Influence of the delta-nucleon interaction on the pion-deuteron breakup process

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In order to evaluate the influence on the pion-deuteron breakup process due to short-range ΔN interaction which was previously determined through its effects on the elastic scattering observables, we calculate the contributions to the helicity amplitudes for the process $\pi d \rightarrow \Delta N$ due to the ΔN short-range interaction in the final state.

INTRODUCTION

In a series of papers[1-5] we have treated the influence on πd elastic scattering of the short-range ΔN interaction which is not accounted for in the existing Faddeev calculations.[6,7] We have shown that, taking as a basis the amplitudes obtained by Garcilazo,[7] the theoretical results can be improved by the inclusion of this interaction to such a point that all essential discrepancies between theory and experiment disappear.[4,5] In that way we have arrived at a determination of ΔN scattering parameters which was remarkably stable and showed a smooth energy dependence. It turned out that only the ${}^5S_2$ and ${}^5P_3$ ΔN states show strong effects and that, except for the ΔN threshold region in the ${}^5S_2$ state, the required inelasticity is small.

The discrepancies between the theory and experiment[8] in the breakup reaction $\pi d \rightarrow \pi NN$ are less dramatic than in πd elastic scattering, but nevertheless there are statistically significant deviations. We, therefore, found it important to investigate the influence of the short-range ΔN interaction also on the breakup reaction. The most important contribution to this process in the energy region considered ($T_\pi = 140-350$ MeV) is the formation of a Δ resonance, according to the graph of Fig. 1.

The Δ particle may interact with the other nucleon in the final state, and hence we have a contribution whose skeleton diagram is shown in Fig. 2.

This contribution, whose short-range part is not accounted for in the existing Faddeev calculations, can be evaluated using the same techniques as we have used in elastic πd scattering.[1-5] In the final state the Δ decays into $N\pi$ and the resulting amplitude may be added to the Faddeev amplitudes. The justification for the simple addition of the amplitudes is the same as given in the πd elastic scattering case (see Ref. 3, Appendix).

In order to evaluate the contribution of the diagram in Fig. 2 we use the same techniques and make the same approximations as explained in Refs. 2 and 3. Thus we only take into account the S-wave part of the deuteron wave function and neglect the influence of the Fermi motion inside the deuteron on the Δ-formation amplitude. It has been shown in Ref. 1 that these approximations are indeed well justified. (We recall that the expected contribution of diagram 2 is a rather small correction to the main contributions given by diagram 1.)

ΔN INTERACTION IN DEUTERON BREAKUP

We consider the formation of a state with given isospin I, spin S, orbital angular momentum L, and total angular momentum J, corresponding to the skeleton diagram in Fig. 2. If the direction of the incident deuteron momentum is taken as the projection axis for the spin and the orbital angular momentum, and s_d and s_J are the third components of the spins of the deuteron and of the ΔN system, respectively, the formation amplitude is

$$M^{d\pi;SLJ}_{s_d,s_J}(s) = 2\sqrt{2} g_{\pi N\Delta} \mid \mathbf{q}_d \mid F_L(s) \langle 1,1;s_d,0 \mid S,s_J \rangle \times \langle S,L;s_J,0 \mid J,s_J \rangle \frac{2}{\sqrt{3}} K^{(S)} \quad (1)$$

[as in Eq. (9) of Ref. 2, but with superfluous $\sqrt{\frac{4}{3}}$ eliminated]. Here $g_{\pi N\Delta}$ is the $\pi^+ p \Delta^{++}$ coupling constant ($g^2_{\pi N\Delta}/4\pi = 20.4$ GeV^{-2}), $\mathbf{q}_d$ is the deuteron momentum in the πd c.m. frame and

$$K^{(S)} = \left[\frac{2S-1}{3} \right]^{1/2} . \quad (2)$$

The function $F_L(s)$ (s is the square of the total c.m. energy) is related to the absorptive and dispersive parts of the triangular diagram. Taking the parametrization of the deuteron wave function of McGee,[9] we obtain

$$\mathrm{Abs}F_L(s) = P \sum_{i=0}^{5} c_i \int_{(m_N - m_\pi)^2}^{(\sqrt{s} - m_N)^2} d\mu^2 \frac{(m_\Delta \Gamma_\Delta/\pi) \cdot \sqrt{2L+1}}{(m_\Delta^2 - \mu^2)^2 + m_\Delta^2 \Gamma_\Delta^2} \times Q_L \left[-\frac{A_i^S(s,\mu^2)}{B} \right] \quad (3)$$

and

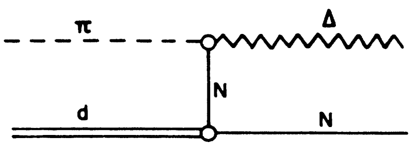

FIG. 1. Diagram for the calculation of the undistorted part of the transition amplitude for $\pi d \to N\Delta$.

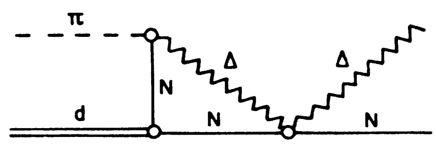

FIG. 2. Diagram for the calculation of the contribution due to the $N\Delta \to N\Delta$ interaction in the final state of the transition $\pi d \to N\Delta$.

$$\mathrm{Disp}F_L(s)=\frac{1}{\pi}⨍\frac{\mathrm{Abs}F_L(s')}{s'-s}ds' , \tag{4a}$$

and then

$$F_L(s)=\mathrm{Disp}F_L(s)+i\ \mathrm{Abs}F_L(s) . \tag{4b}$$

We define

$$P=N_d m_\Delta m_N^2/(2\pi\sqrt{s}\ |\mathbf{q}_d|) , \tag{5a}$$

$$E_1=(s+m_N^2-\mu^2)/(2\sqrt{s}),\quad |\mathbf{q}_1|=(E_1^2-m_N^2)^{1/2} , \tag{5b}$$

$$E_d=(s+m_d^2-m_\pi^2)/(2\sqrt{s}),\quad |\mathbf{q}_d|=(E_d^2-m_d^2)^{1/2} , \tag{5c}$$

and then the quantities B and A_i^S (with $i=0,\ldots,5$) are given by

$$B(s)=2|\mathbf{q}_1|\,|\mathbf{q}_d| \tag{6a}$$

and

$$A_i^S(s,\mu^2)=m_d^2+m_N^2-2E_dE_1-m_i^2 , \tag{6b}$$

where

$$m_i^2=m_N^2+2(\beta_i^2-\beta_0^2) . \tag{7}$$

The quantities Q_L represent the Legendre functions of the second kind, while the constants N_d, c_i, and β_i $(i=0,\ldots,5)$ are parameters of the McGee wave function.[9]

Since s_d coincides with the helicity of the incoming deuteron, we have $s_d\equiv\lambda_d$. We now rotate the final-state angular momentum projection axis into the direction of the scattered (i.e., final) nucleon as seen in the ΔN (and πd) center of mass frame. Calling λ_J the projection of the spin of the ΔN system over this direction, we have

$$M^{d\pi;SLJ}_{\lambda_d,\lambda_J}(s,\theta)=\sum_{s_J} d^J_{s_J,\lambda_J}(\theta)M^{d\pi;SLJ}_{\lambda_d,s_J}(s) . \tag{8}$$

Including in the final state the ΔN interaction given by the ΔN amplitude leading to a final state of total angular momentum J and helicities λ_N,λ_Δ, we obtain the expression

$$M^{d\pi;N\Delta;J}_{\lambda_d;\lambda_N,\lambda_\Delta}(s,\theta)=\sum_{S,S'}\sum_{LL'}\sum_{\lambda_J}M^{d\pi;SLJ}_{\lambda_d,\lambda_J}(s,\theta)(2L'+1)^{1/2}2$$
$$\times\langle\tfrac{1}{2},\tfrac{3}{2};\lambda_N,-\lambda_\Delta|S',\lambda_J\rangle\cdot\langle S'L';\lambda_J0|J,\lambda_J\rangle i\mathcal{M}^{SL;S'L';J}_{\Delta N\to\Delta N}(s)\langle\tfrac{1}{2}\tfrac{3}{2};T_N,T_\Delta|1,T_\pi\rangle . \tag{9}$$

Here T_NT_Δ and T_π are the isospin components of the N, Δ, and π particles, respectively, and $\mathcal{M}^{SL;S'L';J}_{\Delta N\to\Delta N}$ is the scattering amplitude for a ΔN, $I=1$, state with initial and final angular momenta L and L', spins S and S', and total angular momentum J. In the zero width limit of the Δ, we write

$$\mathcal{M}^{SL;S'L';J}_{\Delta N\to\Delta N}(s)=\frac{\pi}{m_Nm_\Delta}\frac{\sqrt{s}}{2\,\mathrm{Re}(q_\Delta)}T^J_{LL';SS'}(s) . \tag{10}$$

From our earlier work we have concluded that only the diagonal terms with $L=L'$ and $S=S'$ are important, and thus we use the parametrization

$$T^J_{LL;SS}(s)=\frac{1}{2i}(\eta^J_{L,S}e^{2i\delta^J_{LS}}-1) . \tag{11}$$

Here q_Δ is the (complex) angular momentum of the Δ in the c.m. frame

$$q_\Delta=\frac{1}{2\sqrt{s}}\{[s+(m_\Delta-i\Gamma/2)^2-m_N^2]^2$$
$$-4s(m_\Delta-i\Gamma/2)^2\}^{1/2} , \tag{12}$$

where $m_\Delta-i\Gamma/2$ is the complex Δ mass, with $m_\Delta=1.211$ GeV and $\Gamma=0.1$ GeV. For energies well above the ΔN threshold we can use the zero width approximation $\mathrm{Re}(q_\Delta)=q_\Delta$ $(\Gamma=0)$.

The partial wave amplitude $M^{d\pi;N\Delta,J}_{\lambda_d;\lambda_N-\lambda_\Delta}(s)$ can be extracted from Eq. (9) as the factor multiplying $d^J_{\lambda_d,\lambda_N-\lambda_\Delta}(\theta)$:

$$M^{d\pi;N\Delta;J}_{\lambda_d;\lambda_N,\lambda_\Delta}(s,\theta)=d^J_{\lambda_d,\lambda_N-\lambda_\Delta}(\theta)M^{d\pi;N\Delta;J}_{\lambda_d,\lambda_N-\lambda_\Delta}(s) . \tag{13}$$

By inserting Eq. (8) into (9) and using Eq. (1), with $s_d\equiv\lambda_d$, and Eq. (10), we obtain finally for the partial wave amplitude

$$M^{d\pi;N\Delta;J}_{\lambda_d;\lambda_N-\lambda_\Delta}(s)=8(\tfrac{2}{3})^{1/2}\langle \tfrac{1}{2},\tfrac{3}{2};T_N,T_\Delta \mid 1;T_\pi\rangle g_{\pi N\Delta}\mid \mathbf{q}_d\mid$$
$$\times\sum_{S,S'=1}^{2}\sum_{L=J-S}^{J+S}\sum_{L'=J-S'}^{J+S'}F_L(s)(2L'+1)^{1/2}\langle 1,1;\lambda_d,0\mid S,\lambda_d\rangle$$
$$\times\langle S,L;\lambda_d,0\mid J,\lambda_d\rangle\langle S',L';\lambda_N-\lambda_\Delta,0\mid J,\lambda_N-\lambda_\Delta\rangle$$
$$\times\langle \tfrac{1}{2},\tfrac{3}{2};\lambda_N,-\lambda_\Delta\mid S',\lambda_N-\lambda_\Delta\rangle K^{(S)}\frac{i\pi}{m_N m_\Delta}\frac{\sqrt{s}}{2\,\mathrm{Re}(q_\Delta)}T^J_{LL';SS'}(s)\ . \tag{14}$$

THE UNDISTORTED AMPLITUDE

As an internal consistency check as well as to get an estimate of the order of magnitude of the influence of the final state $N\Delta$ interaction (at least in a certain kinematical region), it is useful to evaluate also the undistorted formation amplitude of Fig. 1, using the same normalizations and the same approximations as in the case of the distorted amplitude.

We may write directly for the amplitude corresponding to the diagram in Fig. 1 the expression (cf. Ref. 1):

$$\hat{M}^{d\pi;N\Delta}_{s_d;s_N s_\Delta}(\mathbf{q}_N,\mathbf{q}_d,\mathbf{q}_\pi)=i(2m_N)\sum_{s_2}M^{dN_2N}_{s_d,s_2 s_N}(\mathbf{q}_N,\mathbf{q}_d-\mathbf{q}_N)M^{\pi N_2;\Delta}_{s_2,s_\Delta}(\mathbf{q}_d-\mathbf{q}_N,\mathbf{q}_d-\mathbf{q}_N+\mathbf{q}_\pi)\frac{1}{(\mathbf{q}_d-\mathbf{q}_N)^2-m_N^2+i\epsilon}\ . \tag{15}$$

Here $\mathbf{q}_d$, $\mathbf{q}_\pi$, $\mathbf{q}_\Delta$, $\mathbf{q}_N$, $\mathbf{q}_d-\mathbf{q}_N$ are the momentum of the deuteron, pion, Δ resonance, final nucleon, and intermediate nucleon, respectively. The spin components are defined analogously.

For the deuteron-nucleon-nucleon vertex function we use

$$M^{dN_2N}_{s_d,s_2s_N}(\mathbf{q}_N,\mathbf{q}_d-\mathbf{q}_N)=\chi^{s_2}\left\{F_S((\mathbf{q}_d-\mathbf{q}_N)^2)(\sigma\cdot\hat{\xi}^{s_d})+\frac{3}{\sqrt{2}}F_D((\mathbf{q}_d-\mathbf{q}_N)^2\left[\frac{(\sigma\cdot\hat{\mathbf{q}}_N)(\hat{\mathbf{q}}_N\cdot\hat{\xi}^{s_d})}{|\hat{\mathbf{q}}_N|^2}-\frac{1}{3}(\sigma\cdot\hat{\xi}^{s_d})\right]\right\}(i\sigma_2)\chi^{s_N}\ . \tag{16}$$

In the above expression $\hat{\mathbf{q}}_N$ and $\hat{\xi}^{s_d}$, the deuteron spin vector, refer to the rest frame of the deuteron. For $F_s((\mathbf{q}_d-\mathbf{q}_N)^2)$ and $F_D((\mathbf{q}_d-\mathbf{q}_N)^2)$, which are the S- and D-wave deuteron form factors, we take the McGee wave function.[9]

For the $\pi N\Delta$ vertex we use the nonrelativistic expression

$$M^{\pi N_2;\Delta}_{s_2,s_\Delta}(\mathbf{q}_d-\mathbf{q}_N,\mathbf{q}_d-\mathbf{q}_N+\mathbf{q}_\pi)=g_{\pi N\Delta}\langle\tfrac{1}{2},\tfrac{3}{2};T_N,T_\Delta\mid 1;T_\pi\rangle\left[\frac{-2}{\sqrt{3}}\right]\langle\tfrac{1}{2},1;s_2,s_\Delta-s_2\mid\tfrac{3}{2};s_\Delta\rangle(\xi^{*s_\Delta-s_2}\cdot\mathbf{q}_\pi^+)\ , \tag{17}$$

where the vector $\mathbf{q}_\pi^+$ represents the pion momentum in the Δ-rest frame. The internal polarization vector $\xi^{s_\Delta-s_2}$ is used to represent the spin of the Δ particle and T_N,T_Δ,T_π are the isospin components of the nucleon, Δ particle, and π meson, respectively.

For low energies we may use Galilei transformations and obtain

$$\hat{\mathbf{q}}_N=\mathbf{q}_N-\tfrac{1}{2}\mathbf{q}_d,\quad \mathbf{q}_\pi^+=\mathbf{q}_\pi+\frac{s+m_\pi^2-m_d^2}{s+m_\Delta^2-m_\pi^2}\cdot\mathbf{q}_N\ . \tag{18}$$

Three vectors without specifications refer to the $\pi-d$ c.m. frame.

We have shown in Ref. 1 that the deuteron D-wave component and the "*recoil term*" proportional to $\mathbf{q}_\pi^+-\mathbf{q}_\pi$ are of little importance for the final-state interaction, but for the direct contribution (Fig. 1) there may be kinematical regions where those contributions are important. However, in view of the purpose of the calculations presented at the end of the paper, which aim to demonstrate only qualitatively the importance of the final state ΔN interaction corrections, we neglect in the following both the recoil correction $q_\pi^+-q_\pi$ and the deuteron D-wave contribution.

Using the relation

$$\chi^{s_2}(\sigma\cdot\hat{\xi}^{s_d})(i\sigma^2)\chi^{s_N}=\sqrt{2}\langle\tfrac{1}{2},\tfrac{1}{2};s_N,s_2\mid s_d\rangle\ , \tag{19}$$

we obtain within the approximations mentioned above,

$$\hat{M}^{d\pi;N\Delta}_{s_d;s_Ns_\Delta}(\mathbf{q}_N,\mathbf{q}_d,\mathbf{q}_\pi)=(i)(2m_N)\frac{F_s((\mathbf{q}_d-\mathbf{q}_N)^2)}{(\mathbf{q}_d-\mathbf{q}_N)^2-m_N^2+i\epsilon}\cdot\frac{(-2\sqrt{2})}{\sqrt{3}}\cdot g_{\pi N\Delta}$$
$$\times\langle\tfrac{1}{2},\tfrac{3}{2};T_N,T_\Delta\,|\,1,T_\pi\rangle\cdot\langle\tfrac{1}{2},\tfrac{1}{2};s_N,s_d-s_N\,|\,1,s_d\rangle\langle\tfrac{1}{2},1;s_d-s_N,s'\,|\,\tfrac{3}{2};s_\Delta\rangle\cdot(\xi^{*s'}\cdot\mathbf{q}_\pi)\ , \tag{20}$$

with $s'=s_\Delta-s_d+s_N$.

As in the distorted case, we may choose the direction of the outgoing nucleon as the angular momentum projection axis for the final-state particles, and then the helicities of the nucleon and the delta are given by $\lambda_N=s_N$, $\lambda_\Delta=-s_\Delta$. We then introduce the deuteron helicity by a Wigner rotation

$$\hat{M}^{d\pi;N\Delta}_{\lambda_d;\lambda_N,\lambda_\Delta}(s,\theta)=\sum_{s_d}d^1_{s_d,\lambda_d}(-\theta)\hat{M}^{\pi d;N\Delta}_{s_d;\lambda_N,\lambda_\Delta}(s,\theta)\ , \tag{21}$$

where θ is the angle from the deuteron to the nucleon momentum direction.

We may change to an LS coupling in the following way. We express

$$(\xi^{m*}\cdot\mathbf{q}_\pi)=d^1_{-m,0}(-\theta)\cdot|\mathbf{q}_\pi|\ , \tag{22}$$

and use for the product of d functions $d^1_{m,0}(\theta)d^1_{s_d,\lambda_d}(\theta)$ the well-known addition formula

$$d^{j_1}_{m_1m_1'}(\beta)d^{j_2}_{m_2m_2'}(\beta)=(-1)^{m_2'-m_2}\sum_j\langle j_1,j_2;m_1,-m_2\,|\,j,m\rangle\langle j_1,j_2;m_1',-m_2'\,|\,j,m'\rangle d^j_{mm'}(\beta)\ , \tag{23a}$$

and use the formula

$$\sum_{s_d}\langle\tfrac{1}{2},\tfrac{1}{2};\lambda_N,s_d-\lambda_N\,|\,1,s_d\rangle\langle 1,\tfrac{1}{2};\lambda_N-\lambda_\Delta-s_d,s_d-\lambda_N\,|\,\tfrac{3}{2},-\lambda_\Delta\rangle$$
$$\times\langle 1,1;s_d-(\lambda_N-\lambda_\Delta),-s_d\,|\,S,-(\lambda_N-\lambda_\Delta)\rangle=K^{(S)}\langle\tfrac{1}{2},\tfrac{3}{2};\lambda_N,-\lambda_\Delta\,|\,S,\lambda_N-\lambda_\Delta\rangle \tag{23b}$$

to sum over Clebsch-Gordan coefficients. We have

$$\sum_{s_d}\langle\tfrac{1}{2},\tfrac{1}{2};s_N,s_d-s_N\,|\,1,s_d\rangle\langle\tfrac{1}{2},1;s_d-s_N,s'\,|\,\tfrac{3}{2}s_\Delta\rangle(\xi^{*s'}\cdot\mathbf{q}_\pi)d^1_{s_d\lambda_d}(-\theta)$$
$$=\sum_{S=1,2}\langle 1,1;\lambda_d,0\,|\,S,\lambda_d\rangle\langle\tfrac{1}{2},\tfrac{3}{2};\lambda_N,-\lambda_\Delta\,|\,S;\lambda_N-\lambda_\Delta\rangle\cdot K^{(S)}d^S_{\lambda_N-\lambda_\Delta,\lambda_d}(-\theta)\ , \tag{24}$$

where $K^{(S)}$ is given by Eq. (2), and we then obtain

$$\hat{M}^{d\pi;N\Delta}_{\lambda_d;\lambda_N,\lambda_\Delta}(s,\theta)=(-i)(2m_N)\frac{F_S((\mathbf{q}_d-\mathbf{q}_N)^2)}{(\mathbf{q}_d-\mathbf{q}_N)^2-m_N^2}\cdot g_{\pi N\Delta}(-\sqrt{\tfrac{4}{3}}\langle\tfrac{1}{2},\tfrac{3}{2};T_N,T_\Delta\,|\,1;T_\pi\rangle)\,|\mathbf{q}_\pi|\cdot\sqrt{2}$$
$$\times\sum_{S=1}^{2}\langle 1,1;\lambda_d,0\,|\,S,\lambda_d\rangle\langle\tfrac{1}{2},\tfrac{3}{2};\lambda_N,-\lambda_\Delta\,|\,S,\lambda_N-\lambda_\Delta\rangle K^{(S)}d^S_{\lambda_d,\lambda_N-\lambda_\Delta}(\theta)\ . \tag{25}$$

We project out the partial wave amplitude by performing the integration

$$\hat{M}^{d\pi;N\Delta;J}_{\lambda_d;\lambda_N,\lambda_\Delta}(s)=\frac{2J+1}{4\pi}\int\hat{M}^{d\pi;N\Delta}_{\lambda_d;\lambda_N,\lambda_\Delta}(s,\theta)d^J_{\lambda_d,\lambda_N-\lambda_\Delta}(\theta)d\Omega \tag{26}$$

and again make use of the addition formula Eq. (23) to simplify the results. We then obtain

$$\hat{M}^{\pi d;N\Delta;J}_{\lambda_d;\lambda_N,\lambda_\Delta}=\int(-i)(2m_N)\,|\mathbf{q}_\pi|\,\frac{F_S((\mathbf{q}_d-\mathbf{q}_N)^2)}{(\mathbf{q}_d-\mathbf{q}_N)^2-m_N^2}g_{\pi N\Delta}(-\sqrt{\tfrac{4}{3}}\langle\tfrac{1}{2},\tfrac{3}{2};T_N,T_\Delta\,|\,1,T_\pi\rangle)$$
$$\times\sum_{S=1}^{2}\sum_{L=|J-S|}^{J+S}P_L(\theta)\langle 1,1;\lambda_d,0\,|\,S,\lambda_d\rangle\langle\tfrac{1}{2},\tfrac{3}{2};\lambda_N,-\lambda_\Delta\,|\,S,\lambda\rangle$$
$$\times K^{(S)}\frac{(2J+1)}{4\pi}(-1)^{\lambda-\lambda_d}(-1)^{2S-\lambda-\lambda_d}\left[\frac{2L+1}{2J+1}\right]$$
$$\times\langle S,L;\lambda_d,0\,|\,J,\lambda_d\rangle\langle S,L;\lambda,0\,|\,J,\lambda\rangle d\Omega\ , \tag{27}$$

with $\lambda\equiv\lambda_N-\lambda_\Delta$.

Using the McGee representation[9]

$$F_S(q^2) = -N_d \sum \frac{c_i}{q^2 - m_i^2}(q^2 - m_N^2) \tag{28}$$

[see Refs. (9) or (3) for the definitions and values of the parameters c_i and m_i], we can perform the integration over $\cos\theta$ which leads to Legendre functions of the second kind. In the zero Δ-width approximation they are related to the function $\mathrm{Abs}F_L(s)$ similarly to Eq. (3), but with zero width for the Δ.

We finally obtain for the undistorted (kind of impulse approximation) partial wave amplitude the expression

$$\begin{aligned}\hat{M}^{d\pi;N\Delta;J}_{\lambda_d;\lambda_N,\lambda_\Delta}(s) = {} & -i2\sqrt{2}g_{\pi N\Delta}\left(-\sqrt{\tfrac{4}{3}}\langle \tfrac{1}{2},\tfrac{3}{2};T_N,T_\Delta \mid 1,T_\pi\rangle\right)\frac{1}{\mathrm{Re}(q_\Delta)}\frac{\pi\sqrt{s}\,|\mathbf{q}_d|}{m_\Delta m_N} \\ & \times \sum_{S=1,2}\sum_{L=|J-S|}^{J+S}\mathrm{Abs}F_L(s)\sqrt{2L+1}\langle 1,1;\lambda_d,0 \mid S,\lambda_d\rangle \\ & \times\langle S,L;\lambda_d,0 \mid J,\lambda_d\rangle\langle \tfrac{1}{2},\tfrac{3}{2};\lambda_N,-\lambda_\Delta \mid S,\lambda\rangle\cdot\langle S,L;\lambda,0 \mid J,\lambda\rangle\cdot K^{(S)}\,,\end{aligned} \tag{29}$$

with $\lambda \equiv \lambda_N - \lambda_\Delta$.

CONSISTENCY CHECK: WATSON'S THEOREM

An efficient check of the internal consistency of our calculations can be made through the Watson theorem, since our distorted amplitude is a result of the final-state interaction correction to the simple formation amplitude. Adding together the distorted and undistorted expressions, we should obtain the characteristic phase multiplication (in the limit of small phase values) of Watson's theorem. We have

$$\begin{aligned}\hat{M}^{d\pi;N\Delta;J}_{\lambda_d;\lambda_N,\lambda_\Delta} + M^{d\pi;N\Delta;J}_{\lambda_d;\lambda_N,\lambda_\Delta} = {} & 2\sqrt{2}\frac{\pi\sqrt{s}}{m_N m_\Delta \mathrm{Re}(q_\Delta)}g_{\pi N\Delta}\sqrt{\tfrac{4}{3}}\langle \tfrac{1}{2},\tfrac{3}{2};T_N,T_\Delta \mid 1,T_\pi\rangle\,|\mathbf{q}_d| \\ & \times K^{(S)}\langle 1,1;\lambda_d,0 \mid S,\lambda_d\rangle\langle S,L;\lambda_d,0 \mid J,\lambda_d\rangle\cdot\sqrt{2L+1} \\ & \times\langle \tfrac{1}{2},\tfrac{3}{2};\lambda_N,-\lambda_\Delta \mid S,\lambda\rangle\langle S,L;\lambda,0 \mid J,\lambda\rangle\left[i\,\mathrm{Abs}F_L(s) + iF_L(s)\frac{e^{2i\delta}-1}{2i}\right].\end{aligned} \tag{30}$$

If we keep only the "on-shell" part of $F_L(s)$, this expression yields for the large squares expression the value

$$i\,\mathrm{Abs}F_L(s)\left[1 + \frac{e^{2i\delta}-1}{2}\right].$$

For small δ this is approximately

$$\begin{aligned} i\,\mathrm{Abs}F_L(s)\left[1 + \frac{e^{2i\delta}-1}{2}\right] & \approx i\,\mathrm{Abs}F_L(s)\{1+i\delta\} \\ & \approx i\,\mathrm{Abs}F_L(s)e^{i\delta}\,. \end{aligned} \tag{31}$$

We thus have

$$\hat{M} + M \approx \hat{M}e^{i\delta} \tag{32}$$

which is Watson's theorem.

NUMERICAL RESULTS

Using the ΔN amplitudes determined in Ref. 5 we have evaluated the distorted part of the helicity partial waves for $J=2$ and 3. Their values are given in Table I for incident pion lab energies of 0.256 and 0.325 GeV. Accordingly with our previous experience with elastic πd scattering, we expect that these quantities, when combined with the appropriate amplitudes obtained from a Faddeev calculation, will lead to the correct description of πd breakup at these energies. With the purpose of demonstrating the relative importance of the short-range ΔN interaction effects in πd breakup, we present some results for the $\pi d \to \Delta N$ differential cross section, using, instead of the full Faddeev amplitudes, the undistorted amplitudes $\hat{M}^{\pi d;\Delta N}_{\lambda_d;\lambda_N,\lambda_\Delta}(s,\theta)$ given by Eq. (25). The full cross section can be obtained from the sum of these quantities with the contributions of the ΔN interaction to the angular dependent helicity amplitudes

$$M^{\pi d;N\Delta}_{\lambda_d;\lambda_N,\lambda_\Delta}(s,\theta) = \sum_J M^{\pi d;\Delta N;J}_{\lambda_d;\lambda_N,\lambda_\Delta}(s)d^J_{\lambda_d;\lambda_N-\lambda_\Delta}(\theta)\,, \tag{33}$$

with the partial wave amplitudes being given by Eq. (14).

In the zero-width approximation the differential cross section is given by

$$\begin{aligned}\frac{d\sigma}{d\Omega} = {} & \frac{4m_N\cdot m_\Delta}{64\pi^2\cdot s}\frac{|\mathbf{q}_\Delta|}{|\mathbf{q}_\pi|}\frac{1}{3} \\ & \times\sum_{\lambda_d}\sum_{\lambda_N\lambda_\Delta}|M^{\pi d;N\Delta}_{\lambda d;\lambda_N,\lambda_\Delta}(s,\theta) + \hat{M}^{\pi d;N\Delta}_{\lambda d;\lambda_N,\lambda_\Delta}(s,\theta)|^2\,.\end{aligned} \tag{34}$$

However, this zero-width approximation for the Δ particle in the final state would lead to an unrealistic energy dependence near threshold, as we must take into account the finite width of the produced Δ, in the same way as it

TABLE I. Values (in GeV^{-1}) of the real and imaginary parts of the partial wave helicity amplitudes given by Eq. (14) for the pion lab energies 0.256 and 0.325 GeV. The 24 amplitudes are reduced to 12 due to the symmetry $\lambda_d,\lambda_N,\lambda_\Delta \rightarrow -\lambda_d,-\lambda_N,-\lambda_\Delta$. As can be observed from the table, there are only seven independent amplitudes for each J value.

			$T=0.256$ GeV			
$2\lambda_d$	$2\lambda_N$	$2\lambda_\Delta$	$J=2$		$J=3$	
0	1	−3	−18.500	10.986	−11.244	11.524
0	1	−1	−16.022	9.514	−12.317	12.624
0	1	1	−13.082	7.768	−10.667	10.933
0	1	3	−9.250	5.493	−7.111	7.288
2	−1	−3	−8.011	4.757	−5.806	5.951
2	−1	−1	−11.329	6.728	−8.709	8.926
2	−1	1	−13.875	8.240	−10.057	10.307
2	−1	3	−16.022	9.514	−9.181	9.409
2	1	−3	−16.022	9.514	−9.181	9.409
2	1	−1	−13.875	8.240	−10.057	10.307
2	1	1	−11.329	6.728	−8.709	8.926
2	1	3	−8.011	4.757	−5.806	5.951
			$T=0.325$ GeV			
$2\lambda_d$	$2\lambda_N$	$2\lambda_\Delta$	$J=2$		$J=3$	
0	1	−3	−10.979	3.198	−1.367	1.806
0	1	−1	−9.508	2.770	−1.497	1.978
0	1	1	−7.763	2.262	−1.296	1.713
0	1	3	−5.489	1.599	−0.864	1.142
2	−1	−3	−4.754	1.385	−0.706	0.933
2	−1	−1	−6.723	1.959	−1.059	1.399
2	−1	1	−8.234	2.399	−1.222	1.615
2	−1	3	−9.508	2.770	−1.116	1.475
2	1	−3	−9.508	2.770	−1.116	1.475
2	1	−1	−8.234	2.399	−1.222	1.615
2	1	1	−6.723	1.959	−1.059	1.399
2	1	3	−4.754	1.385	−0.706	0.933

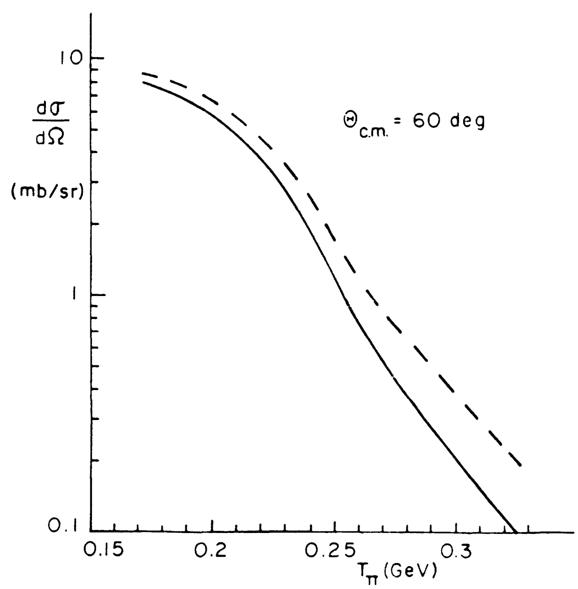

FIG. 3. Comparison of the cross section values at 60° c.m. angle for the process $\pi d \rightarrow N\Delta$, obtained with the undistorted amplitude (dashed line) due to the diagram in Fig. 1, and with the distorted amplitude which takes into account the contribution of the ΔN interaction of short range (solid line).

has been done for the intermediate state Δ [cf. Eq. (3)]. We then evaluate a cross section

$$\sigma(s,\theta,\mu^2) ,$$

where μ^2 is the varying mass of the final state Δ, and describe the experimental cross section using a Breit-Wigner shape for μ. Thus

$$\sigma(s,\theta)=\int_{m_\Delta^2-\alpha}^{m_\Delta^2+\alpha} d\mu^2 \frac{(m_\Delta \Gamma_\Delta/\pi)}{(m_\Delta^2-\mu^2)^2+m_\Delta^2\Gamma_\Delta^2}\sigma(s,\theta,\mu) . \tag{35}$$

The quantity α corresponds to the range of invariant masses, in which a πN system is experimentally counted as a Δ resonance. In the numerical calculations we have taken $\alpha=\Gamma/2=0.050$ GeV.

We are well aware that the model calculation for the full cross section should only be used as a demonstration for expected effects of the ΔN final-state interaction. For small angles of the outgoing pions, the NN final-state interaction (after the decay of the Δ resonance) will play an important role[10] while for large angles the deuteron D-wave and the recoil corrections become important. To avoid such unfavorable circumstances, we present in Fig.

3 the energy dependence of the differential cross section only at the intermediate angle of 60°, where the model is at least qualitatively reliable. We observe in the whole energy range a decrease in the value of the differential cross section, which ranges from about 10% to about 50%. These results show that the short-range ΔN interaction is a non-negligible ingredient for the proper description of the πd breakup cross sections.

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