Nucleon-nucleon correlation effects on deeply inelastic lepton scattering in the region $x > 1$

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(Received 12 April 1988)

Short-range nucleon-nucleon correlations are introduced into the convolution treatment of deeply inelastic lepton-nucleus scattering. The nuclear momentum distribution is calculated using a Jastrow-type wave function to lowest order in the cluster expansion. The excitation energy of 1p-2h final states is included. This leads to smaller structure functions in the region $x > 1$ than those found by Akulinichev and Shlomo. The dependence of the nuclear structure function on Q^2 is also discussed for that region.

I. INTRODUCTION

The observation of significant nuclear dependence of structure functions[1,2] [the European Muon Collaboration (EMC) effect] has lead to continuing interest in deeply inelastic scattering (DIS) of leptons on nuclei. Various explanations have been put forward,[3] all of which can fit the data for $x < 1$. However the region $x > 1$ is also of interest, and preliminary analyses of data from Bologna-CERN-Dubna-Munich-Saclay (BCDMS) (Ref. 4) and Stanford Linear Accelerator Center (SLAC) (Ref. 5) are already available.

In the region $x > 1$ the struck quark carries more momentum than it could in a single nucleon. It has been speculated that multiquark clusters could contribute significantly in this region.[6] However, scattering from a pair of interacting nucleons will also contribute.

Proposed explanations of the EMC effect include changes in the scale of nucleons, multiquark clusters, as well as treatments in terms of conventional degrees of freedom:[7–16] nucleons, pions, and deltas. The latter are usually implemented as a convolution of the free hadron structure functions with the momentum distributions for each type of hadron in the nucleus.[17] Keeping only the nucleon term in the convolution, the nuclear structure function $F_2^A(x)$ has the form

$$F_2^A(x)=\int_x^\infty dz\, f^A(z)F_2^N(x/z)\ , \tag{1}$$

where $F_2^N(x)$ is the free-nucleon structure function, and $f^A(z)$ is the momentum distribution of nucleons in the nucleus. The latter may be written as

$$f^A(z)\simeq \sum_\alpha \int d^3\mathbf{p}\,|\phi_\alpha(\mathbf{p})|^2\delta^3\left[z-\frac{E^A-E_\alpha^{A-1}-\mathbf{p}\cdot\mathbf{n})}{M_N}\right]\ , \tag{2}$$

where $\phi_\alpha(\mathbf{p})$ is the single-nucleon overlap[18] between the A-particle ground state and the $(A-1)$-particle state α. The vector $\mathbf{n}=\mathbf{q}/\nu$ lies along the direction of the three-momentum transfer and can be taken to be the unit vector $\mathbf{q}/|\mathbf{q}|$ in the Björken limit, where we write the four-momentum transfer as $q=(\nu,\mathbf{q})$ (see Sec. III).

Most of these studies of DIS from nuclei have used an independent-particle model of the nucleus, although two groups have attempted to include effects from NN correlations in the nuclear momentum distribution.[19,20] Here we use Jastrow-type wave functions to calculate the nuclear momentum distributions. The short-range correlations in this wave function correspond to 2p-2h excitations where the two excited nucleons can have momenta well above the Fermi momentum. This gives the momentum distribution a long tail, of the kind used in Refs. 19 and 20, and so leads to a larger structure function for $x > 1$ than Fermi momentum alone would produce.

However, when the virtual photon in DIS is absorbed by one of these nucleons it leaves the nucleus in a 1p-2h state. The excitation energy for this final state must come from the virtual photon. Like the separation energy in the EMC effect this tends to shift the momentum distributions to smaller values of x, and so tends to decrease the structure function at large x.

Here we carry out a detailed calculation of the NN correlation contribution to DIS for $x > 1$, in order to examine the relative importance of these effects (increased momentum and greater excitation energy).

II. NUCLEON-NUCLEON CORRELATIONS

In previous calculations of the nucleon momentum distribution an independent-particle wave function has been used. The overlap functions are then just the occupied single-particle wave functions; here we use a Fermi gas model and so they are plane waves. The final states in this case are 1h states with separation energies (including the rest mass of the struck nucleon) given by

$$e_{\rm sep}=E^A-E^{A-1}=M_N+\frac{\mathbf{p}^2}{2M}+V\ , \tag{3}$$

where V is the average single-particle potential. In the usual way[7,10–12] this gives a single-particle contribution to the momentum distribution of the form

$$f_{\rm sp}^A(z)=\sum_{s,t}\int^{k_F} d^3\mathbf{k}\,\delta^3\left[z-\frac{M_N+\mathbf{p}^2/2M_N+V-\mathbf{p}\cdot\mathbf{n}}{M_N}\right]\ . \tag{4}$$

For the Fermi momenta we use phenomenological values which fit quasi-elastic electron scattering.[21]

To lowest order in the cluster expansion, correlations introduce pieces in the wave function where a pair of nucleons is excited into levels $(\mathbf{k},\mathbf{l})$ above the Fermi sea leaving holes in the levels $(\mathbf{m},\mathbf{n})$. If the virtual photon is absorbed by one of these nucleons the $(A-1)$-particle nucleus is left in an excited state with separation energy

$$e_{\mathrm{sep}}=\varepsilon_m+\varepsilon_n-\varepsilon_l \ , \tag{5}$$

where the single-particle energies are $\varepsilon_{\mathbf{m}}=(M_N^2+\mathbf{m}^2)^{1/2}+V$, etc. Note that the momenta of the correlated nucleons can be comparable with the nucleon mass. We have therefore used the relativistic expression for the nucleon kinetic energy.

The correlation contribution to the momentum distribution can be written as a sum of direct and exchange terms

$$f_{\mathrm{cor}}(z)=f_{\mathrm{dir}}(z)+f_{\mathrm{exch}}(z) \ . \tag{6}$$

The direct piece is given by the following nine-dimensional integral:

$$f_{\mathrm{dir}}(z)=\sum_{s,t}\int d^3\mathbf{k}\,d^3\mathbf{l}\,d^3\mathbf{m}\,d^3\mathbf{n}\,\delta(\mathbf{k}+\mathbf{l}-\mathbf{m}-\mathbf{n})\hat{F}(|\mathbf{k}-\mathbf{m}|)^2\theta(k_F-|\mathbf{m}|)\theta(k_F-|\mathbf{n}|)$$
$$\times\theta(|\mathbf{k}|-k_F)\theta(|\mathbf{l}|-k_F)\,\delta\left[z-\frac{1}{M_N}(\varepsilon_m+\varepsilon_n-\varepsilon_l+k_3)\right] , \tag{7}$$

where k_F is the Fermi momentum and $\hat{F}(q)$ is the Fourier transform of the correlation function $F(r)$ for nuclear matter. The exchange term is similar, but with $\hat{F}[\,|\mathbf{k}-\mathbf{m}|\,]^2$ replaced by $\hat{F}[\,|\mathbf{k}-\mathbf{n}|\,]\hat{F}[\,|\mathbf{k}-\mathbf{m}|\,]$. The implied sum over spin and isospin is also different for the exchange term: for symmetric nuclear matter it gives a factor of $\frac{1}{4}$ compared to the direct term.

We adopt a parametrization of the radial correlation function $F(r)$ used in the Jastrow-type calculations of Ref. 22. These correlation functions were derived for simple potentials with hard core radii of $r_c=0.6$ fm or $r_c=0.4$ fm, which fit the following low-energy NN data: the binding energy of the deuteron, the triplet n-p scattering length, the singlet n-p scattering length, and the singlet n-p effective range. The correlation function for the NN potential OMY has the form[22]

$$F(r)=\begin{cases}0 & \text{for } r<r_c\\ \{1-\exp[-\mu(r-r_c)]\}\{1+\gamma\exp[-\mu(r-r_c)]\} & \text{for } r\ge r_c\end{cases} , \tag{8}$$

where the hard-core radius is $r_c=0.6$ fm, and different sets of (μ,γ) are used for different Fermi momenta. It satisfies the following normalization constraint:

$$\int[F^2(r)-1][1-\tfrac{1}{4}l^2(k_F\cdot r)]dr=0 \ , \tag{9}$$

where $l(x)=3[\sin(x)-x\cos(x)]/x^3$. We also calculated the single-nucleon momentum distribution for this correlation function and checked that it gives reasonable agreement with recent calculations of the nucleon momentum distributions in nuclear matter.[23,24]

The total distribution function $f(z)$ should be normalized to give the correct number of nucleons,

$$\int dz\,f(z)=\int dz[f_{\mathrm{sp}}(z)+f_{\mathrm{cor}}(z)]=A \ . \tag{10}$$

We enforce this constraint by adjusting the normalization of the single-particle piece $f_{\mathrm{sp}}(z)$.

III. RESULTS AND DISCUSSION

In Eq. (7), the correlated distribution function is written as a nine-dimensional integral in momentum space. Three of the integrations can be performed analytically to leave a six-dimensional nontrivial integral over a rather complicated region in the remaining variables (the direct term could be reduced further to a five-dimensional integral, but at the cost of further complicating the region of integration). We have evaluated this numerically using

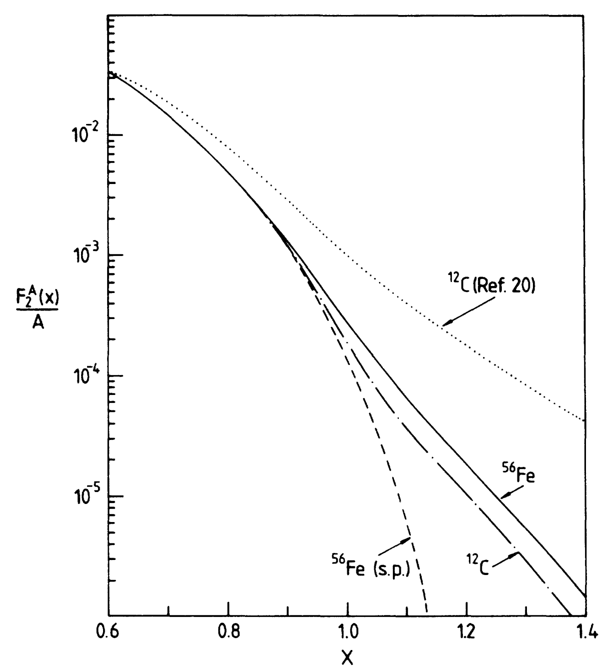

FIG. 1. Nuclear structure functions as a function of x. The solid and dash-dotted lines are our results for $Q^2=50$ GeV.[2] The dashed line shows the results of Akulinichev and Shlomo (Ref. 20). For comparison, the dashed line shows the single-particle result, without short-range correlations.

two multidimensional integration techniques: the advanced Korobov-Conroy method and the iterative Cronord-Gauss method. We have checked that these methods agree within the requested accuracy (1 part in 10^2).

We have calculated structure functions for uniform nuclear matter with Fermi momenta of $k_F = 220$ MeV ($e_{\rm sep} = 32$ MeV) and $k_F = 260$ MeV ($e_{\rm sep} = 30$ MeV). According to the work of Moniz *et al.*[21] these correspond to ^{12}C and ^{56}Fe, respectively. The results are plotted in Fig. 1. We have used several different (Q^2-dependent) parametrizations[25] of the free-nucleon structure function, to check that our results and conclusions do not depend on it. The curves in Fig. 1 are for the parameter set 1 of Duke and Owens. For smaller values of x our structure functions are in reasonable agreement with the observed EMC ratio.[1,2]

In the region $z > 1$, the exchange term in $f_{\rm cor}(z)$ has a similar form to the direct term, and tends to decrease the distribution by about 20%. It has a somewhat larger effect for $z < 1$, but there the correlated piece $f_{\rm cor}(z)$ is less important than the single-particle piece. The contributions from high-momentum nucleons ($|\mathbf{k}| > M_N$) can make the distribution $f(z)$ nonzero for negative z. This probably reflects the fact that we are not using a consistent relativistic description of the nuclear wave function. If we calculate $F_2^A(x)$ only for physical values of x (i.e., positive x) the negative-z region does not contribute to the convolution in Eq. (1). In fact the tail of the nucleon momentum distribution with $z < 0$ is very small, provided we use relativistic kinetic energies in Eqs. (5)–(7); it contributes less than 2% of the normalization integral (10).

We can compare our results with those of Akulinichev and Shlomo[20] for ^{12}C, which are in good agreement with the preliminary analysis from BCDMS.[4] They calculated the structure function including correlation effects, but neglecting the excitation energies of the final 1p-2h states, as well as the exchange term. This allowed them to write $f(z)$ directly in terms of the nucleon momentum distribution. We find that this approximation shifts the peak in $f_{\rm cor}(z)$ towards greater z by about $\Delta z = 0.2$. Such a shift significantly increases the predicted structure function in the region $x > 1$, as can be seen in Fig. 1. Our results are about an order of magnitude less than those of Akulinechev and Shlomo for $x = 1.2$, and this difference is even greater for larger values of x. This indicates that the effects of greater nucleon momenta due to correlations tend to be cancelled by the greater excitation energies of the final states. Even so we do find significant enhancement over the pure single-particle calculation in this region, although the resulting structure function is very small for $x > 1.3$.

We have also examined the Q^2 dependence of the nuclear structure function for $x > 1$. In Fig. 2 we show the effects of the standard (logarithmic) dependence due to the quantum chromodynamics (QCD) evolution of the nucleon structure function. In addition to this there are kinematic effects[13] in the nuclear structure function which become significant for small Q^2. The vector $\mathbf{n}$ in Eq. (2) reduces to the unit vector $\mathbf{q}/|\mathbf{q}|$ only in the Björken limit. For finite Q^2, the usual approximation should be modified by multiplying the space component of the momentum by a factor

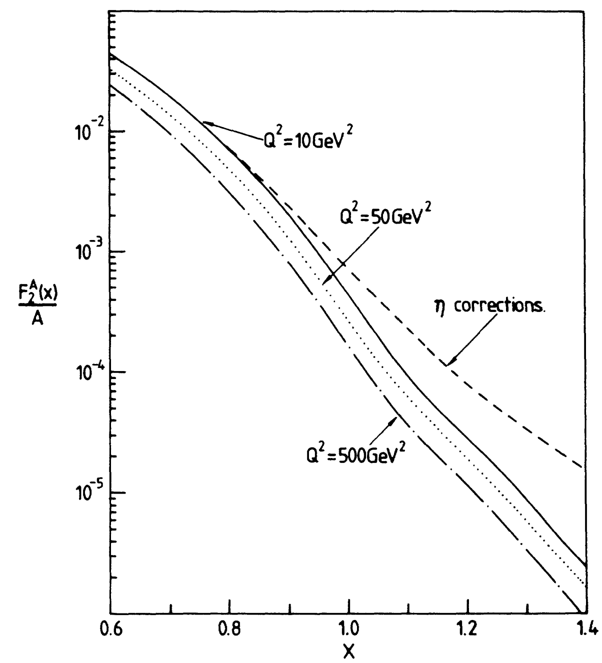

FIG. 2. Dependence of the nuclear structure function on Q^2. We show results using the Q^2-dependent nucleon structure function for $Q^2 = 10$, 50, and 500 GeV^2. We also show the effect of the kinematic correction discussed in the text for $Q^2 = 10$ GeV^2.

$$\eta = \left[\frac{Q^2 + \nu^2}{\nu^2} \right]^{1/2} = \left[1 + \frac{4M^2 x^2}{Q^2} \right]^{1/2} . \tag{11}$$

In the scaling limit this reduces to unity, but for small Q^2 it can have important effects, especially at large x. Also in Fig. 2 we show the effect of including it for $Q^2 = 10$ GeV^2. For higher values of Q^2 it rapidly becomes unimportant.

If the preliminary analysis from BCDMS (Refs. 4 and 20) are confirmed by future experiments, our results would suggest that the nucleons-only picture is insufficient to explain DIS for $x > 1$, and other objects such as multiquark clusters may be present. However, such a conclusion would be premature since there are other mechanisms which may contribute. In particular our calculations include only terms which can be written as a convolution over free-nucleon structure functions. We have thus neglected any interactions between the slow-moving debris of the struck nucleon, as well as quark exchanges between nucleons.[26] Both correspond to DIS from two (or more) interacting nucleons, and so may give significant contributions in the region $x > 1$.

We are grateful to J. M. Irvine and M. Modarres for useful discussions.

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[1]J. J. Aubert *et al.*, Phys. Lett. **123B**, 275 (1983).
[2]R. G. Arnold *et al.*, Phys. Rev. Lett. **52**, 727 (1984).
[3]For reviews of the various approaches, see C. H. Llewellyn Smith, Nucl. Phys. **A434**, 35c (1985); R. P. Bickerstaff, M. C. Birse, and G. A. Miller, Phys. Rev. D **33**, 3228 (1986).
[4]I. A. Savin, Proceedings of the Sixth International Seminar on High-Energy Physics, Dubna, 1981 (unpublished), pp. 223–237.
[5]D. Day, Nucl. Phys. **A478**, 397c (1988).
[6]H. J. Pirner and J. P. Vary, Phys. Rev. Lett. **46**, 1376 (1981).
[7]C. H. Llewellyn Smith, Phys. Lett. **128B**, 107 (1983).
[8]M. Ericson and A. W. Thomas, Phys. Lett. **128B**, 112 (1983).
[9]E. L. Berger, F. Coester, and R. B. Wiringa, Phys. Rev. D. **29**, 398 (1984); E. L. Berger and F. Coester, *ibid.* **32**, 1071 (1985).
[10]S. V. Akulinichev, S. A. Kulagin, and G. M. Vagradov, Phys. Lett. **158B**, 485; J. Phys. G **11**, L245 (1986); S. V. Akulinichev *et al.*, Phys. Rev. Lett. **55**, 2239 (1985).
[11]G. V. Dunne and A. W. Thomas, Phys. Rev. D **33**, 2061 (1986); Nucl. Phys. **A455**, 701 (1986).
[12]R. P. Bickerstaff and G. A. Miller, Phys. Lett. **108B**, 409 (1986); M. Staszel, J. Rozynek, and G. Wilk, Phys. Rev. D **29**, 2638 (1984).
[13]J. Rozynek, in Proceedings of High Energy Physics Conference, Kazimierz, Warsaw, Poland, 1986, edited by Z. Adjuk (unpublished), p. 465.
[14]B. L. Birbrair *et al.*, Phys. Lett. **166B**, 119 (1986).
[15]L. L. Frankfurt and M. I. Strickman, Phys. Lett. **183B**, 254 (1987).
[16]M. B. Johnson and J. Speth, Nucl. Phys. **A470**, 488 (1987).
[17]G. B. West, Ann. Phys. (N.Y.) **74**, 646 (1972); W. B. Atwood and G. B. West, Phys. Rev. D **7**, 773 (1973); A. Bodek and J. L. Ritchie, *ibid.* **23**, 1070 (1981); **24**, 1400 (1981); L. L. Frankfurt and M. I. Strickman, Phys. Rep. **76C**, 215 (1981).
[18]C. F. Clement, Nucl. Phys. **A213**, 469, 493 (1973); M. C. Birse and C. F. Clement, Nucl. Phys. **A351**, 112 (1981).
[19]H. Araseki and T. Fujita, Nucl. Phys. **A439**, 681 (1985).
[20]S. V. Akulinichev and S. Shlomo, Phys. Rev. C **33**, 1551 (1986).
[21]E. J. Moniz *et al.*, Phys. Rev. Lett. **26**, 445 (1971).
[22]D. A. Chakkalakal, C. -H. Yang, and J. W. Clark, Nucl. Phys. **A271**, 185 (1976).
[23]S. Fantoni and V. R. Pandharipande, Nucl. Phys. **A427**, 473 (1984).
[24]J. G. Zabolitzky and W. Ey, Phys. Lett. **76B**, 527 (1978).
[25]M. Gluck, E. Hoffman, and E. Reya, Z. Phys. C **13**, 119 (1982); D. W. Duke and J. F. Owens, Phys. Rev. D **30**, 49 (1984).
[26]R. L. Jaffe, in *Workshop on Nuclear Chromodynamics: Quarks and Gluons in Particles and Nuclei,* edited by S. Brodsky and E. Moniz (World-Scientific, Singapore, 1986), p. 215; P. Hoodbhoy and R. L. Jaffe, Phys. Rev. D **35**, 113 (1987).