Excitation of the Δ resonance in the ^{12}C(e,e') reaction

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The inclusive (e,e') scattering from nuclei is studied with the assumption that the electromagnetic production of pions from the nucleon can be described by a model consisting of the conventional Born term and a Δ-excitation term. We show that the global features of the recent data of the inclusive ^{12}C(e,e') reaction in the GeV energy region can be described by the following procedures: (1) the parameters of the constructed one-body and two-body current operators are fitted to the total cross sections of $\gamma N \to \pi N$, $p(e,e')$, and $\gamma d \to np$; (2) the medium effect on the Δ propagation is treated according to the information extracted from the Δ-hole model calculation. It is found that the two-body mechanism of Δ annihilation by the nuclear medium improves the fit to the data in the "dip" region. The predicted magnitudes in the Δ region are lower than the data, indicating the importance of more complicated multinucleon mechanisms.

In this paper we report on a study of the Δ-excitation mechanism in the inclusive ^{12}C(e,e') reaction. Our primary interest is to examine the extent to which the recent data[1] of this reaction in the GeV energy region can be related to the elementary $\Delta \leftrightarrow \pi N$ decay and the $N\Delta \leftrightarrow NN$ transition mechanism.

Qualitatively, the approach taken in the present study is similar to that of Laget.[2] The starting point is a model of the one-body current operator for describing the electromagnetic production of pions from the nucleon. It contains a Born term deduced from a field theoretical Lagrangian and a Δ-excitation term. The parameters of the model are suitably adjusted to fit the data of $\gamma N \to \pi N$ and $p(e,e')$ reactions. The constructed one-body current operator can induce a two-body mechanism that the produced pion is absorbed by a second nucleon in the nucleus. The strength of the constructed two-body operator is then determined by a πNN form factor, which is adjusted directly to fit the data of the $\gamma d \to np$ reaction. By integrating the matrix elements of these two current operators over the momentum distribution of the nucleus, one can obtain the main features of the inclusive (e,e') cross section. To compare with the data, it is necessary to include the medium effects on the Δ propagation.

The considered one-body current operator for pion production [Fig. 1(a)] can be defined by its matrix element in momentum space

$$\langle \mathbf{k}'\mathbf{p}' \mid [J_1^\mu(0) \mid \mathbf{p}\rangle = \langle \mathbf{k}'\mathbf{p}' \mid [J_1^\mu(0)]_{\text{Born}} \mid \mathbf{p}\rangle + \langle \mathbf{k}'\mathbf{p}' \mid [J_1^\mu(0)]_\Delta \mid \mathbf{p}\rangle \ , \qquad (1)$$

where $\mathbf{k}'$ and $\mathbf{p}'$ are, respectively, the momentum of the pion and the nucleon. For a given incident photon four momentum $q^\mu = (\omega, \mathbf{q})$, the allowed final πN states are restricted by $\mathbf{k}' + \mathbf{p}' = \mathbf{q} + \mathbf{p}$. All particles except the photon are always kept on their mass shell; i.e., $E_N(p) = (m^2 + p^2)^{1/2}$ and $E_\pi(\mathbf{k}) = (\mu^2 + \mathbf{k}^2)^{1/2}$ for the nucleon and the pion, respectively. This formulation is consistent with the Δ-hole model,[3,4] which will be used later to introduce a procedure for describing the medium effects on the Δ propagation.

For simplicity, we take the Born term derived by Laget,[2] but regularize it with a form factor to account for the final πN interaction

$$\langle \mathbf{k}'\mathbf{p}' \mid [J_1^\mu(0)]_{\text{Born}} \mid \mathbf{p}\rangle = \langle \mathbf{k}'\mathbf{p}' \mid [J_1^\mu(0)]_{\text{Born}} \mid \mathbf{p}\rangle_{\text{Laget}} \left[\frac{\Lambda_B^2}{\Lambda_B^2 + \mathbf{K}^2} \right] , \qquad (2)$$

where $\mathbf{K}$ is the πN relative momentum, and Λ_B is an adjustable parameter. This modification is found to be needed in fitting the γN data since in an approach consistent with the Δ-hole model our parametrization of the Δ term is significantly different from Laget's form.

The Δ-excitation term is expressed in terms of two vertex functions

$$\langle \mathbf{k}'\mathbf{p}' \mid [\mathbf{J}(0)]_\Delta \mid \mathbf{p}\rangle = h^*_{\pi N,\Delta}(\mathbf{K}) \frac{1}{E - E_\Delta(\mathbf{p}_\Delta) - \Sigma_\Delta(\mathbf{p}_\Delta, E)} \mathbf{F}_{\gamma N,\Delta}(q_\mu^2) \ , \qquad (3a)$$

$$\langle \mathbf{k}'\mathbf{p}' \mid [J_0(0)]_\Delta \mid \mathbf{p}\rangle = 0 \ , \qquad (3b)$$

where E is the total energy of the system, $\mathbf{q} = \mathbf{k}' + \mathbf{p}' - \mathbf{p}$, $\mathbf{p}_\Delta = \mathbf{k}' + \mathbf{p}'$, and $E_\Delta(\mathbf{p}_\Delta) = (m_\Delta^2 + \mathbf{p}_\Delta^2)^{1/2}$ is the energy of the bare Δ with a mass m_Δ. The Δ self-energy Σ_Δ is also determined from the $\pi N \leftrightarrow \Delta$ vertex function

$$\Sigma_\Delta(\mathbf{p}_\Delta, E) = \int \frac{| h_{\pi N,\Delta}(K) |^2 K^2 dK}{\omega_0(E, \mathbf{p}_\Delta) - E_N(K) - E_\pi(K) + i\epsilon} \ , \qquad (4)$$

where $\omega_0(E, \mathbf{p}_\Delta) = (E^2 - \mathbf{p}_\Delta^2)^{1/2}$ is the total energy in the c.m. frame and $\mathbf{K}$ is the πN relative momentum. By fitting the πN P_{33} phase shifts, we have $m_\Delta = 1310$ MeV

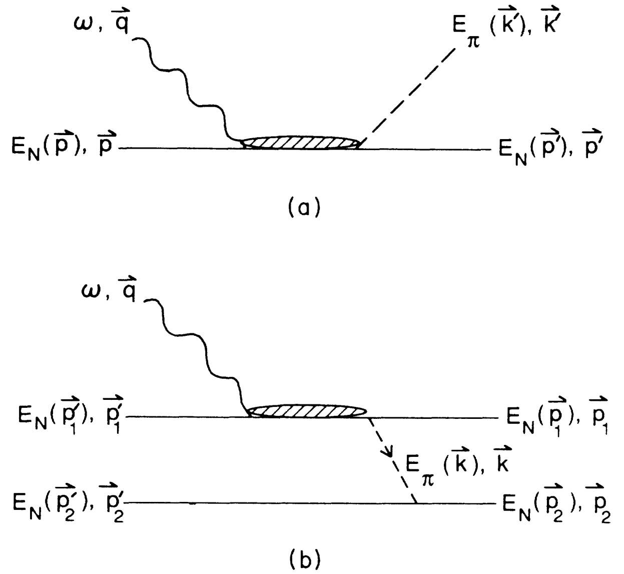


FIG. 1. (a) One-body mechanism of pion production from the nucleon, (b) two-body $\gamma NN \rightarrow NN$ mechanism.

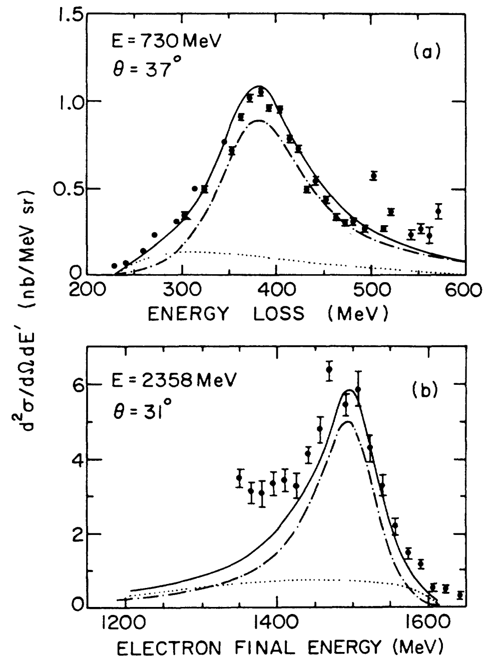


FIG. 2. The calculated $p(e,e')$ cross sections (solid curves) are compared with the data (Ref. 6). The dash-dotted (dotted) curves are the individual contributions from the Δ excitation (Born) term.

and

$$h_{\pi N,\Delta}(\mathbf{K}) = \frac{1}{(2\pi)^{3/2}} \frac{i f_{\pi N,\Delta}}{\sqrt{2E_\pi(K)}} \left[\frac{\Lambda_\Delta^2}{\Lambda_\Delta^2 + K^2} \right] \frac{1}{\mu} (\mathbf{S}\cdot\mathbf{K})(I T_{-I}) , \tag{5}$$

where $f_{\pi N,\Delta}=6.01$, $\Lambda_\Delta=358$ MeV, and I is the z component of the pion isospin. The $\Delta \leftrightarrow N$ transition spin operator **S** is defined by the following reduced matrix (in the convention of Edmonds[5]):

$$\langle \tfrac{3}{2} \| \mathbf{S} \| \tfrac{1}{2} \rangle = -\langle \tfrac{1}{2} \| \mathbf{S}^{+} \| \tfrac{3}{2} \rangle = 2 . \tag{6}$$

The transition isospin operator **T** is also similarly defined. The $\gamma N \rightarrow \Delta$ vertex is parametrized, according to the nonrelativistic quark model, as

$$\mathbf{F}_{\gamma N,\Delta}(q_\mu^2) = F_\Delta(q_\mu^2) \frac{1}{(2\pi)^3} \frac{i f_{\gamma N,\Delta}}{m_\Delta} (\mathbf{S}\times\mathbf{q}) T_3 \tag{7}$$

with

$$F_\Delta(q_\mu^2) = F_N(q_\mu^2)(1+q_\mu^2/t^2) , \tag{8}$$

where $F_N(q_\mu^2)=[1-q_\mu^2/(0.71 \text{ GeV}/c)^2]^{-2}$ is the usual nucleon form factor.

The range Λ_B of the Born term [Eq. (2)] and the strength $f_{\gamma N,\Delta}$ of the Δ-excitation term [Eq. (7)] are adjusted to fit the data of the total γN cross section. The quality of our fits is comparable to that of Laget.[2] The resulting parameters are $\Lambda_B=350$ MeV/c and $f_{\gamma N,\Delta}=3.6$. The range t of the $\gamma N \leftrightarrow \Delta$ form factor [Eq. (8)] is determined by fitting the data of the inclusive $p(e,e')$ reaction. It is found that the existing data[6] can be reasonably described with the value $t=6$ GeV/c. In Fig. 2 we show two fits to the data at $E=730$ and 2358 MeV. It is seen that the cross sections are dominated by the Δ excitation (dash-dotted curves), but the contributions from the Born term (dotted curves) are significant at all energies. This completes the construction of our model of the one-body current operator for pion production.

The two-body mechanism $\gamma NN \rightarrow NN$ [Fig. 1(b)] is assumed to be dominated by the coupling to the πNN intermediate state. It is defined by the following matrix element (spin-isospin indices are suppressed)

$$\langle \mathbf{p}_1 \mathbf{p}_2 | J_2^\mu(0) | \mathbf{p}'_1 \mathbf{p}'_2 \rangle = h^*_{\pi N,N}(K) \frac{1}{E - E_N(\mathbf{p}_1) - E_N(\mathbf{p}'_2) - E_\pi(\mathbf{k}) + i\epsilon} \langle \mathbf{p}_1 \mathbf{k} | J_1^\mu(0) | \mathbf{p}'_1 \rangle \tag{9}$$

with

$$\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}'_1 + \mathbf{p}'_2 + \mathbf{q}, \quad \mathbf{k} = (\mathbf{p}'_1 + \mathbf{q}) - \mathbf{p}_1 = \mathbf{p}_2 - \mathbf{p}'_2 .$$

The πNN form factor $h_{\pi N,N}(K)$ is defined by

$$h_{\pi N,N}(K) = \frac{1}{(2\pi)^{3/2}} \frac{i f_{\pi NN}}{\sqrt{2E_\pi(K)}} \left[\frac{\Lambda_{\pi NN}^2}{\Lambda_{\pi NN}^2 + K^2} \right] \frac{1}{\mu} (\boldsymbol{\sigma}\cdot\mathbf{K}) I \tau_{-I} , \tag{10}$$

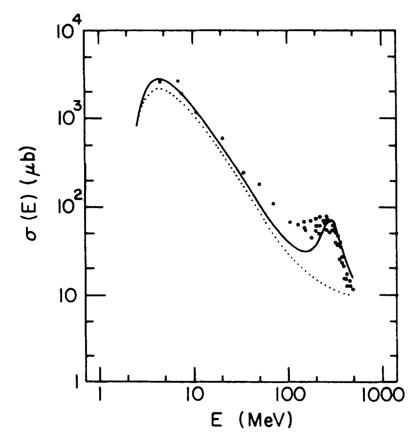


FIG. 3. The calculated total cross sections of the $\gamma d \to np$ reaction are compared with the data (Ref. 11).

where K is the πN relative momentum evaluated from $\mathbf{k}$ and $\mathbf{p}_2'$ and $f^2_{\pi NN}/4\pi = 0.081$.

The parameter $\Lambda_{\pi NN}$ is adjusted to fit the total cross section of the $\gamma d \to np$ reaction. The calculation is done by integrating the usual one-body nucleonic current matrix element (also given explicitly in Ref. 2) and the two-body matrix element Eq. (9) over the deuteron ground-state wave function. With $\Lambda_{\pi NN} = 1000$ MeV/c, our fit is shown in Fig. 3. The model can give a reasonable description of the data in the Δ-excitation energy region $E_\gamma \simeq 250$ MeV. The discrepancies in the lower energy region indicate the need of a more accurate description of the final πNN dynamics. It probably needs a unitary πNN calculation[7] to resolve the problem. For our present purpose, the fit shown in Fig. 3 justifies the use of the model in the study of the Δ excitation in nuclei. In this way we have determined the two-body current operator from the real photon limit. If we assume that the model is also valid for the virtual photon case, we can proceed to calculate the ^{12}C(e,e') cross section.

Our main objective is to examine the extent to which the inclusive ^{12}C(e,e') reaction can be understood in terms of the elementary $\gamma N \to N$ transition and the pionic mechanisms defined above. We start with the impulse approximation and use the closure approximation to sum the one-hole and two-hole final nuclear states. The inclusive ^{12}C(e,e') cross section can then be calculated from the following equation:

$$\frac{d^2\sigma}{d\Omega\, dE'} = \left[\frac{d^2\sigma}{d\Omega\, dE'}\right]_N + \left[\frac{d^2\sigma}{d\Omega\, dE'}\right]_{N\pi} + \left[\frac{d^2\sigma}{d\Omega\, dE'}\right]_{NN}, \tag{11}$$

where the subindices N, $N\pi$, and NN denote, respectively, the contributions from the elementary $eN \to e'N'$, $eN \to e'N'\pi$, and $eN_1N_2 \to e'N_1'N_2'$ processes. The first two terms lead to one-hole final nuclear states and can be written explicitly as follows:

$$\left[\frac{d^2\sigma}{d\Omega\, dE'}\right]_N = \sum_{nlj}(2j+1)\int d\mathbf{p}\,\delta\left\{\omega - |\epsilon_{nlj}| - \left[E_N(\mathbf{p}+\mathbf{q}) - m + \frac{\mathbf{q}^2}{2m_A}\right]\right\}\frac{1}{4\pi}|R_{nlj}(p)|^2\left[\left[\frac{d^2\tilde{\sigma}}{d\Omega\, dE'}\right]\right]_{N,\mathbf{p}} \tag{12}$$

and

$$\left[\frac{d^2\sigma}{d\Omega\, dE'}\right]_{\pi N} = \sum_{nlj}(2j+1)\int d\mathbf{p}\frac{1}{4\pi}|R_{nlj}(p)|^2 \times \int d\mathbf{k}\,\delta\left\{\omega - |\epsilon_{nlj}| - \left[E_N(\mathbf{p}+\mathbf{q}-\mathbf{k}) + E_\pi(\mathbf{k}) - m + \frac{\mathbf{q}^2}{2m_A}\right]\right\}\left[\frac{d^3\tilde{\sigma}}{d\Omega\, dE'\, d\mathbf{k}}\right]_{\pi N,\mathbf{p}}, \tag{13}$$

where $R_{nlj}(p)$ is the shell-model harmonic oscillator radial wave function with $b = 1.64$ fm. The single nucleon cross section in the right-hand side of Eq. (12) is evaluated from the standard $\gamma N \to N$ current operator given explicitly in Ref. (2). The single nucleon cross section in Eq. (13) is evaluated from the one-body current operator defined by Eqs. (1)–(8). In both calculations the Fermi motion of the nucleon is taken into account by evaluating the single nucleon cross sections at the kinematics that the initial nucleon is moving with a momentum $\mathbf{p}$. This is essential in describing the widths of the (e,e') cross sections to be discussed later.

The last term in Eq. (11) is evaluated from the two-body matrix element Eq. (9). Separating the relative and the center-of-mass parts of the harmonic wave functions for the initial two nucleons in ^{12}C, we have

$$\left[\frac{d^2\sigma}{d\Omega\, dE'}\right]_{NN} = \sum_{\substack{n_1l_1j_1\\ n_2l_2j_2}}\left\{\sum_{\substack{nlsj\\ \lambda N'NL}}(2j+1)(2\lambda+1)M_\lambda(n_1l_1n_2l_2;NLnl)M_\lambda(n_1l_1n_2l_2;N'Lnl)\left[\frac{2}{1+\delta_{n_1n_2}\delta_{l_1l_2}\delta_{j_1j_2}}\right]\right.$$
$$\times\frac{1}{2l+1}\int d\mathbf{p}R_{NL}\left[\frac{1}{\sqrt{2}}p\right]R_{N'L}\left[\frac{1}{\sqrt{2}}p\right]\frac{1}{4\pi}$$
$$\times\int d\mathbf{k}\,\delta\left[\omega - \epsilon_{n_1l_1j_1} - \epsilon_{n_2l_2j_2} - \frac{(\mathbf{p}+\mathbf{q})^2}{4m} - \frac{k^2}{m} - \frac{\mathbf{q}^2}{2m_A}\right]$$
$$\left.\times\frac{1}{2j+1}\left[\frac{d^3\tilde{\sigma}}{d\Omega\, dE'\, d\mathbf{k}}\right]_{NN,\mathbf{p}(nls)j}\right\}, \tag{14}$$

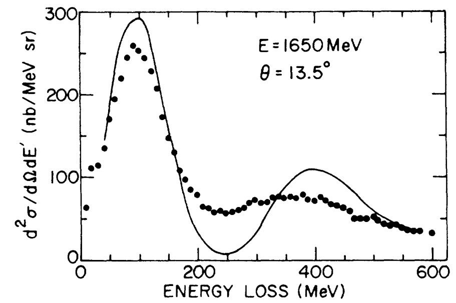


FIG. 4. Impulse approximation calculation of ^{12}C(e,e') cross sections are compared with the data (Ref. 1).

where **k** is the relative momentum of the two outgoing nucleons, M_λ is the coefficient of the Moshinsky transformation. $\mathbf{p}(nls)j$ denotes a pair of nucleons on a state specified by its total momentum **p** and internal orbital-spin quantum numbers: *nlsj*. The binding effect also enters the calculation through the separation energy ϵ_{nlj} of each orbital. With an appropriate change of the initial wave function, the calculation of the elementary cross section in the right-hand side of Eq. (14) is identical to that for the deuteron.

In Fig. 4 we compare the recent data[1] at $E=1650$ MeV with the impulse approximation calculation described above. It is clear that the calculated results reproduce the kinematic features of the data, but not the magnitudes. The medium effects must be included to redistribute the strength of the cross section. Similar results are also seen in the lower energy regions.

We now introduce a simple prescription to describe the medium effects on the Δ-excitation mechanism. We assume that the medium effect on the Δ propagation can be described by changing the Δ self-energy Σ of Eq. (3a)

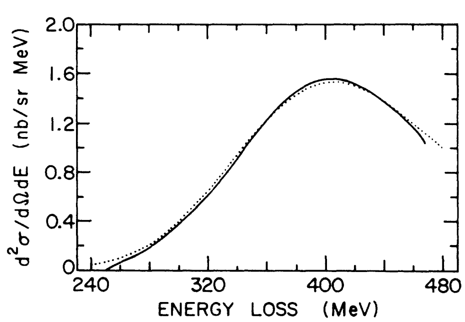


FIG. 5. The calculated contributions of the quasifree Δ production to the ^{12}C(e,e') cross sections at 620 MeV, $\theta=60°$ are compared with that of the Δ-hole model calculation (dotted curve) by Koch and Ohtsuka (Ref. 4).

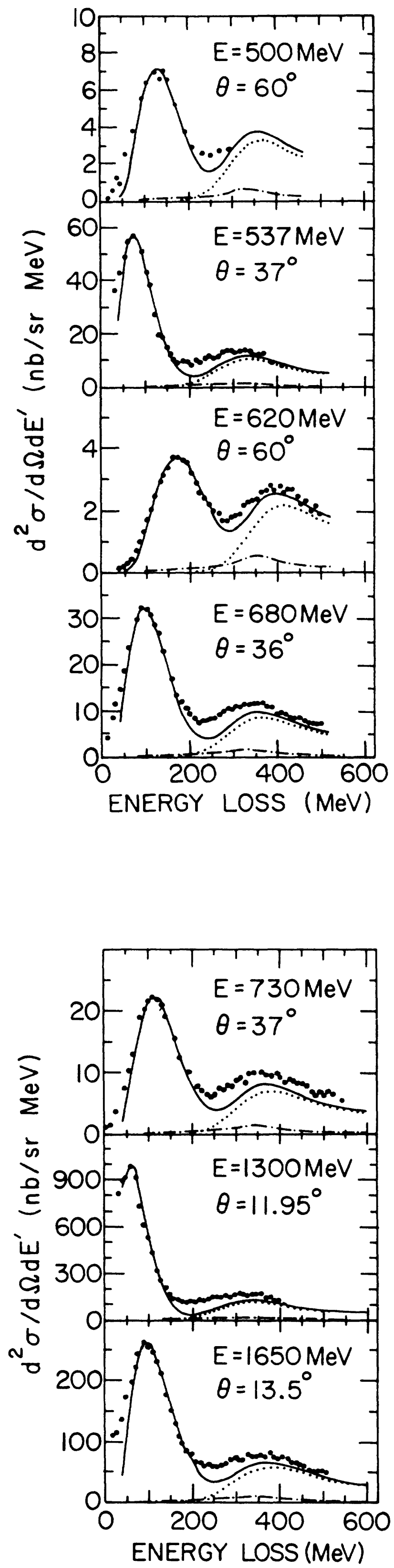
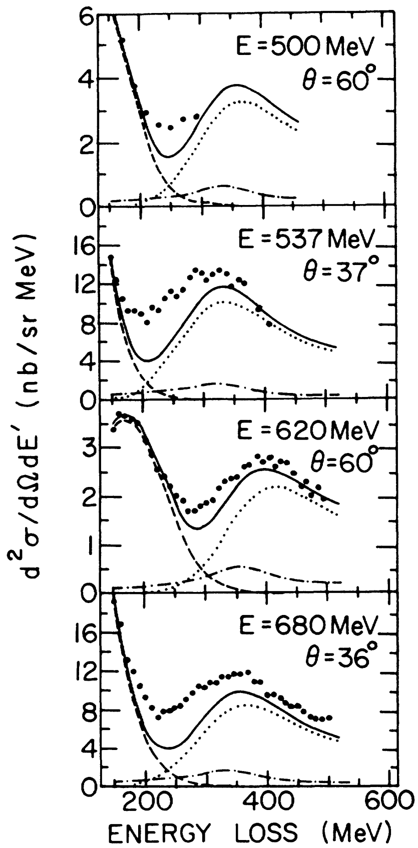


FIG. 6. The calculated ^{12}C(e,e') cross sections are compared with the data (Refs. 1 and 12). See text for the normalization of the calculated cross section.



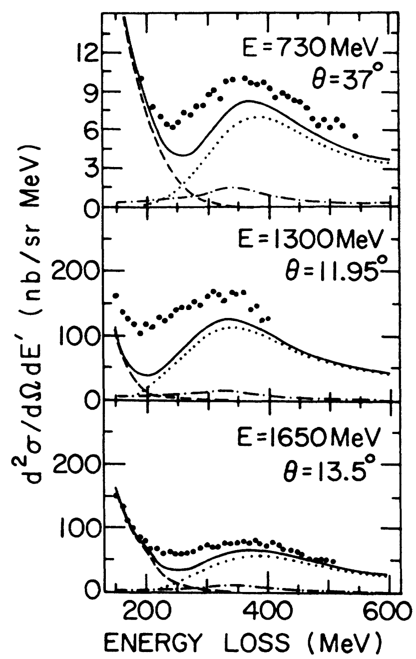


FIG. 7. The calculated ^{12}C(e,e') cross sections (solid curves) are compared with the data (Refs. 1 and 12). The dashed curves are from the quasifree nucleonic process [Eq. (12)] normalized to the nucleon quasifree peaks in $\omega \leq 200$ MeV regions. The dotted curves are from the quasifree Δ production [Eq. (13)]. The dash-dotted curves are from the $\gamma NN \rightarrow NN$ two-body process [Eq. (14)].

from its value in free space. This procedure is known to be not very accurate in the study[8] of pion nucleus scattering because of the large contributions from the Pauli and coherent scattering from the ground state. But, as found by Koch and Ohtsuka[4] these two terms tend to cancel each other in the (e,e') kinematics and the dominant medium effect is from the spreading potential[8] which describes the annihilation of the Δ by the nuclear matter. In this case the simple procedure of shifting the Δ self-energy is expected to work. In Fig. 5 we show that if we add a shift $V_\Delta = (-30-i40)$ MeV in the Δ propagator, our calculation of the quasifree Δ production reproduces that of Koch and Ohtsuka.[4]

With the above simple procedure of treating medium effects, we obtain the results shown in Fig. 6. In addition to the new data[1] at GeV energies, we also compare our predictions with the data[12] at lower energies. In these calculations, we normalize the pure nucleonic contribution [Eq. (12)] to the data at the quasifree peaks in the energy region below the pion production threshold. The needed normalization factors are about 0.8 as can be seen in Fig. 4. This procedure is known[9] to be inadequate in providing a correct microscopic interpretation of the nucleon quasifree process, but it should be sufficient for providing a qualitative estimate of its contribution relative to the contribution from the Δ excitation. We see that the main discrepancies are in the "dip" region. The calculated magnitudes and shapes in the Δ regions are reasonable in the comparisons with the data.

We now show in Fig. 7 the contributions from each term of Eq. (11). The dashed curves are the contributions from the pure nucleonic quasifree process [Eq. (12)]. The cross sections are clearly dominated by the quasifree Δ production (dotted curves), obtained from keeping only the Δ term in the calculation of Eq. (12). It is interesting to note that the two-body mechanism (dash-dotted), mainly due to the $\gamma NN \rightarrow N\Delta \rightarrow NN$ annihilation mechanism in the calculation of Eq. (14), helps to shift the positions of the Δ peaks and to reduce the discrepancies with the data in the "dip" region. The calculated widths are also comparable to the data. However, the calculated magnitudes are lower than the data in all cases. The problem in the "dip" region is not resolved.

In summary, we have shown that it is possible to relate the elementary $\Delta \rightarrow \pi N$ decay and $N\Delta \rightarrow NN$ processes to the inclusive (e,e') reaction from nuclei. By appropriately determining the parameters of the Born term and the Δ-excitation term from the fit to the γN and γd data, one can reproduce the main features of the inclusive (e,e') data. The simple procedure of shifting the Δ self-energy by $(-30-i40)$ MeV, suggested by the result of the Δ-hole calculation,[3,4] seems sufficient to account for the medium effect on the Δ propagation. The remaining discrepancies seen in Figs. 6 and 7 suggest that the multinucleon $\pi(\Delta)$ absorption mechanism probably needs to be considered. The same observation was also made in the study of pion absorption by nuclei.[10]

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