

## 10 $\hbar\omega$ shell-model description of the $T=0$ spectrum of ${}^4\text{He}$ using modified Sussex matrix elements

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The  $T=0$  states of  ${}^4\text{He}$  are investigated in complete  $N\hbar\omega$  shell-model spaces with  $3 \leq N \leq 10$  using a realistic two-body interaction based on the Sussex matrix elements. It is found that it is possible to describe the ground-state properties and the  $T=0$  excited spectrum of this nucleus in a consistent manner provided the model space is enlarged up to  $10\hbar\omega$  of oscillator excitation. In particular, the lowest  $1^-$  state appears in our  $10\hbar\omega$  spectrum with an excitation energy comparable to the energy of 24.1 MeV favored by recent experimental investigations of the reaction  ${}^2\text{H}(d,p){}^3\text{H}$ . Moreover, the  $D$ -state admixture of the large-basis ground-state wave function amounts to 4.7%, in fair agreement with the value deduced from recent analyzing power measurements of the reaction  ${}^2\text{H}(d,\gamma){}^4\text{He}$ .

### I. INTRODUCTION

Most nuclear structure investigations rely on the assumption that atomic nuclei can be described as systems of nucleons interacting via pairwise forces. In this picture the specification of a two-nucleon potential suitable to the description of a particular nucleus is unavoidably ambiguous and, accordingly, many effective interactions have been utilized. Nuclear structure calculations are nevertheless more credible when the model Hamiltonian is based to a large extent on the two-nucleon scattering and bound state experimental data which, after all, are our most reliable source of information about the nuclear interaction. In this respect, the set of matrix elements proposed by the Sussex group<sup>1</sup> is a well-adapted starting point for calculations carried out in the framework of the harmonic oscillator shell model.

The capabilities of the Sussex matrix elements (SME) have been extensively tested, especially in shell-model studies of very light nuclei.<sup>2-6</sup> It is found that most of the relative energies in the calculated spectra are reasonably well reproduced but that, as for other realistic nucleon-nucleon forces, the theoretical binding energies are much too small. Different modifications of the SME have been proposed to correct this defect.<sup>7-9</sup> In this problem it is very difficult to evaluate the part played by the truncation of the model space. Our aim in the present work is to shed some light on this question by investigating  ${}^4\text{He}$  in shell-model bases extended beyond  $4\hbar\omega$  and  $5\hbar\omega$  of oscillator excitation which are the usual computational limits in this type of calculations for the positive and negative parity states, respectively.

We have first calculated the binding energy of  ${}^4\text{He}$  in a four-nucleon harmonic oscillator basis including all the configurations up to  $10\hbar\omega$  excitation energy using the bare SME. From the results obtained it is clear that the problem of underbinding is inherent in the matrix elements rather than in the truncation of the basis. Therefore we decided to modify the Sussex interaction according to a procedure proposed by Bevelacqua and Philpott.<sup>7</sup>

We have thus continued our calculations using the SME multiplied by a single strength factor which was adjusted in the various model spaces to yield the correct binding energy. These modified SME were then utilized to calculate the excited  $T=0$  spectrum as well as the charge radii and the  $P$ - and  $D$ -state probabilities of both the ground state and the first  $0^+$  excited state.

We wish to emphasize that the present study is relevant to shell-model calculations in general. It is well known that nowadays much work is devoted to the development of shell-model codes designed to handle huge configuration spaces. It is thus useful to show explicitly on an example what might be gained from such an effort. For this purpose,  ${}^4\text{He}$  appears as a good testing ground since it is the lightest nucleus showing an excited spectrum; moreover, its binding energy per nucleon is comparable to that of heavier nuclei. Of course, by basing our work on the SME we make the assumption underlying implicitly most nuclear structure investigations, that a real understanding of the short-range correlations which arise from the presence of a repulsive core in the free nucleon-nucleon potential is not required to describe the nuclear spectra reliably. In fact, the correct description of these correlations remains an open problem since even modern nucleon-nucleon interactions underbind  ${}^4\text{He}$  by several Mev (Ref. 10). The capabilities of these interactions for the description of excited states are even more uncertain. Therefore, we believe that approaches which describe both the ground state and the excited spectrum in a consistent manner in terms of a few adjustable parameters are still useful steps towards the understanding of low-energy nuclear properties. In this context we stress that besides the usual oscillator parameter, the only parameter in our model is a single strength parameter whose value is determined by the observed binding energy, so that the calculation of all the other observables is entirely parameter free.

Our paper is organized as follows. The method of calculation is outlined in Sec. II. In Sec. III the modified SME are used to investigate the properties of the ground

state and the lowest  $0^+$  excited state as functions of the total number of oscillator quanta included in the shell-model basis. Results concerning the  $T=0$  negative parity spectrum are discussed in Sec. IV. Concluding remarks are presented in Sec. V.

## II. METHOD OF CALCULATION

The operator matrices were constructed in four-nucleon shell-model bases including all the configurations up to  $N_0$  harmonic oscillator quanta with  $N_0=10$ . As in our previous works,<sup>11,12</sup> the basis states were taken of the form

$$\mathcal{A}[[q_1 q_2]_{j_1} [q_3 q_4]_{j_2}]_{j_0}$$

where  $q_i$  represents the usual single nucleon harmonic oscillator quantum numbers  $n_i l_i j_i$ ;  $\mathcal{A}$  is the antisymmetrizer and the square brackets are used to denote angular momentum and isospin coupling as well as normalization. It is worth remembering that the basis states of this type with three or four particles in the same orbit must be orthogonalized. Actually, for  $N_0=10$  there are only five pairs of nonorthogonal states. The orbits and the intermediate quantum numbers defining these pairs are given in Table I.

As the model Hamiltonian is translationally invariant, the eigenstates of its matrix representation in complete harmonic oscillator bases are automatically eigenstates of the center-of-mass (c.m.) Hamiltonian with eigenvalues  $E_{c.m.}$  equal to

$$(2n_{c.m.} + l_{c.m.} + \frac{3}{2})\hbar\omega .$$

In addition, the eigenstates of the energy matrices calculated in different  $N_0\hbar\omega$  spaces and describing the same internal structure of  ${}^4\text{He}$  are degenerate. These properties make it possible to identify the eigenstates characterized by  $n_{c.m.}=0$  and, via  $E_{c.m.}$ , to determine the corresponding values of  $l_{c.m.}$ . These quantities are of practical interest since in a basis coupled to total angular momentum equal to zero,  $l_{c.m.}$  is precisely equal to the internal angular momentum quantum number  $J$  of the system under study. To put it in another way, in our calculations

TABLE I. Basis states of the form  $\mathcal{A}[[q_1 q_2]_{j_1} [q_3 q_4]_{j_2}]_{j_0}$  up to  $10\hbar\omega$  excitation energy with three or four particles in the same orbit.

orbits	$j$	$t$
$(0d_{\frac{5}{2}})^4$	0	1
	1	0
$(0d_{\frac{5}{2}})^3(0f_{\frac{5}{2}})$	0	1
	1	0
$(0d_{\frac{5}{2}})^3(0f_{\frac{7}{2}})$	1	0
	3	0
$(0d_{\frac{5}{2}})^3(1d_{\frac{7}{2}})$	0	1
	1	0
$(0d_{\frac{5}{2}})^3(0g_{\frac{7}{2}})$	1	0
	3	0

the eigenstates of the energy matrices characterized by  $n_{c.m.}=0$  and a given value of  $l_{c.m.}$  in the  $N_0\hbar\omega$  space are at the same time good states ( $E_{c.m.}=\frac{3}{2}\hbar\omega$ ) in bases including all the configurations coupled to angular momentum  $J=l_{c.m.}$  and of maximum excitation energy equal to  $N\hbar\omega$  with  $N=N_0-J$ . In this way we were able to extract from a single energy matrix good eigenstates coupled to different values of  $J$ . On the other hand, it is easily seen that these states have internal parities equal to  $(-1)^J$  or  $(-1)^{J+1}$  according as to whether  $N_0$  is even or odd.

The charge radii of the ground state and the lowest  $0^+$  excited state were calculated from the expression

$$R_{ch} = \left[ \frac{1}{4} \langle \phi_{SM} | \sum_{i=1}^4 r_i^2 | \phi_{SM} \rangle + R_N^2 - \frac{3}{8} b^2 \right]^{1/2} .$$

In this formula  $b$  is the usual oscillator length parameter  $(\hbar/m\omega)^{1/2}$  and  $\phi_{SM}$  denotes good shell-model states. In our calculations  $\hbar^2/m$  was taken equal to 41.4723 MeV fm<sup>2</sup>. The nucleon radius  $R_N=0.772$  fm was extracted from the nucleon form factor given by Janssens *et al.*<sup>13</sup>

To calculate the  $P$ - and  $D$ -state probabilities of these levels, we have taken advantage of the fact that for  $J=0$  the internal orbital quantum number  $L$  is equal to the total spin quantum number  $S$  ( $L=S=0,1,2$ ). Consequently, the probabilities  $P_L$  of the states characterized by definite values of  $L$  can be obtained by solving the following set of equations:

$$\langle \phi_{SM} | S^2 | \phi_{SM} \rangle = \sum_L L(L+1)P_L ,$$

$$\langle \phi_{SM} | (S^2)^2 | \phi_{SM} \rangle = \sum_L L^2(L+1)^2P_L ,$$

$$\sum_L P_L = 1 .$$

The construction of the matrix representation of the spin operator  $S^2$  is straightforward. In fact, one has only to replace the relative two-body matrix elements in the four-nucleon Hamiltonian matrix by  $(-1)^{s+l} \delta_{nn'} \delta_{ll'} \delta_{ss'}$ , where  $s, l$ , and  $n$  are, respectively, the spin, the orbital angular momentum, and the radial quantum numbers characterizing the relative wave function of a pair of nucleons.

The Coulomb interaction is not included in the model Hamiltonian. We compare our theoretical results with a Coulomb-corrected experimental spectrum obtained by assuming that the Coulomb repulsion gives a contribution of 0.7 MeV to the ground state energy and of 0.5 MeV to the energies of the excited states. Variational calculations of resonant states in  ${}^4\text{He}$  by Carlson *et al.*<sup>14</sup> indicate that this procedure is accurate within 0.1 MeV.

## III. MODIFIED SUSSEX MATRIX ELEMENTS

As was shown by Elliott *et al.*,<sup>1</sup> the matrix elements of the internucleon potential deduced from the nucleon-nucleon scattering data are subject to the constraint,

$$(4n + 2l + 3)\hbar\omega < 330 \text{ MeV} .$$

TABLE II. Extrapolated values (in MeV) of  ${}^3D_1$  matrix elements involved in  $8\hbar\omega$  and  $10\hbar\omega$  calculations.

$b = 1.6$ fm		$b = 1.8$ fm	
$\langle 2^3D_1   V   3^3D_1 \rangle$	$\langle 3^3D_1   V   3^3D_1 \rangle$	$\langle 3^3D_1   V   4^3D_1 \rangle$	$\langle 4^3D_1   V   4^3D_1 \rangle$
4.28	4.40	3.50	3.64

Therefore, the investigation of nuclear properties in a complete  $N\hbar\omega$  model space relying solely on these data cannot be achieved unless the oscillator length parameter is larger than some value which for  $N=6, 8,$  and  $10$  is equal to about 1.4, 1.6, and 1.8 fm, respectively. In fact, for  $b$  equal to 1.6 and 1.8 fm a few  ${}^3D_1$  matrix elements needed in our  $8\hbar\omega$  and  $10\hbar\omega$  calculations are not tabulated in Ref. 1. They were estimated by extrapolating the known SME to lower  $b$ . The values thus obtained are given in Table II. We have checked that these additional matrix elements have little effect on the calculated spectra. For instance, in a  $8\hbar\omega$  calculation with  $b = 1.6$  fm, the variation of the binding energy of  ${}^4\text{He}$  is equal to about 0.2 MeV when the matrix elements  $\langle 2^3D_1 | V | 3^3D_1 \rangle$  and  $\langle 3^3D_1 | V | 3^3D_1 \rangle$  are both varied from zero to 10 MeV.

The theoretical energies  $E_N(\text{g.s.})$  and  $E_N(0^+)$  of the ground state and the lowest  $0^+$  excited state of  ${}^4\text{He}$  calculated in the various  $N\hbar\omega$  spaces using the bare SME are displayed in Fig. 1 as functions of the oscillator length parameter. It is apparent that the decrease of  $E_N(\text{g.s.})$

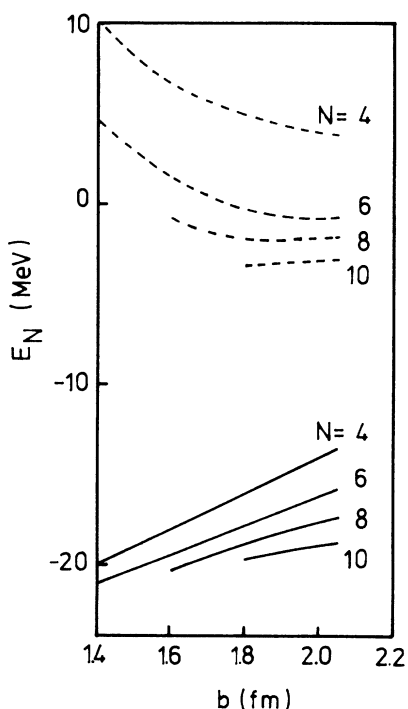


FIG. 1. Theoretical energies of the ground state (solid curves) and the lowest  $0^+$  excited state (dashed curves) obtained in the various  $N\hbar\omega$  spaces as functions of the oscillator length parameter using the bare SME.

when the model space is enlarged up to  $10\hbar\omega$ , though important, is not sufficient to bring this energy in agreement with the Coulomb-corrected experimental value of  $-29$  MeV. Likewise,  $E_N(0^+)$  remains much greater than the Coulomb-corrected experimental value of  $-8.7$  MeV. Actually, the preceding inequality defining the accessible orbits makes the agreement of  $E_N(\text{g.s.})$  and  $E_N(0^+)$  with experiment impossible, even in arbitrarily large model spaces. For instance, the lowest possible value of  $E_N(\text{g.s.})$  in the  $10\hbar\omega$  space ( $-19.66$  MeV for  $b = 1.8$  fm) is greater than its counterpart in the  $8\hbar\omega$  space ( $-20.31$  MeV for  $b = 1.6$  fm). Note that in the  $6\hbar\omega$  and  $8\hbar\omega$  model spaces the excitation energy

$$E_{x,N}(0^+) = E_N(0^+) - E_N(\text{g.s.})$$

of the lowest  $0^+$  excited state coincides with the Coulomb-corrected experimental value of 20.3 MeV for  $b$  equal to about 1.6 fm, a typical value for the oscillator length parameter in shell-model investigations of  ${}^4\text{He}$ ; however, when the model space is enlarged further, it is no longer possible to bring  $E_{x,N}(0^+)$  in agreement with experiment as for  $N=10$  the highest value of this energy drops to less than 17 MeV.

It turns out that a correct shell-model description of the binding energy of  ${}^4\text{He}$  can be achieved using sets of two-body matrix elements obtained by changing the SME within limits compatible with the expected uncertainties in these quantities. Bevelacqua and Philpott<sup>7</sup> have proposed a very simple procedure to generate such sets. It merely consists in defining an effective interaction of the form  $\lambda V^{\text{Sussex}}$  where  $\lambda$  is an adjustable parameter. These authors obtained a good description of the ground state properties of the  $A=3$  and  $A=4$  systems in  $4\hbar\omega$  model spaces by taking  $\lambda=1.168$  and  $b=1.6$  fm. The lowest  $0^+$  excited state of  ${}^4\text{He}$ , on the contrary, was still found much too high in energy.

We have employed the same method in our large-basis calculations. The values of  $\lambda$  defined in the various  $N\hbar\omega$  spaces by the condition  $E_N(\text{g.s.}) = -29$  MeV are given in Table III for  $b = 1.6$  and 1.8 fm together with the corresponding excitation energies  $E_{x,N}(0^+)$ . It is clear that the extension of the model space from  $N=4$  to  $N=10$  improves, considerably, the position of the lowest  $0^+$  excited state. In a  $10\hbar\omega$  calculation, the discrepancy between the theoretical energy and the Coulomb-corrected experimental value is reduced to practically 1 MeV for  $b = 1.8$  fm. We wish also to emphasize that for a given value of the oscillator length parameter, the strength factor is noticeably shifted towards unity with increasing  $N$ . In fact,  $E_N(\text{g.s.})$  and  $E_{x,N}(0^+)$  are sensitive functions of  $\lambda$ : For all the values of  $N$  and  $b$  considered, we found that when  $\lambda$  is increased by 1%,  $E_N(\text{g.s.})$  and  $E_{x,N}(0^+)$  de-

TABLE III. Strength factor  $\lambda$  determined in each  $N\hbar\omega$  space by the constraint  $E_N(\text{g.s.}) = -29$  MeV for  $b = 1.6$  and  $1.8$  fm, and excitation energies of the lowest  $0^+$  excited state calculated using the interaction  $\lambda V^{\text{Sussex}}$ ; corresponding values of the charge radii and the  $D$ -state probabilities of both the ground state and the lowest  $0^+$  excited state.

$b$	$N$	$\lambda$	$E_{x,N}(0^+)$ (MeV)	$R_{\text{ch}}(\text{g.s.})$ (fm)	$R_{\text{ch}}(0^+)$ (fm)	$P_D(\text{g.s.})$ (%)	$P_D(0^+)$ (%)
1.6	4	1.166	29.43	1.70	2.37	3.84	3.80
	6	1.142	25.34	1.68	2.58	4.43	3.66
	8	1.124	24.03	1.68	2.72	4.71	4.34
1.8	4	1.225	26.78	1.78	2.60	3.12	2.72
	6	1.182	23.26	1.73	2.82	3.90	2.59
	8	1.154	22.40	1.72	2.91	4.35	3.37
	10	1.137	21.37	1.71	3.05	4.70	3.31

crease roughly by 0.7 and 0.3 MeV, respectively. Let us note, by the way, that these energy shifts are of the same order of magnitude as the contributions of the Coulomb repulsion.

In Table III we have also displayed the charge radii and the  $D$ -state probabilities of the ground state and the lowest  $0^+$  excited state calculated using the modified SME; the  $P$ -state probabilities were found negligibly small and, accordingly, they are not presented. It is seen that in all the model spaces considered, the charge radius of the lowest  $0^+$  excited state is significantly greater than the charge radius of the ground state, and that both these states have comparable  $D$ -state probabilities. Our results thus support the usual breathing-mode interpretation of the lowest  $0^+$  excitation of  ${}^4\text{He}$ . It is also apparent that the ground state charge radius, whose experimental value is equal to 1.68 fm (Ref. 15), is best described by the  $b = 1.6$  fm calculations. Moreover, for this oscillator length parameter the  $D$ -state admixture of the  $8\hbar\omega$  ground state wave function is in fair agreement with the value of 4.8% deduced from recent tensor analyzing power measurements of the reaction  ${}^2\text{H}(d,\gamma){}^4\text{He}$  by Weller *et al.*<sup>16</sup> However, as noted above, these calculations are restricted to  $N \leq 8$  model spaces, which prevents a correct description of the lowest  $0^+$  excited state. Actually, the transition to the  $10\hbar\omega$  space is rewarding as, for  $b = 1.8$  fm, the resulting reduction of  $E_{x,N}(0^+)$  relative to the lowest value of this energy in the  $8\hbar\omega$  space amounts to 2.66 MeV, whereas the charge radius and the  $D$ -state admixture are only slightly changed.

#### IV. THE $T=0$ NEGATIVE PARITY SPECTRUM

According to the survey done by Fiarman and Meyerhof<sup>17</sup> in 1973 the only  $T=0$  excited levels whose characteristics are firmly established are, besides the breathing-mode excitation, the  $0^-$  and  $2^-$  states with excitation energies equal to 21.1 and 22.1 MeV, respectively. The  $1^-$  isoscalar state, which belongs to the same  $SU(4)$  supermultiplet, is mentioned as uncertain and is located 8.9 MeV above the  $2^-$  state. On the other hand, more recently Gruebler *et al.*<sup>18</sup> have reported strong evidence for a  $1^-$   $T=0$  state at 24.1 MeV. The existence of such a level at a comparable energy (24.4 MeV) results also from a  $R$ -matrix analysis of four-body reactions by

Hale and Dodder.<sup>19</sup> It is thus of great interest to see how the  $T=0$  negative parity spectrum comes out in large shell-model spaces. It must be realized, however, that the specification of the model Hamiltonian relevant to the negative parity spectrum is somewhat ambiguous when binding energy restrictions are imposed, as shell-model states belong to  $N\hbar\omega$  spaces with  $N$  even or odd according as to whether the parity is positive or negative. Consequently, to calculate the negative parity spectrum in the  $N\hbar\omega$  space, there is no obvious choice between the interactions determined either in the  $(N-1)\hbar\omega$  or in the  $(N+1)\hbar\omega$  space. Fortunately, this ambiguity about the interaction causes uncertainties in the calculated energies of the negative parity levels which, for  $N > 5$ , become less than 1 MeV. The spectra presented in this section were calculated by using in the  $N\hbar\omega$  space the strength parameter determined in the  $(N+1)\hbar\omega$  space.

As the breathing-mode excitation is best represented in the  $10\hbar\omega$  space by  $b = 1.8$  fm calculations, we have used the same harmonic oscillator parameter to investigate the  $T=0$  negative parity spectrum. The theoretical excitation energies obtained in the various  $N\hbar\omega$  spaces are compared in Fig. 2 with the Coulomb-corrected experimental energies. As was outlined in Sec. II, our computing procedure makes it possible to extract from the energy matrices, besides the  $0^+$  spectrum, the  $0^-$  and  $1^-$  eigenvalues up to  $N=9$  but the  $2^-$  eigenvalues only up to  $N=7$ . However, given the regular dependence of these energies upon  $N$ , it is obvious that a reliable estimate of the  $2^-$  eigenvalue in the  $9\hbar\omega$  space can be obtained by extrapolating the eigenvalues calculated for  $N \leq 7$ . The eigenvalue obtained in this way is indicated in Fig. 2 between brackets.

It is seen from Fig. 2 that the theoretical energies of the negative parity states are strongly reduced with increasing  $N$ , and that in the  $9\hbar\omega$  space the  $0^-$  and  $2^-$  states are predicted close to their respective experimental counterparts. Concerning the  $1^-$  state, our large-basis calculations yield results which are compatible with the conclusions of the experimental investigations of the  ${}^2\text{H}(d,p){}^3\text{H}$  reaction by Gruebler *et al.*<sup>18</sup> and the  $R$ -matrix analysis by Hale and Dodder.<sup>19</sup> In this context, the fact that the theoretical energy of this level in our  $3\hbar\omega$  calculation is close to the value mentioned as uncer-

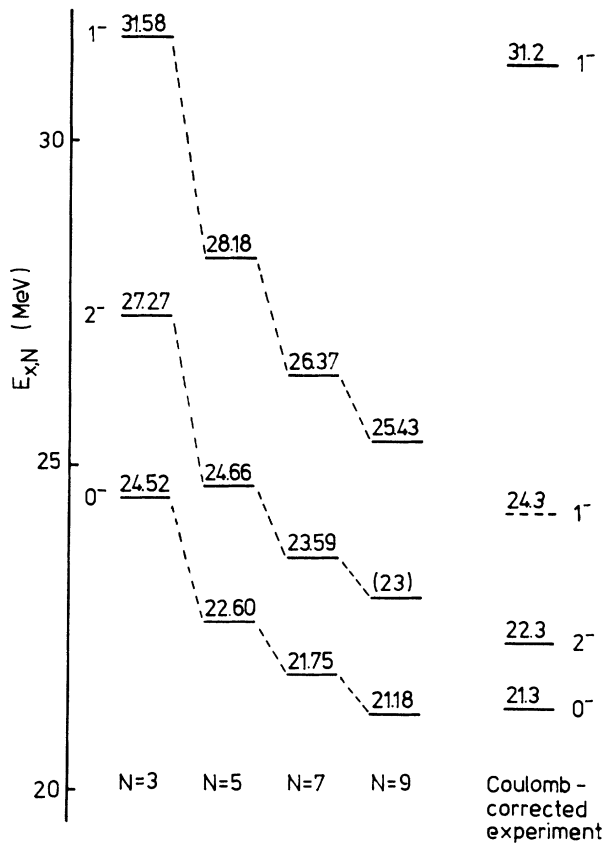


FIG. 2. Theoretical  $T=0$  negative parity spectra obtained using modified SME determined in each  $N\hbar\omega$  space by the constraint  $E_{N+1}(\text{g.s.}) = -29$  MeV for  $b = 1.8$  fm. The  $9\hbar\omega$  excitation energy of the  $2^-$  state mentioned within brackets was obtained by extrapolating to  $N=9$  the energies calculated for  $N \leq 7$ . The Coulomb-corrected experimental excitation energies (see the text) were deduced from the experimental data presented in Ref. 17 (solid lines) and Ref. 18 (dashed line).

tain in the survey by Fiarman and Meyerhof<sup>17</sup> is obviously a mere coincidence and is by no means to be considered as an argument supporting the existence of a  $1^-$   $T=0$  state at high excitation energy.

Finally, attention must be drawn to the fact that the excitation energy of the breathing mode calculated in the  $4\hbar\omega$  space (26.78 MeV) is located between the energy of the  $2^-$  state calculated in the  $3\hbar\omega$  (27.27 MeV) and its counterpart in the  $5\hbar\omega$  space (24.66 MeV). Thus, in too restricted shell-model bases the order of these states cannot be defined properly. Actually, the ambiguity inherent in comparing positive with negative parity shell-model spectra blurs most shell-model studies, irrespective of the

previously mentioned uncertainties connected with the specification of the modified SME for odd values of  $N$ . Clearly, only an enlargement of the configuration space beyond the usual computational limits is capable of reducing significantly this deficiency.

## V. CONCLUDING REMARKS

We have shown that it is possible to achieve an acceptable shell-model description of the  $T=0$  spectrum of  $^4\text{He}$ , including binding energy restrictions, provided the configuration space is extended up to  $10\hbar\omega$  of oscillator excitation. The model Hamiltonian used in our calculations is based on internucleon matrix elements obtained by multiplying the SME by a single scale factor of order unity. It is likely that our results could be improved by varying individual SME. However, such a procedure would necessarily introduce a good deal of arbitrariness in the calculations, especially in extended model spaces as in this case the bulk of numerous configurations, whose weights taken separately are extremely small, contributes significantly to the lowering of the calculated energies.

Actually, our results seem to indicate that in large-basis shell-model calculations, complicated multinucleon effects can to a large extent be simulated by increasing the strength of the free nucleon-nucleon potential by less than 15%. It would be interesting to test this idea starting from the Reid softcore, Paris, or other sophisticated potentials. In fact, we chose the popular SME among other "realistic" interactions merely to have the possibility to compare our  $4\hbar\omega$  results with those reported by other authors. Of course, as was illustrated in our previous works,<sup>11,12</sup> any two-body interaction having finite matrix elements in a harmonic oscillator basis can be used as input of our computer program.

It would also be worth extending our study to heavier systems. In this respect, we mention that our calculations, which required the handling of a shell-model basis comprising 2765 four-body antisymmetrized states constructed so as to have good total angular momentum and isospin, were made feasible with reasonable economy on a modest computer (an IBM 4341) thanks to an elaborate procedure based on the storage of numerous geometrical quantities. Given the rapid development of computer technology, the extension of this procedure to the study of the lightest  $p$ -shell nuclei is thus not unthinkable.

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