Isospin-forbidden positron decay of ^{48}V and time-reversal invariance

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A large number of β-γ circular polarization experiments have been performed to obtain the asymmetry coefficient $\tilde{A}$ for the $^{48}V \rightarrow ^{48}Ti$ decay. The Fermi to Gamow-Teller mixing ratio y can be deduced from such measurements, and is important since the time-reversal violating amplitude is proportional to $y/(1+y^2)$. Unfortunately, owing to the scatter in the experimentally deduced value of y, nothing can be said about the time-reversal property of this decay. Our theoretical calculation using the Nilsson model yields values of y consistent with four of the more recent experiments and also consistent with time-reversal invariance.

INTRODUCTION

Isospin-forbidden beta decays ($J \neq 0$, $\Delta J = 0$, $\Delta T = \pm 1$, and no parity change) have been extensively studied.[1] They are of interest in the study of both isospin impurity and time-reversal invariance. For such studies, the experimental measurement is the asymmetry parameter $\tilde{A}$ from either polarized nuclei or β-γ circular polarization correlations in unpolarized nuclei. These experiments are difficult and the numerical results from different workers tend to differ tremendously.

However, two important physical quantities could be deduced from such measurements.[1] First, we can obtain the Fermi to Gamow-Teller (GT) mixing ratio

$$y = \frac{C_v M_F}{C_A M_{GT}} , \tag{1}$$

where C_v and C_A are the vector and axial-vector coupling constants. This quantity is important because the size of the time-reversal violating amplitude is proportional[2] to $y/(1+y^2)$. Secondly, we can deduce the value of the Fermi nuclear matrix element M_F which is a measure of the isospin impurities due to charge-dependent forces since contributions from virtual pion states should be zero in the conserved vector current (CVC) theory.

The positron decay from the 4^+ ground state of ^{48}V to the 2.2956 MeV 4^+ state of ^{48}Ti has been well studied.[3-10] Figure 1 gives eight independent measurements of y as a function of time. These experimentally deduced values of y are rather scattered with three values consistent with time-reversal invariance (the latest two measurements and that of Mann *et al.* when the value of y could be zero). The aim of this paper is to obtain a theoretical value of y and discuss the value so obtained in relation to time-reversal invariance.

CALCULATION AND RESULTS

A previous shell-model calculation[11] of the positron decay of ^{48}V using an effective nucleon interaction yielded a value for the Coulomb matrix element of 14 keV. If we take[1] ΔE as 5316 keV and $\log ft$ as 6.18, we obtain

$$y = -0.117 \tag{2}$$

which is consistent with two (Daniel *et al.* and Nooijen *et al.*) of the eight independent measurements and indicates T violation. However, in their calculations, as a check on the reliability of the wave functions which they used, they also calculated the Gamow-Teller $\log ft$ values. They found that although the experimental transition rates were slower than calculated ones by about 2–3 for the Sc and Mn transitions, the ^{48}V transition was off by a factor of 30. Therefore, the above theoretical value of y is unreliable. Moreover, from Fig. 1, we feel that the earlier experimental values of y of Refs. 3, 4, and 8 are most probably wrong and possibly also that of Ref. 6.

Recently,[12-14] we have used the Nilsson model[15] with a one-body spheroidal Coulomb potential to obtain the M_F of a number of transitions. As the results show reasonably good agreement between theory and experiment, we shall use the same approach.

We assume that the deformed nucleus ^{48}V has the rotational band $K = 4$ and that the deformed ^{48}Ti has $K = 0$ as shown in Fig. 2, where $|G\rangle$, $|P\rangle$, $|A\rangle$, and $|T_<\rangle$

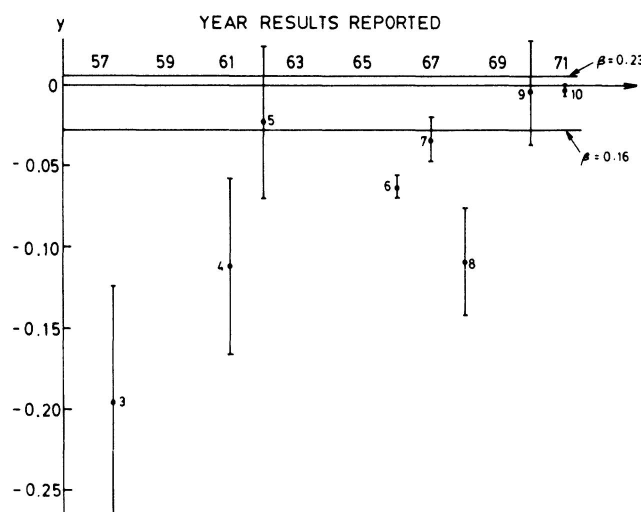

FIG. 1. Plot of all experimental values of the Fermi to Gamow-Teller mixing ratio y that have been reported. The numbers that label the data points refer to references. The two horizontal lines are theoretical values of y for $\beta = 0.16$ and $\beta = 0.23$.

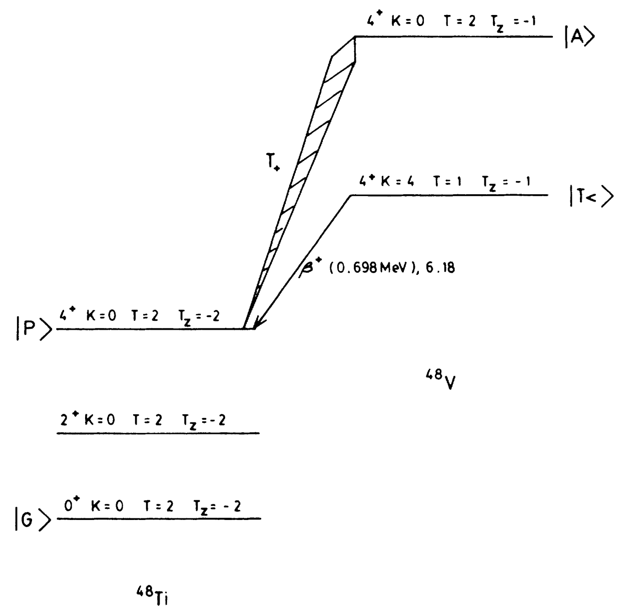

FIG. 2. Partial decay scheme of ^{48}V.

are the ground state, the parent state, the analog state, and the antianalog state, respectively. By the K-selection rule for beta decay of $\Delta K \leq 1$, the beta matrix elements with $K=4$ vanish and thus the experimentally observed decay is due to the admixture of other K bands to the $K=4$ ground state of ^{48}V and to the $K=0$ excited state of ^{48}Ti. Assuming axially symmetric prolate deformation, the initial state is

$$| i \rangle = | J=4, M, K=4, T=1, T_z=-1 \rangle + \bar{\alpha}_1 | J=4, M, K=1, T=1, T_z=-1 \rangle + \bar{\alpha}_4 | J=4, M, K=4, T=2, T_z=-1 \rangle + \cdots \quad (3)$$

and the final state is

$$f \rangle = | J=4, M, K=0, T=2, T_z=-2 \rangle + a_3 | J=4, M, K=3, T=2, T_z=-2 \rangle + a_4 | J=4, M, K=4, T=2, T_z=-2 \rangle + \cdots , \quad (4)$$

where $\bar{\alpha}_1$ is the admixture amplitude of $K=1$ in the initial state, a_3 and a_4 are those of $K=3$ and $K=4$ in the final state, respectively, and $\bar{\alpha}_4$ is the isospin impurity amplitude given by

$$\bar{\alpha}_4 = -\frac{\langle K=4, T=1, T_z=-1 | V_c | K=4, T=2, T_z=-1 \rangle}{\Delta E} , \quad (5)$$

where ΔE is the separation energy and V_c the Coulomb potential. The Fermi matrix element is

$$M_f = \langle f | T_- | i \rangle = 2\bar{\alpha}_4 a_4 \quad (6)$$

and the Gamow-Teller matrix element is calculated from the relation

$$M_{\rm GT}^2 = \frac{1}{2J+1} \sum_{\mu, M_i, M_f} | \langle J, M_f, K_f, T_f, T_{zf} | D_{\rm GT}(\mu) | J, M_i, K_i, T_i, T_{zi} \rangle |^2 . \quad (7)$$

When the operator $D_{\rm GT}(\mu)$ is transformed into the body-fixed coordinate system, we obtain

$$M_{\rm GT}^2 = \left| \frac{\bar{a}_1}{\sqrt{2}} \langle \chi_0 \chi_{T_z=-1}^{T=2} | D'_{\rm GT}(-1) | \chi_1 \chi_{T_z=-1}^{T=1} \rangle + \frac{a_3}{\sqrt{5}} \langle \chi_3 \chi_{T_z=-2}^{T=2} | D'_{\rm GT}(-1) | \chi_4 \chi_{T=-1}^{T=1} \rangle + \frac{2a_4}{\sqrt{5}} \langle \chi_4 \chi_{T_z=-2}^{T=2} | D'_{\rm GT}(0) | \chi_4 \chi_{T_z=-1}^{T=1} \right|^2 , \quad (8)$$

where $| \chi_{K_i} \chi_{T_{zi}}^{T_i} \rangle$ and $| \chi_{K_f} \chi_{zf}^{T_f} \rangle$ are the intrinsic states, which depend on the deformation parameter β. However, it was found that for all values of β, the value of the third term of Eq. (8) is much larger than the other two, so that by neglecting the first two terms we have a relation between $M_{\rm GT}$ and a_4, from which the value of a_4 could be calculated if the value of $M_{\rm GT}$ is known.

The value of $M_{\rm GT}$ can be obtained from the following relation[1]

$$| M_{\rm GT} | = \frac{C_v}{C_A} \left[\frac{2 \text{ ft (superallowed)}}{\text{ft (decay under study)}} \right]^{1/2} \times \frac{1}{(1+y^2)^{1/2}} . \quad (9)$$

Owing to the smallness of the experimental value of y, we shall obtain essentially the same value of $M_{\rm GT}$ irrespective of whichever experimental value[3–10] of y we use.

In order to obtain values for M_F of Eq. (6), we need to

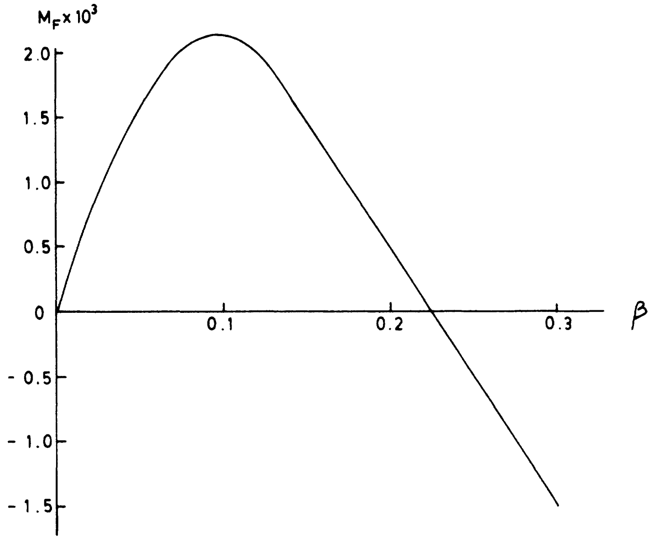

FIG. 3. Variation of the Fermi matrix element M_F as a function of the deformation parameter β.

calculate the isospin impurity amplitude $\bar{\alpha}_4$. For the calculation of the isospin impurity as given by Eq. (5) we take V_c to be the one-body spheroidal Coulomb potential given by[16]

$$V_c = \frac{(Z-1)e^2}{R}\left[\tfrac{3}{2}-\tfrac{1}{2}(r/R)^2\right]+a(r/R)^2 Y_{20}, \quad \text{for } r<R ,$$

$$= \frac{(Z-1)e^2}{r}+a(R/r)^3 Y_{20} , \quad \text{for } r>R , \tag{10}$$

where R is the nuclear radius and a is related to the Bohr deformation parameter β by

$$a = \tfrac{3}{5}\beta(Z-1)e^2/R . \tag{11}$$

The calculations were carried out for various values of β and the final results are presented in Fig. 3. Although the experimental value of β is not available, a recent theoretical calculation[17] gives $\beta \approx 0.23$. With this value of β, we obtain $M_F = -0.25\times 10^{-3}$ from Fig. 3, from which we deduce $y = 5.5\times 10^{-3}$. However, an earlier calculation[18] yields $\beta \approx 0.16$ giving $y = -0.028$. In Fig. 1, we have drawn the lines corresponding to y for $\beta = 0.16$ and $\beta = 0.23$, and for this range our calculation is in agreement with the experimental results of Refs. 5, 7, 9, and 10. It is also consistent with time-reversal invariance if β is around 0.22.

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