cyclic and anticyclic permutation operators, $\Psi_\alpha^D(pq)$ is the direct term generated from the solution of the Faddeev equations, and $\Psi_\alpha^{\rm ex}(pq)$ is the exchange term involving the permutation operator. The partial-wave states $\phi_\alpha(\hat{\mathbf{p}}\hat{\mathbf{q}})$ are chosen to be the Jj coupling states, denoted by

$$\alpha=\{[(LS)J,(l\tfrac{1}{2})j]\mathcal{J}\mathcal{J}_z;(T\tfrac{1}{2})\mathcal{T}\mathcal{T}_z\}\ ,$$

where capital letters refer to the "pair" nucleons and small letters refer to the "spectator" nucleon. The detailed expressions for $\Psi_\alpha^D(pq)$, $\Psi_\alpha^{\rm ex}(pq)$, and $\phi_\alpha(\hat{\mathbf{p}}\hat{\mathbf{q}})$ are given in Appendix A. Explicit expressions of the partial-wave decomposition of Eq. (6) for the charge form factors in the Jj coupling scheme are given in Appendix B.

Three wave functions are generated for the Bonn OBEPQ potential corresponding to the Faddeev amplitudes with $J\le 1$ (5-channel, $N=5$), $J\le 2$ (18-channel, $N=18$), and $J\le 4$ (34-channel, $N=34$) two-nucleon partial-wave states. This will allow us to study the sensitivity of the first minimum and the secondary maximum of $|F_{\rm ch}^{^3{\rm He}}(Q^2)|$ to higher partial-wave components of the wave function. For the Reid soft-core potential, we generated a 5-channel wave function. The antisymmetrization of the wave function was carried out to 34-channel states for the exchange term ($M=34$) in Eq. (7) for all four wave functions. The pair and spectator momenta p and q have ranges $0\le p<\infty$ and $0<q\le q_{\rm max}$, where $q_{\rm max}$ is specified for each case later. Our calculated 34-channel ^{3}H binding energy is $E_b(^3{\rm H})=8.34$ Mev with the Bonn OBEPQ potential as in Ref. 32. The calculated probabilities of each partial-wave state are summarized in Table I. The Derrick-Blatt probabilities[6,58] for our Bonn 34-channel wave function are as follows. We have for the symmetric S states, $P(S)=92.57\%$, the mixed symmetric S states, $P(S')=1.25\%$, the P states, $P(P)=0.05\%$, and the D states, $P(D)=6.13\%$.

To show the convergence of $F_{\rm ch}(Q^2)$ as a function of

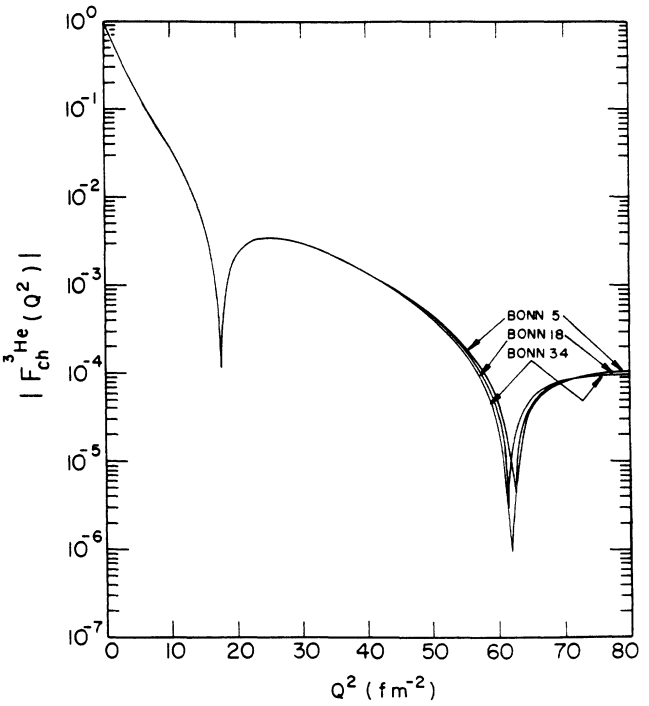

FIG. 1. Convergence of $|F_{\rm ch}^{^3{\rm He}}(Q^2)|$ calculated with the 5-, 18-, and 34- channel Bonn OBEPQ potential wave functions using point nucleons and with the restriction $0<q\le q_{\rm max}=5.5$ fm^{-1} for both the direct and exchange terms of Eq. (7).

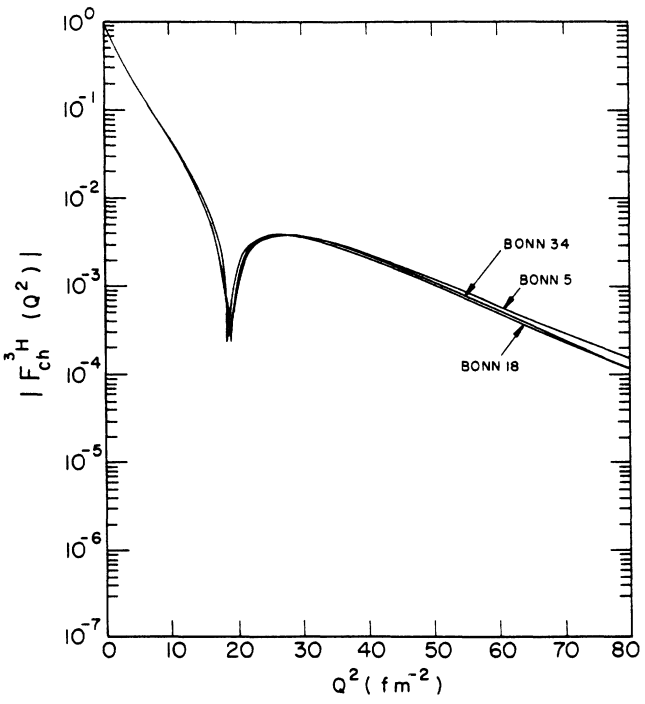

FIG. 2. Convergence of $|F_{\rm ch}^{^3H}(Q^2)|$ calculated with the 5-, 18-, and 34- channel Bonn OBEPQ potential wave functions using point nucleons and with the restriction $0<q<q_{\rm max}=5.5$ fm^{-1} for both the direct and exchange terms of Eq. (7).

the number of two-nucleon partial-wave states used, the results of $F_{ch}^{^3He}(Q^2)$ and $F_{ch}^{^3H}(Q^2)$ calculated with $J \leq 1$ (5-channel), $J \leq 2$ (18-channel), and $J \leq 4$ (34-channel) using the Bonn OBEPQ potential and $0 < q < q_{max} = 5.5$ fm^{-1} [for both the direct and exchange terms of Eq. (7)] are compared in Figs. 1 and 2, respectively. For all cases of Figs. 1 and 2, the point nucleon form factors are used. As can be seen from Figs. 1 and 2, the 18-channel and 34-channel cases already appear to be identical except near the very sensitive regions around the diffraction minimum. All three cases are very similar, and hence the 5-channel case can essentially be considered to be converged.

With the nucleon form factors of Blatnik and Zovko[57] and of Iachello *et al.*,[56] we have calculated $|F_{ch}(Q^2)|$ for ^{3}He and ^{3}H using the Bonn OBEPQ potential with $J \leq 1$ (5-channel) and $J \leq 4$ (34-channel) and also using the RSC potential with $J \leq 1$ (5-channel). For both the direct and

TABLE I. Partial-wave components and their probabilities.

State label	Partial-wave component						Probability (%)	
α	L	S	J	l	j	T	RSC	Bonn OBEPQ
1	0	0	0	0	$\frac{1}{2}$	1	4.388(+1)	4.598(+1)
2	1	1	0	1	$\frac{1}{2}$	1	1.118(+0)	9.168(−1)
3	1	0	1	1	$\frac{1}{2}$	0	1.444(−1)	9.827(−2)
4	1	0	1	1	$\frac{3}{2}$	0	2.644(−1)	2.156(−1)
5	1	1	1	1	$\frac{1}{2}$	1	1.224(+0)	1.038(+0)
6	1	1	1	1	$\frac{3}{2}$	1	5.259(−1)	4.279(−1)
7	0	1	1	0	$\frac{1}{2}$	0	4.440(+1)	4.621(+1)
8	0	1	1	2	$\frac{3}{2}$	0	1.067(+0)	8.697(−1)
9	2	1	1	0	$\frac{1}{2}$	0	3.113(+0)	2.000(+0)
10	2	1	1	2	$\frac{3}{2}$	0	2.300(−1)	9.619(−2)
11	2	0	2	2	$\frac{3}{2}$	1	3.795(−1)	2.317(−1)
12	2	0	2	2	$\frac{5}{2}$	1	5.823(−1)	3.671(−1)
13	2	1	2	2	$\frac{3}{2}$	0	1.399(−1)	1.191(−1)
14	2	1	2	2	$\frac{5}{2}$	0	9.982(−2)	9.183(−2)
15	1	1	2	1	$\frac{3}{2}$	1	1.190(−1)	5.844(−2)
16	1	1	2	3	$\frac{5}{2}$	1	1.104(+0)	6.297(−1)
17	3	1	2	1	$\frac{3}{2}$	1	3.994(−1)	1.378(−1)
18	3	1	2	3	$\frac{5}{2}$	1	4.070(−2)	8.040(−3)
19	3	0	3	3	$\frac{5}{2}$	0	1.173(−2)	5.913(−3)
20	3	0	3	3	$\frac{7}{2}$	0	1.398(−2)	9.159(−3)
21	3	1	3	3	$\frac{5}{2}$	1	5.329(−2)	3.500(−2)
22	3	1	3	3	$\frac{7}{2}$	1	4.730(−2)	3.236(−2)
23	2	1	3	2	$\frac{5}{2}$	0	2.309(−1)	1.453(−1)
24	2	1	3	4	$\frac{7}{2}$	0	1.605(−1)	6.098(−2)
25	4	1	3	2	$\frac{5}{2}$	0	6.293(−2)	1.999(−2)
26	4	1	3	4	$\frac{7}{2}$	0	4.295(−2)	1.594(−2)
27	4	0	4	4	$\frac{7}{2}$	1	8.362(−2)	2.662(−2)
28	4	0	4	4	$\frac{9}{2}$	1	9.567(−2)	3.272(−2)
29	4	1	4	4	$\frac{7}{2}$	0	1.619(−2)	1.055(−2)
30	4	1	4	4	$\frac{9}{2}$	0	1.212(−2)	9.089(−3)
31	3	1	4	3	$\frac{7}{2}$	1	2.000(−2)	2.284(−3)
32	3	1	4	5	$\frac{9}{2}$	1	2.132(−1)	7.213(−2)
33	5	1	4	3	$\frac{7}{2}$	1	9.412(−2)	2.453(−2)
34	5	1	4	5	$\frac{9}{2}$	1	1.018(−2)	1.265(−3)

exchange terms of Eq. (7), we restrict q to $0<q\leq q_{\max}=5.5$ fm^{-1} for the Bonn OBEPQ potential and for the RSC potential. Due to the presence of $\mathbf{q}'=\mathbf{q}+\frac{2}{3}\mathbf{Q}$ in the final-state wave function $\Psi^*_A(\mathbf{p},\mathbf{q}')$ in Eq. (6), $\Psi^*_A(\mathbf{p},\mathbf{q}')$ is set to zero whenever q' exceeds $q_{\max}$ when carrying out the actual integration. The above truncation is expected to introduce increasing numerical inaccuracies as Q increases. However, if we assume that $\Psi_A(\mathbf{p},\mathbf{q})$ is negligible beyond $q>5.5$ fm^{-1} (a reasonable approximation), then the numerical integration of Eq. (6) up to $q=q_{\max}=5.5$ fm^{-1} is expected to give accurate results for $Q<\frac{3}{2}q_{\max}=\frac{3}{2}(5.5)$ fm$^{-1}=8.25$ fm^{-1} or $Q^2<68$ fm^{-2}. To check this accuracy, we extended $q_{\max}$ to 11.5 fm^{-1} for the 5-channel wave functions and found negligible changes in our results. Our calculated values of $|F_{\rm ch}(Q^2)|$ for ^{3}He and ^{3}H are plotted in Figs. 3 and 4 and are compared with the data points of McCarthy, Sick, and Whitney[38] and Arnold *et al.*[46] for ^{3}He and Juster *et al.*[43] for ^{3}H. In these figures we also compare our calculated results which use the nucleon form factors of Iachello *et al.*[56] with our calculated results which use the nucleon form factors of Blatnik and Zovko.[57] This demonstrates the sensitivity of our calculated values of $F^{^3\rm He}_{\rm ch}(Q^2)$ and $F^{^3\rm H}_{\rm ch}(Q^2)$ to the nucleon form factors used for large values of Q^2. We note that for $Q^2\geq 20$ fm^{-2}, the experimental data[46] are for $|A^{1/2}(Q^2)|$, which is defined as

$$A(Q^2)=[\,|F^{^3\rm He}_{\rm ch}(Q^2)|^2+\mu^2_{^3\rm He}\tau\,|F^{^3\rm He}_{\rm mag}(Q^2)|^2]/(1+\tau)$$

with $\mu_{^3\rm He}=-3.2$ nuclear magnetons and $\tau=Q^2/4M^2_A$ (M_A is ^{3}He mass), rather than for $F^{^3\rm He}_{\rm ch}(Q^2)$. Since there are no available data for $F^{^3\rm He}_{\rm ch}(Q^2)$ [or $F^{^3\rm He}_{\rm mag}(Q^2)$], $Q>6$ fm^{-1}, comparison of the calculated $F^{^3\rm He}_{\rm ch}(Q^2)$ and the experimental $A^{1/2}(Q^2)$ may be premature, but we assume that $A(Q^2)\approx|F^{^3\rm He}_{\rm ch}(Q^2)|^2$ for 9 fm$^{-1}>Q\geq 6$ fm^{-1}. The above shows the importance of measuring $F^{^3\rm He}_{\rm mag}(Q^2)$, $Q\geq 6$ fm^{-1} in the future.

Figures 3 and 4 show that the charge form factors of ^{3}He and ^{3}H, $|F^{^3\rm He}_{\rm ch}(Q^2)|_{\rm Bonn}$ and $|F^{^3\rm H}_{\rm ch}(Q^2)|_{\rm Bonn}$, generated from the Bonn potential decrease more slowly at small Q^2 than $|F^{^3\rm He}_{\rm ch}(Q^2)|_{\rm Reid}$ and $|F^{^3\rm H}_{\rm ch}(Q^2)|_{\rm Reid}$ generated from the RSC potential. This observation is consistent with the binding energy predictions[10,15–19,31,32,59] of the two potentials as tighter binding is expected to give smaller rms charge radii. However, the slow decrease of $|F^{^3\rm He}_{\rm ch}(Q^2)|_{\rm Bonn}$ causes the first minimum to occur at $Q^2_{\min}\approx 17.5$ fm^{-2} for the Bonn OBEPQ case, away from the observed $Q^2_{\min}\approx 11.5$ fm^{-2}. Similarly, the first

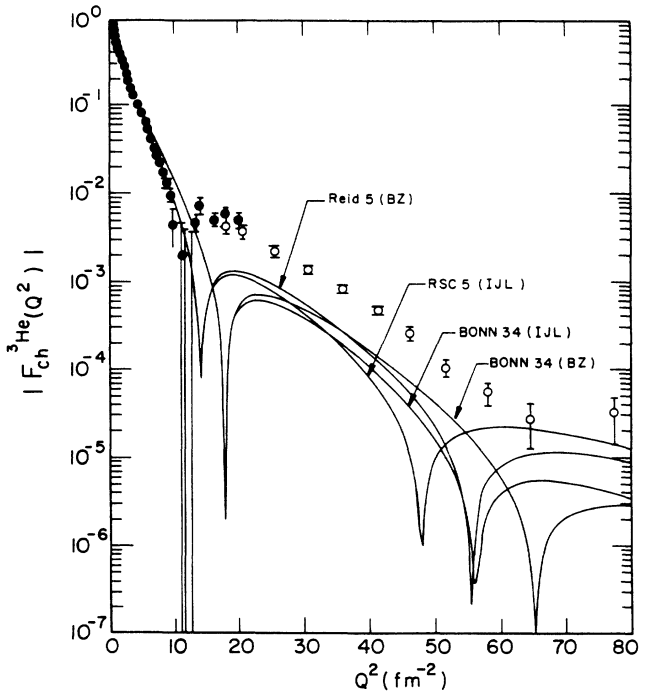

FIG. 3. Comparison of the 34-channel Bonn OBEPQ potential wave function result of $|F^{^3\rm He}_{\rm ch}(Q^2)|$ with the data points of McCarthy, Sick, and Whitney (Ref. 38) and Arnold *et al.* (Ref. 46) and also with the 5-channel RSC result. The nucleon form factors of Blatnik and Zovko (Ref. 57) and Iachello *et al.* (Ref. 56) are used. For the Bonn OBEPQ and RSC potential wave functions, q is restricted to $0<q\leq 5.5$ fm^{-1} for both the direct and exchange terms of Eq. (7).

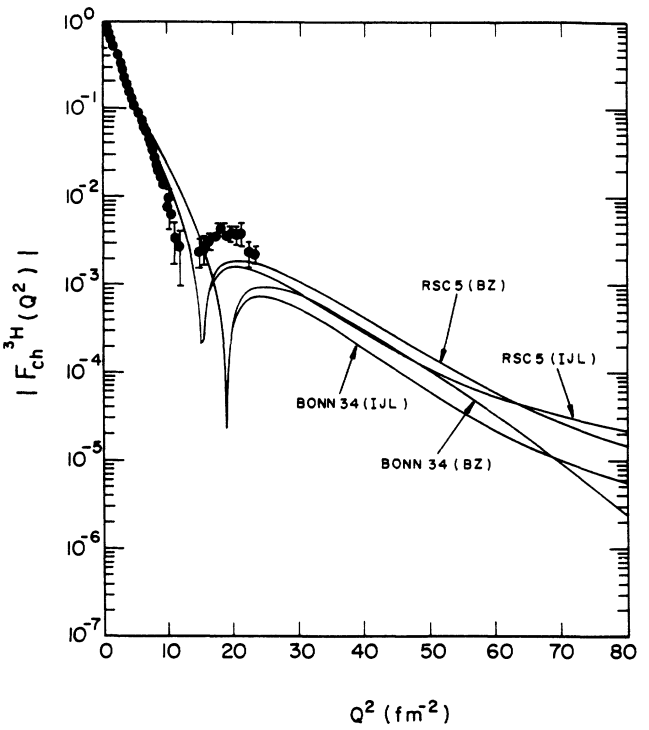

FIG. 4. Comparison of the 34-channel Bonn OBEPQ potential wave function result of $|F^{^3\rm H}_{\rm ch}(Q^2)|$ with the data points of Juster *et al.* (Ref. 43) and also with the 5-channel RSC result. The nucleon model of Blatnik and Zovko (Ref. 57) and Iachello *et al.* (Ref. 56) are used. The restrictions for q are the same as those used in Fig. 3.

minimum of $|F_{ch}^{^3H}(Q^2)|_{Bonn}$ occurs at $Q^2_{min}\approx 18.5$ fm^{-2} as compared to the observed value[43] of $Q^2_{min}\approx 12.6$ fm^{-2}. Figures 3 and 4 also show that the experimental value $|F_{ch}^{^3He}(Q^2_{max})|_{exp}\approx 6\times 10^{-3}$. We obtain the ratio

$$R=|F_{ch}^{^3He}(Q^2_{max})|_{exp}/|F_{ch}^{^3He}(Q^2_{max})|\approx 9$$

for the Bonn potential as compared to $R\approx 4$ for the RSC potential.

As expected, the effects of the Darwin-Foldy and spin-orbit terms of Eq. (2) are found to be small for $Q^2<20$ fm^{-2}. For example, these terms leave the first minimum of $|F_{ch}^{^3He}(Q^2)|$ relatively unchanged, and their contribution to the secondary maximum is approximately 20–25 %.

Our $|F_{ch}^{^3He}(Q^2)|_{Bonn}$ calculation shows that the first minimum and the secondary maximum are insensitive to the higher partial-wave components. For example, the secondary maximum varies less than 1% going from 5-channel to 18-channel states. At high momentum transfer, higher partial-wave components of the wave function are expected to come into play. However, we believe that the secondary minimum of $|F_{ch}^{^3He}(Q^2)|$ is a poor place to look for the effect of higher partial waves as we are dealing with very small high-partial-wave contributions there as indicated by the results in Figs. 1 and 2, and also dealing with large uncertainties in the nucleon form factors (in particular, the neutron charge form factor).

Figure 5 shows the isoscalar $F_{ch}^S(Q^2)$ and isovector $F_{ch}^V(Q^2)$ decompositions of the trinucleon charge form factors which are defined as

$$F_{ch}^S(Q^2)=F_{ch}^{^3He}(Q^2)+\tfrac{1}{2}F_{ch}^{^3H}(Q^2) \quad (8a)$$

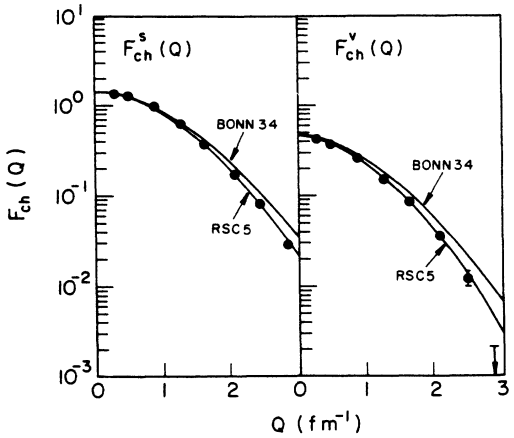

FIG. 5. Comparison of the data (Ref. 60) and the calculated results of $F_{ch}^S(Q^2)$ and $F_{ch}^V(Q^2)$ using the 34-channel Bonn OBEPQ potential wave function and the 5-channel RSC potential wave function. For both cases, the nucleon form factor of Blatnik and Zovko (Ref. 57) is used. The q restriction is the same as those used in Figs. 3 and 4.

and

$$F_{ch}^V(Q^2)=F_{ch}^{^3He}(Q^2)-\tfrac{1}{2}F_{ch}^{^3H}(Q^2)\ . \quad (8b)$$

They are compared to the recent measurements by Beck *et al.*[60] The figure shows that $F_{ch}^S(Q^2)$ and $F_{ch}^V(Q^2)$ calculated with the Reid soft-core potential compare better with the experimental data than those with the Bonn potential in the impulse approximation.

TABLE II. The current experimental status of the root mean square (rms) charge radii for ^{3}He and ^{3}H.

Reference	Q^2_{min} (fm^{-2})	Q^2_{max} (fm^{-2})	$r_c(^3\text{He})$ (fm)	$r_c(^3\text{H})$ (fm)
Collard *et al.*[a]	1.0	8.0	1.87±0.05[j]	1.70±0.05[j]
McCarthy *et al.*[b]	0.347	20.0	1.88±0.05[l]	
Szalata *et al.*[c]	0.032	0.34	1.89±0.05[j]	
Dunn *et al.*[d]	0.7	11.0	1.935±0.03[l]	
Retzlaff and Skopik[e]	0.885	3.20	1.877±0.019[j]	
Beck *et al.*[f]	0.0477	9.0	1.87±0.03[j]	
	0.0477	9.0	1.88±0.02[k]	
	0.0477	2.96		1.68±0.03[j]
Juster *et al.*[g]	0.3	22.9		1.76±0.04[l]
Otterman *et al.*[h]	0.20	3.7	1.976±0.015[j]	
			1.956±0.020[l]	
Martino[i]	0.3	22.9	1.93±0.03[l,n]	1.81±0.05[l,m]

[a]Reference 37.
[b]Reference 38.
[c]Reference 39.
[d]Reference 40.
[e]Reference 41.
[f]Reference 42.
[g]Reference 43.
[h]Reference 44.
[i]Reference 47.
[j]Power-series analysis.
[k]Fourier-Bessel analysis.
[l]Sum of Gaussian analysis.
[m]The data of Juster *et al.* (Ref. 43) are used.
[n]The data used are from Refs. 37–46.

III. ROOT MEAN SQUARE RADII

As pointed out by Dreher *et al.*[61] and Friar and Negele,[62] there are difficulties in determining the rms charge radii from data as regards normalization of different data sets, propagation of uncertainties, finite extent of data in momentum space and the completeness of finite truncated expansion series.

Table II shows a summary of the experimentally extracted values of $r_c(^3\mathrm{He})$ and $r_c(^3\mathrm{H})$ as well as their extraction methods. We first note that the extracted values of $r_c(^3\mathrm{H})$ vary from 1.65 to 1.815 fm. Even for the most recent measurement of $r_c(^3\mathrm{H})$, Juster *et al.*[43] have extracted $r_c(^3\mathrm{H})=1.76\pm0.04$ cm, while Martino[47] has obtained $r_c(^3\mathrm{H})=1.81\pm0.05$ fm from the same electron-scattering data.[43] Furthermore, despite the relative abundance of experimental data, the extracted values of $r_c(^3\mathrm{He})$ also vary widely, $1.87\leq r_c(^3\mathrm{He})\leq 1.98$ fm.

One method of obtaining $r_c(^3\mathrm{He})$ and $r_c(^3\mathrm{H})$ is by fitting $F_{\mathrm{ch}}(Q^2)$ to a sum of analytic functions over the entire observed momentum transfer Q and then using a power series for small Q^2,

$$F_{\mathrm{ch}}(Q^2)=1-\tfrac{1}{6}Q^2\langle r_c^2\rangle+\tfrac{1}{120}Q^4\langle r_c^4\rangle-\cdots, \qquad (9)$$

to extract $\langle r_c^2\rangle$. Another commonly used fitting method is based on the Fourier-Bessel expansion of the underlying charge and current densities, which involves the spherical Bessel functions.[63] Still another method is based on a series of the Gaussian functions.[64] Beck's calculations[63] indicate that using these fitting procedures over the entire available range of Q may result in a rather poor fit at small Q, the region in which data are scarce or do not exist and to which $r_c(^3\mathrm{He})$ and $r_c(^3\mathrm{H})$ are most sensitive. Therefore, accurate determination of $F_{\mathrm{ch}}(Q^2)$ at very small Q is absolutely necessary to extract r_c, and this no doubt is a challenging problem for experimentalists.

In this paper we choose to extract the rms charge radii using Eq. (9), and expand our calculated $F_{\mathrm{ch}}^{^3\mathrm{H}}(Q^2)$ and $F_{\mathrm{ch}}^{^3\mathrm{He}}(Q^2)$ in a power series in Q^2 over the interval $0<Q^2\lesssim 0.06$ fm^{-2} and extract $\langle r_c^2\rangle$ from the expansion coefficients. Our results calculated with various nucleon form factors (point nucleon, Blatnik-Zovko,[57] Iachello-Jackson-Lande,[56] and Janssens *et al.*[65]) are summarized and compared with previous calculations in Table III. As can be seen from the table, our values are in reasonable agreement with the previous calculations of Refs. 6, 16, and 17 for the RSC potential and Ref. 33 for the Bonn potential. Our calculated values of the rms radii are smaller by ~ 0.03 fm compared with the calculated values given in Refs. 16, 17, and 19. This discrepancy may be due to the differences in the antisymmetrization schemes used or may indicate our level of accuracy for calculating the rms radii of ^{3}He and ^{3}H. The table also shows that the rms charge radii predictions are insensitive to the nucleon form factor models, as at very small Q the proton charge form factors are accurately known and the neutron charge form factors are close to 0. The table also shows that our Bonn potential charge radii compare fairly well with those of Ref. 33, $r_c(^3\mathrm{H})=1.73$ fm and $r_c(^3\mathrm{He})=1.91$ fm, obtained from the coordinate version of the Bonn potential (OBEPR).[29] Our calculated results with the Bonn OBEPQ potential in Table II also show that the convergence of the Faddeev partial-wave series

TABLE III. The calculated rms charge radii of ^{3}H and ^{3}He in fermis with the nucleon form factors of Blatnik and Zovko (Ref. 57) (BZ), Iachello, Jackson, and Lande (Ref. 56) (IJL), and Janssens, Hofstadter, Hughes, and Yearian (Ref. 65) (JHHY).

	This work			Other work			
NN potential (No. of channel)	Nucleon form factors	^{3}H	^{3}He	Nucleon form factors	^{3}H	^{3}He	Reference
RSC	Point	1.66	1.86	Point	1.70(1.698[c])	1.89(1.933[c])	16,19,24[c]
(5)	BZ	1.80	2.01	BZ		2.04	17
	IJL	1.78	2.02				
	JHHY	1.79	2.03	JHHY	1.83	1.96	6[a]
OBEPQ	Point	1.55	1.70				
(5)	BZ	1.70	1.86				
	IJL	1.68	1.87				
	JHHY	1.69	1.88				
OBEPQ	Point	1.57	1.70				
(18)	BZ	1.71	1.86				
	IJL	1.69	1.87				
	JHHY	1.70	1.88				
OBEPQ	Point	1.56	1.70				
(34)	BZ	1.71	1.86				
	IJL	1.69	1.87				
	JHHY	1.70	1.88	JHHY	1.73	1.91	33[b]

[a]Darwin-Foldy and spin-orbit terms in the impulse approximation Eq. (2) are omitted.

[b]The Bonn OBEPR potential (Ref. 29) with $J\leq 4$ (34 channel) is used. Reference 33 does not state whether Darwin-Foldy and spin-orbit terms of Eq. (2) are included or not.

[c]Reference 24 includes the Coulomb interaction for ^{3}He. All other cases in this table neglect the Coulomb interaction.

for the rms radii is rather fast as are the cases of the charge form factors (Figs. 1 and 2) and of the binding energy of ^{3}H.[32]

Comparison of Table III with Table II shows that our Bonn OBEPQ result of $r_c(^3\mathrm{H})=1.71$ fm compares very well with the experimentally extracted values of Collard *et al.*,[37] and Beck *et al.*,[42] but not with Martino's value[47] of $r_c(^3\mathrm{H})=1.81\pm0.05$ fm. Also our Bonn OBEPQ result of $r_c(^3\mathrm{He})=1.86$ fm agrees well with the experimentally extracted values of Refs. 37–39, 41, and 42, but not well with Refs. 40, 44, and 47. Corrections due to the Coulomb force are yet to be made but are expected to give ≈ 0.04 fm increase.[24] When the estimated Coulomb corrections of ~ 0.04 fm is added, our calculated values of $r_c(^3\mathrm{He})$ may become consistent with those of Refs. 40 and 47 but not with that of Ref. 44. The experimental values of $r_c(^3\mathrm{He})$ by Dunn *et al.*,[40] Otterman *et al.*,[44] and Martino[47] are ~ 0.1 fm larger than those of Refs. 37–39, and 41–43 as shown in Table II. Thus, we conclude that the Bonn OBEPQ potential is capable of reproducing the experimental values[37–39,41–43] of the rms charge radii of ^{3}H and ^{3}He within the experimental uncertainties.

IV. SUMMARY AND CONCLUSIONS

We have calculated the charge form factors and the rms charge radii of ^{3}H and ^{3}He using the wave functions generated from the solutions of the Faddeev equations in momentum space with the new Bonn OBEPQ potential and, for comparison, with the RSC potential.

For the charge form factors, as discussed in Sec. II we find that the partial-wave expansion converges very rapidly. However, our calculated results with the Bonn OBEPQ potential for $F_{\mathrm{ch}}^{^3\mathrm{H}}(Q^2)$ and $F_{\mathrm{ch}}^{^3\mathrm{He}}(Q^2)$ do not improve the previous RSC results. A similar situation exists for the results calculated with the addition of three-nucleon forces.[27] Since the meson-exchange current corrections have not been calculated in a consistent scheme[66–68] yet for the Bonn OBEPQ potential as well as other potentials, it may be premature to assess definite merits of the Bonn OBEPQ and other potentials in reproducing the trinucleon charge form factor data for intermediate values of $Q<M\approx 5$ fm^{-1}. For $Q>M$, on the other hand, both the conventional meson-exchange currents (which involve $1/M$ expansion) and the nonrelativistic potential model are expected to be invalid. In this regard, it may be a desirable fact that our impulse approximation results for $F_{\mathrm{ch}}^{^3\mathrm{H}}(Q^2)$ and $F_{\mathrm{ch}}^{^3\mathrm{He}}(Q^2)$ are substantially smaller than the data for large Q, since we will be able to add contributions from relativistic models and/or the QCD-motivated quark models with appropriate currents associated with them for $Q>M$. Therefore, the failure of the Bonn OBEPQ potential in reproducing the charge form factors for $Q>M$ is not necessarily a disadvantage from a much broader and fundamental point of view.

As expected from the successful prediction of ^{3}H binding energy, the Bonn OBEPQ potential yields the correct rms charge radii of ^{3}H and ^{3}He as discussed in Sec. III. There we also noted that some of the rms charge-radii-extraction methods may not be reliable.

In summary, the Bonn OBEPQ potential is shown to reproduce the rms charge radii of ^{3}H and ^{3}He, but does not improve the charge form factors at least in the impulse approximation. It has been now shown to reproduce the following trinucleon properties: (1) the binding energy of ^{3}H (Refs. 31 and 32), (2) the rms charge radii of ^{3}H and ^{3}He (this work), (3) the asymptotic normalization constant for ^{3}H (Ref. 35), and (4) the binding energy difference between ^{3}H and ^{3}He (Ref. 25). In contrast, other realistic potentials[13,20–23] fail to reproduce the above properties.[27] It remains to be seen whether the Bonn OBEPQ potential is also capable of reproducing the spin-doublet *nd* scattering length.[26–28]

Even though all terms in the Bonn OBEPQ potential originate from a set of Feynman diagrams, their nonrelativistic reduction to static potential forms inevitably introduce approximations with adjustable parameters, thus losing the original "fundamental" nature of the strong interaction. However, since the Bonn OBEPQ potential is capable of reproducing both the two-nucleon data and most of the low-energy trinucleon data better than any other existing realistic potentials,[13,20–23] it can be regarded as a new successful semiphenomenological potential model for low-energy nuclear physics. This, in turn, will allow us to introduce additional corrections due to contributions from the relativistic models and/or from subnucleonic degrees of freedom such as the QCD-motivated quark models[69,70] for higher energy regime and processes.

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APPENDIX A

Here we present our results for the antisymmetrization of the trinucleon wave functions. Our antisymmetric wave function in Jacobi coordinates is defined as [Eq. (7)]

$$\Psi_A(\mathbf{pq})=(1+P)\Psi_F(\mathbf{pq})$$

$$=\sum_{\alpha=1}^{N}\Psi_\alpha^D(pq)\phi_\alpha(\hat{\mathbf{p}}\hat{\mathbf{q}})$$

$$+\sum_{\alpha=1}^{M}\Psi_\alpha^{\mathrm{ex}}(pq)\phi_\alpha(\hat{\mathbf{p}}\hat{\mathbf{q}})\ ,$$

where α represents the quantum numbers in the Jj coupling,

$$\alpha=\{[(L_1S_1)J_1,(l_1\tfrac{1}{2})j_1]\tfrac{1}{2}\mathscr{T}_z;(T_1\tfrac{1}{2})\tfrac{1}{2}\mathscr{T}_z\}\ ,$$

and

$$\phi_\alpha(\hat{\mathbf{p}}\hat{\mathbf{q}})=\{[\mathbf{Y}_{L_1}(\hat{\mathbf{p}})\times\boldsymbol{\chi}_{S_1}]_{J_1}\times[\mathbf{Y}_{l_1}(\hat{\mathbf{q}})\times\boldsymbol{\chi}_{1/2}]_{j_1}\}_{1/2\mathscr{J}_z}$$
$$\times(\boldsymbol{\eta}_{T_1}\times\boldsymbol{\eta}_{1/2})_{1/2\mathcal{T}_z}\cdot$$

P is the sum of the cyclic and anticyclic permutation operators which act upon our system of three identical nucleons using the standard isotopic spin formalism. The direct term is obtained from the solution to the momentum space Fadeev equations,

$$\Psi_\alpha^D(pq)=\int\int\Psi_F(\mathbf{pq})\phi_\alpha(\hat{\mathbf{p}}\hat{\mathbf{q}})d\hat{\mathbf{p}}\,d\hat{\mathbf{q}}\ ,$$

and the exchange term $\Psi_\alpha^{\rm ex}(pq)$ is then determined from the permutation operator by

$$\Psi_\alpha^{\rm ex}(pq)=\int\int[P\Psi_F(\mathbf{pq})]\phi_\alpha(\hat{\mathbf{p}}\hat{\mathbf{q}})d\hat{\mathbf{p}}\,d\hat{\mathbf{q}}\ .$$

After some lengthy algebra, we have

$$\Psi_\alpha^{\rm ex}(pq)=\sum_{\alpha_3}\sum_{\Lambda\lambda}\sum_{rr_1r_2}\sum_{\mathcal{L}_3\mathcal{S}_3}(-1)^{1+l_1+r+\mathcal{L}_3}(\hat{L}_1\hat{S}_1\hat{J}_1\hat{T}_1\hat{l}_1\hat{j}_1)(\hat{L}_3\hat{S}_3\hat{J}_3\hat{T}_3\hat{l}_3\hat{j}_3)(\hat{r}\,\hat{r}_1\hat{r}_2\hat{\mathcal{L}}_3\hat{\mathcal{S}}_3)^2$$
$$\times\left[\int_{-1}^{1}dx\,P_r(x)\frac{\Psi^D_{\alpha_3}(p_3q_3)}{(p_3)^{L_3}(q_3)^{l_3}}\right](-\tfrac{1}{2}p)^\Lambda(\tfrac{3}{4}q)^{L_3-\Lambda}(-p)^\lambda(-\tfrac{1}{2}q)^{l_3-\lambda}$$
$$\times\left[[2(L_3-\Lambda)+1]\begin{pmatrix}2L_3+1\\2\Lambda\end{pmatrix}[2(l_3-\lambda)+1]\begin{pmatrix}2l_3+1\\2\lambda\end{pmatrix}\right]^{1/2}$$
$$\times\begin{pmatrix}r&r_1&L_1\\0&0&0\end{pmatrix}\begin{pmatrix}r&r_2&l_1\\0&0&0\end{pmatrix}\begin{pmatrix}\Lambda&\lambda&r_1\\0&0&0\end{pmatrix}\begin{pmatrix}L_3-\Lambda&l_3-\lambda&r_2\\0&0&0\end{pmatrix}$$
$$\times\begin{Bmatrix}\frac{1}{2}&\frac{1}{2}&S_3\\\mathcal{S}_3&\frac{1}{2}&S_1\end{Bmatrix}\begin{Bmatrix}\frac{1}{2}&\frac{1}{2}&T_3\\\frac{1}{2}&\frac{1}{2}&T_1\end{Bmatrix}\begin{Bmatrix}r_1&r_2&\mathcal{L}_3\\l_1&L&r\end{Bmatrix}\begin{Bmatrix}L_1&S_1&J_1\\l_1&\frac{1}{2}&j_1\\\mathcal{L}_3&\mathcal{S}_3&\frac{1}{2}\end{Bmatrix}$$
$$\times\begin{Bmatrix}L_3&S_3&J_3\\l_3&\frac{1}{2}&j_3\\\mathcal{L}_3&\mathcal{S}_3&\frac{1}{2}\end{Bmatrix}\begin{Bmatrix}\Lambda&L_3-\Lambda&L_3\\\lambda&l_3-\lambda&l_3\\r_1&r_2&\mathcal{L}_3\end{Bmatrix},$$

where

$$\alpha_3=\{[(L_3S_3)J_3,(l_3\tfrac{1}{2})j_3]\tfrac{1}{2}\mathscr{J}_z;(T_3\tfrac{1}{2})\tfrac{1}{2}\mathcal{T}_z\}\ ,$$

and

$$p_3=(\tfrac{1}{4}p^2+\tfrac{9}{16}q^2-\tfrac{3}{4}pqx)^{1/2},\quad q_3=(p^2+\tfrac{1}{4}q^2+pqx)^{1/2}\ ,\quad \hat{L}=(2L+1)^{1/2}\ ,$$

and $P_r(x)$ are the Legendre polynomials.

APPENDIX B

We present a partial-wave decomposition of Eq. (6). First we write Eq. (6) as (the subscript i for particle index is suppressed)

$$F_{\rm ch}(Q^2)=\frac{3}{Z}\int\int d\mathbf{p}\,d\mathbf{q}\,\Psi_A^*(\mathbf{p},\mathbf{q}')[\rho_{\rm I}(Q^2)+\rho_{\rm II}(Q^2)]\Psi_A(\mathbf{p},\mathbf{q})\equiv F_{\rm ch}^{\rm I}(Q^2)+F_{\rm ch}^{\rm II}(Q^2)$$

with $\rho_{\rm I}(Q^2)=[1-(Q^2/8M^2)]\hat{e}(Q^2)$, and $\rho_{\rm II}(Q^2)=(-i/4M^2)\mathbf{Q}\cdot\boldsymbol{\sigma}\times\mathbf{k}[2\hat{\mu}(Q^2)-\hat{e}(Q^2)]$, where the operators $\hat{e}(Q^2)$ and $\hat{\mu}(Q^2)$ are given by Eqs. (3) and (4).

$F_{\rm ch}^{\rm I}(Q^2)$ and $F_{\rm ch}^{\rm II}(Q^2)$ can easily be evaluated by employing the spherical tensor algebra. The spin-isospin and angular part of the ground-state wave function $\phi_\alpha(\hat{p},\hat{q})$ in Eq. (7) is given in Appendix A in terms of the spherical tensor product. By expanding $\Psi_A(\mathbf{p},\mathbf{q}')\equiv\Psi_A(\mathbf{p},\mathbf{q}+\frac{2}{3}\mathbf{Q})$ in terms of the basis states of $\hat{p}$ and $\hat{q}$, we obtain

$$\Psi_A(\mathbf{p},\mathbf{q}')=\sum_{\beta,\Gamma_1,\Gamma_{\alpha'}}\kappa q^{(l_0-\lambda)}(\tfrac{2}{3}Q)^\lambda\left[\frac{1}{2}\int_{-1}^{1}dz\frac{\psi_\beta(p,q')P_r(z)}{q'^{\,l_0}}\right][\phi_{\alpha'}(\hat{p},\hat{q})\times\mathbf{Y}_{r_1}(\hat{Q})]\mathscr{J}\mathscr{J}_z\ ,$$

where

$$\beta=\{[(L_1,S_1)J_1,(l_0,\tfrac{1}{2})j_0]\mathscr{J}\mathscr{J}_z;(T,\tfrac{1}{2})\tfrac{1}{2},\tau_z\}\ ,$$

$$\alpha'=\{[(L_1,S_1)J_1,(l_1,\tfrac{1}{2})j_1]\mathscr{J}'\mathscr{J}'_z;(T,\tfrac{1}{2})\tfrac{1}{2},\tau_z\}\ ,$$

$$\Gamma_1=\{r,\lambda,j'_0,r_1\},\quad \Gamma_{\alpha'}=\{l_1,j_1,\mathscr{J}'\}\ ,$$

and

$$\kappa=\sqrt{4\pi}(-)^{l_1+r_1+j_0+j_1+j'_0+J_1+\mathscr{J}}\begin{bmatrix}2l_0+1\\2\lambda\end{bmatrix}^{1/2}[2(l_0-\lambda)+1]^{1/2}\hat{r}^2\hat{r}_1\hat{l}_0\hat{l}_1\hat{j}_1\hat{j}_0$$
$$\times(\hat{j}'_0)^2\hat{\mathscr{J}}'\begin{bmatrix}\lambda & r & r_1\\0 & 0 & 0\end{bmatrix}\begin{bmatrix}r & l_0-\lambda & l_1\\0 & 0 & 0\end{bmatrix}\begin{Bmatrix}\lambda & l_0-\lambda & l_0\\\frac{1}{2} & j_0 & j'_0\end{Bmatrix}\begin{Bmatrix}r & l_0-\lambda & l_1\\\frac{1}{2} & j_1 & j'_0\end{Bmatrix}\begin{Bmatrix}j_1 & r_1 & j_0\\\lambda & j'_0 & r\end{Bmatrix}\begin{Bmatrix}J_1 & j_1 & \mathscr{J}'\\r_1 & \mathscr{J} & j_0\end{Bmatrix},$$

with $\hat{J}=(2J+1)^{1/2}$.

Using the above expressions, we obtain for $F^{\mathrm{I}}_{\mathrm{ch}}(Q^2)$,

$$F^{\mathrm{I}}_{\mathrm{ch}}(Q^2)=\frac{3}{2Z}\left[1-\frac{Q^2}{8M^2}\right]\sum_{\alpha,\lambda}(\tfrac{2}{3}Q)^{\lambda}\frac{l!}{(l-\lambda)!\lambda!}\int\int dp\,dq\,p^2q^{l-\lambda+2}$$
$$\times\psi_\alpha(p,q)\int_{-1}^{1}dz\frac{P_\lambda(z)\psi^*_\alpha(p,q')}{q'^{\,l}}[F^S_{\mathrm{ch}}(Q^2)\pm(-)^T(1-\tfrac{2}{3}T)F^V_{\mathrm{ch}}(Q^2)]\ ,$$

where the $+(-)$ sign is taken for ^{3}He(^{3}H) nucleus, $F^S_{\mathrm{ch}}(Q^2)=\frac{1}{2}[F^p_{\mathrm{ch}}(Q^2)+F^n_{\mathrm{ch}}(Q^2)]$, and $F^V_{\mathrm{ch}}(Q^2)=\frac{1}{2}[F^p_{\mathrm{ch}}(Q^2)-F^n_{\mathrm{ch}}(Q^2)]$.

Evaluation of $F^{\mathrm{II}}_{\mathrm{ch}}(Q^2)$ is more involved due to the presence of the factor, $\mathbf{Q}\cdot\boldsymbol{\sigma}\times\mathbf{k}$. By setting $\mathbf{k}=\mathbf{q}$ in the center-of-mass system, we obtain

$$F^{\mathrm{II}}_{\mathrm{ch}}(Q^2)=\frac{27}{8ZM^2}\sum_{\alpha}\sum_{\Lambda}(-)^{j'_0+j_1}(\tfrac{2}{3}Q)^{\lambda+1}\begin{bmatrix}2l+1\\2\lambda\end{bmatrix}^{1/2}(\hat{l})^2(\hat{l}_1)^2[2(l-\lambda)+1]^{1/2}(\hat{r})^2(\hat{j}_1)^2(\hat{j}'_0)^2$$
$$\times[G^S(Q^2)\pm(-)^T(1-\tfrac{2}{3}T)G^V(Q^2)]\begin{bmatrix}r & l-\lambda & l_1\\0 & 0 & 0\end{bmatrix}\begin{bmatrix}1 & l & l_1\\0 & 0 & 0\end{bmatrix}$$
$$\times\begin{bmatrix}\lambda & r & 1\\0 & 0 & 0\end{bmatrix}\begin{Bmatrix}r & l-\lambda & l_1\\\frac{1}{2} & j_1 & j'_0\end{Bmatrix}\begin{Bmatrix}j_1 & 1 & j\\\lambda & j'_0 & r\end{Bmatrix}\begin{Bmatrix}\lambda & l-\lambda & l\\\frac{1}{2} & j & j'_0\end{Bmatrix}\begin{Bmatrix}j_1 & j & 1\\l_1 & l & 1\\\frac{1}{2} & \frac{1}{2} & 1\end{Bmatrix}$$
$$\times\int\int dp\,dq\,p^2q^{l-\lambda+3}\psi^*_\alpha(p,q)\int_{-1}^{+1}dz\frac{P_r(z)\psi_\alpha(p,q')}{(q')^l}\ ,$$

where $\Lambda=\{r,\lambda,j'_0,l_1,j_1\}$, and $G^{S,V}(Q^2)\equiv 2G^{S,V}_{\mathrm{mag}}(Q^2)-F^{S,V}_{\mathrm{ch}}(Q^2)$ with $G^S_{\mathrm{mag}}(Q^2)\equiv\frac{1}{2}[\mu_pF^p_{\mathrm{mag}}(Q^2)+\mu_nF^n_{\mathrm{mag}}(Q^2)]$ and $G^V_{\mathrm{mag}}(Q^2)\equiv\frac{1}{2}[\mu_pF^p_{\mathrm{mag}}(Q^2)-\mu_nF^n_{\mathrm{mag}}(Q^2)]$.

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